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EXPLORING THE DYNAMICS OF EXHAUST EMISSIONS AND FUEL CONSUMPTION: EVIDENCE FROM REGULATORY TESTS OF A PASSENGER CAR

Summary. The formal scientific description of the operation of combustion engines is a major challenge. This work introduces a novel research framework that treats pollutant emission intensity and fuel consumption as stochastic processes conditioned by a vehicle's velocity course and quantifies their dynamic properties using statistical analysis. The methodological innovation presented in this paper combines standardized laboratory chassis-dynamometer testing with a comprehensive set of statistical characteristics to isolate and compare dynamic effects for multiple regulated and unregulated exhaust compounds. The empirical data are obtained by testing a passenger car with a spark-ignition engine according to the regulatory driving cycle. Beyond the conventional cycle-averaged approach, the study demonstrates that process-level variability (e.g., coefficient of variation, kurtosis, and skewness) can reveal pollutant-specific sensitivities to transient engine operation states. A noteworthy example of methane emission intensity, which has the strongest dynamic properties over the entire cycle, compared to the weakest dynamic properties during high velocity phases, highlights that emissions depend strongly on the velocity and, therefore, are not well captured by standard averaging methods. These findings highlight the significance of supplementing regulatory testing with stochastic characterization, enabling improved interpretation of the outcomes and better alignment of calibration for engine exhaust aftertreatment strategies with transient operating conditions.

1. INTRODUCTION

The adverse implications of transport-related pollution on people and the environment have been a source of growing concern for over the last 70 years. It is widely agreed that a significantly large share of pollution emitted from transport can be attributed to road vehicles, which operate by burning hydrocarbon fossil fuels in internal combustion engines [1]. Various measures have been taken to reduce vehicle emissions in order to counteract environmental degradation, or at least mitigate its effects [2]. Among them, the introduction of emission limits for selected exhaust components has been the most effective [3]. The first exhaust emission standards and the corresponding test procedures were developed

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in the USA at the beginning of the second half of the 20th century. This was followed shortly thereafter by Japan and Europe and then by other countries.

Modern regulations on emissions certification and test procedures for light-duty vehicles are based on the same principle [4]. Vehicles are subjected to testing over a chassis dynamometer driving cycle, which has the form of a velocity process over time. During the test, the composition of diluted exhaust gases is analyzed, and the results are expressed as specific distance emission (unit g/km) and specific distance particle number (unit 1/km). A number of modifications have been made to the details of regulatory emission drive cycles and procedures over time.

For the performance of automotive engines to be evaluated objectively, they must be tested under conditions that are as close as possible to real vehicle operation conditions. For this reason, efforts are being made to develop driving tests with a driving velocity process that is representative of real vehicle use. This was the goal of the Worldwide Harmonized Light Vehicle Test Procedure (WLTP) [5,6], which was implemented in the type-approval processes for light vehicles in various countries globally, including the European Union in September 2017. Under the WLTP, laboratory tests are conducted on a chassis dynamometer in Worldwide Harmonized Light Vehicles Test Cycles (WLTCs) divided into several classes: 1, 2, 3a, and 3b [5–7]. The individual test classes depend on the ratio of the engine rated useful power, N_{eN}, and vehicle mass, m_V, as well as the maximum vehicle velocity, v_{max}. The designation of vehicle classes is presented in Fig. 1.

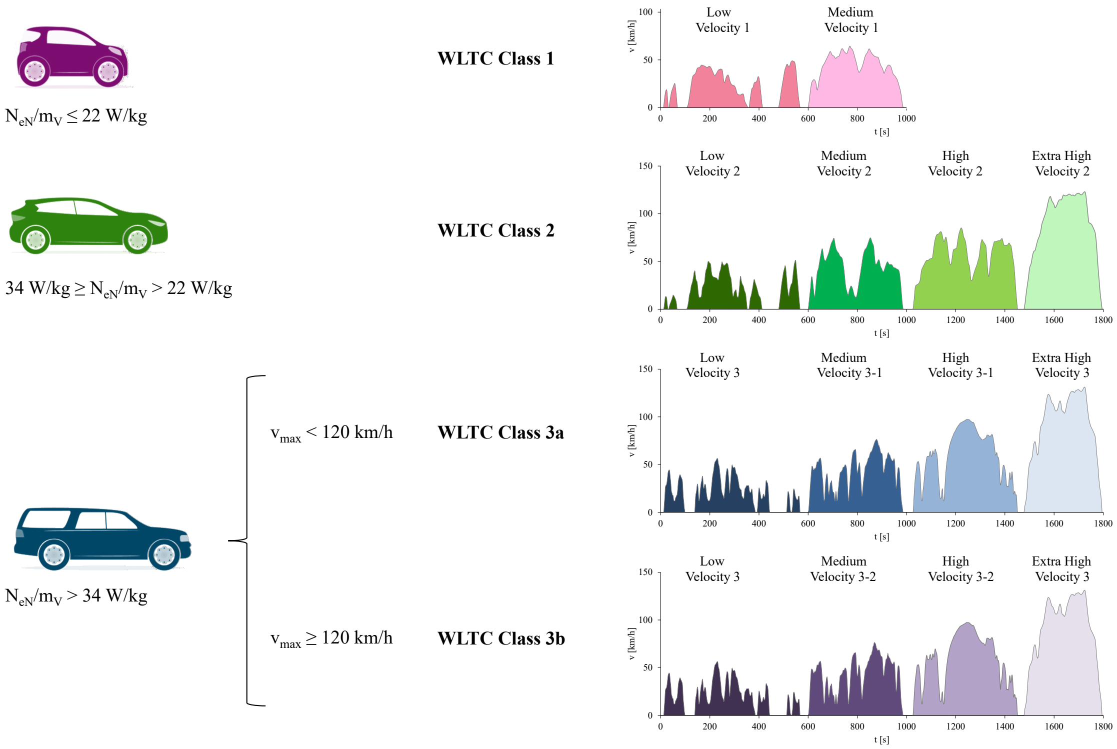

Fig. 1. WLTC test class division

In the European Union, the WLTP replaced the procedure based on the New European Driving Cycle (NEDC) [5,6], which, despite the name suggesting otherwise, was outdated since it did not correspond to the current state of technology in modern vehicles. The WLTC and NEDC tests are fundamentally different. The WLTC test was developed according to the principle of a faithful time simulation of the vehicle velocity [5,6], or, more precisely, it is a combination of sections of driving velocity functions recorded during road tests. On the other hand, the NEDC test is based on the synthesis of requirements for velocity values, such as average velocity, extreme accelerations, probability density of velocity during the test, and the conditions of real vehicle use [5]. The consequences of using various test creation methods are essentially differences in the dynamic properties of the tests stemming from the vehicle velocity process. Velocity processes in tests like the WLTC test usually have stronger dynamic properties than in tests created using a synthesis of requirements for the velocity values [5]. These conclusions are confirmed by research, particularly studies conducted in the frequency domain.

The stronger dynamic properties of velocity functions in tests created based on their simulated recreation in the time domain lead to large disparities between the research results obtained from these tests and results obtained from tests created using a synthesis of requirements set for the velocity values [5,6]. This is because the more dynamic vehicle velocity determines the stronger dynamic properties of engine operating states, such as engine speed and load, the measure of which can be torque or effective power. Above all, more dynamic properties of vehicle velocity promote larger exhaust emissions and higher fuel consumption. For this reason, there is a need to extend the scope of vehicle test results analysis in single tests, which, in the vast majority of scientific publications, is limited to determining average values of, for example, road emissions, for the entire test.

Since the development of the first engine testing procedures, researchers have been interested in phenomena associated with the operation of combustion engines in transient conditions [5]. Numerous studies have indicated the strong sensitivity of actual fuel dosing and emission of particular pollutants to engine operating states. Publications on this subject have become increasingly common since the transition from the NEDC to the more dynamic WLTC in Europe [6]. Following this, the advancement of studies in this field was facilitated by the implementation of a novel type-approval testing procedure incorporating real driving emissions (RDE) to assess vehicle emissions under real road conditions and actual traffic [7–9]. However, only a few researchers have considered the stochastic characteristics of such conditions, treating them as random.

According to Zhai et al. [10], pollutant emissions from motor vehicles can differ significantly even if determined in the same driving conditions. To investigate which factors affect this variability, researchers performed tests on a vehicle using a portable emissions measurement system. They concluded that emissions of carbon dioxide, carbon oxide, nitrogen oxides, and particle number are strongly correlated with certain dynamic properties of vehicle and engine operation (i.e., vehicle velocity and acceleration, engine speed, engine torque, and instantaneous air-fuel ratio).

Shahariar et al. [11] examined the exhaust emissions and fuel efficiency of a light-duty commercial vehicle with a compression ignition engine under a set of six driving cycles developed on their own. The cycles modeled different urban traffic conditions as well as three distinct driving styles, thereby altering the dynamics of the course of vehicle velocity. The conclusions indicated that engine performance is substantially different in real-road scenarios than in steady conditions, leading to considerably elevated emissions of pollutants. The largest impact was observed for driving style: aggressive driving caused a moderate increase in carbon dioxide and nitrogen oxides emissions, a significant increase in carbon oxide emissions, and a significant increase in particle mass emissions, as well as an increase in particle number.

Lujan et al. [12] tested a passenger vehicle with a compression ignition engine in randomly synthesized driving cycles. The proposed driving cycle generator was based on the data from WLTC, employing the transition probability matrix to develop a vehicle velocity process that simulated various dynamic driving conditions. The cycles were then implemented on a chassis dynamometer, with the focus on vehicle nitrogen oxides emissions. The findings showed a notable variation in nitrogen oxide emissions, reaching as much as 60% based on the driving style reflected by the cycle (ranging from gentle to aggressive). The authors highlighted the random aspect of their approach, stressing the unpredictability of the cycles generated using their method.

Stochastic analysis was also used by Zhang et al. [13] to introduce the concept of uncertain, pseudo-random vehicle driving cycles that can better capture the dynamic features of personal driving style. The main goal was to develop a driver-oriented power control system for plug-in hybrid electric vehicles in order to enhance their overall performance, including a reduction in fuel efficiency and pollutant emissions. To achieve this, the researchers recorded individual vehicle trips and classified them according to factors like driving style, traffic conditions, weather, purpose of the trip, and day of the week. Then, a stochastic model was implemented, and simulation studies were conducted. The results showed that the control strategy based on this stochastic model responds better to the dynamic properties of real road conditions than the so-called rule-based control strategy, which compares the current values of vehicle parameters with predefined algorithms in the form of maps.

Similar assumptions motivated Wasserburger and Hametner [14] to introduce a novel approach for constructing driving cycles consisting of an automatic generation of velocity process based on any naturalistic driving data collected. The randomness of the dynamic characteristics of a generated drive cycle was ensured by employing logistic regression to depict the intensity of a vehicle's acceleration or deceleration. The resulting driving cycles were fully compliant with RDE requirements. The authors emphasized that their method did not require conducting cost- and time-consuming road tests using a portable emissions measurement system, as was the case in earlier works, such as that by Claßen et al. [15]. Finally, a recently published paper [8] by the authors of the current study explains the assumptions of a stochastic method of investigating the exhaust emissions and fuel consumption of a motor vehicle in dynamic conditions. As an example, an RDE test was performed on a vehicle with a spark-ignition engine. The results were analyzed in the time domain (statistical characteristics of the processes and correlations between them), in the frequency domain (the power spectral density of the processes), and in the process values domain (probability density of processes). Significant variation was found between the considered processes. Given the need to extend the typical methods used in an investigation of exhaust emissions and fuel consumption, the current study followed the above path by averaging these parameters' values over the distance driven [16]. In fact, under dynamic motor vehicle operation conditions, significant time variability of engine speed and torque was observed, which affected the variability of pollutant emissions and fuel consumption [17]. This makes it advisable to carry out a detailed analysis of the pollutant emission intensity and, especially, its changes over time.

The main objective of the studies reported in this paper was to assess the dynamic aspects of the processes concerning the emission of the most important exhaust gas components and fuel consumption of a combustion engine operating in transient conditions. The empirical data obtained through vehicle testing on a chassis dynamometer were analyzed using a stochastic approach [8]. According to this approach, the seemingly causal conditions of vehicle use in the driving test, corresponding to actual operation, are considered to be random, while the time courses of pollutant emission and fuel consumption are treated as a stochastic process. This is justified by the significant degree of uncertainty of many factors determining the conditions of engine use in transient states. The source of the basic uncertainty in driving tests is the reproduction of the velocity course on a chassis dynamometer, which is primarily related to the accuracy of the vehicle control.

This article contains a detailed statistical analysis of the emission intensity of the following pollutants: carbon monoxide (CO); carbon dioxide (CO_2); nitrogen oxides (NO_x); hydrocarbons (HC), including non-methane hydrocarbons (NMHC) and methane (CH_4); as well as mass fuel consumption intensity. The coefficient of variation was defined as a quantitative measure of the dynamics of the studied quantities. This enabled the selection of quantities with the strongest and weakest dynamic properties.

2. MATERIALS AND METHODS

2.1. Terms and definitions

This section provides definitions for terms used in the paper. Pollutant emission is the mass of pollutants emitted from a test object. Specific distance emission is the derivative of pollutant emission over the distance traveled by the vehicle. Specific distance particle number is the derivative of the particle number over the distance traveled by the vehicle. Specific distance fuel consumption is the derivative of fuel consumption (optionally: mass or volume) over the distance traveled by the vehicle. Pollutant emission intensity is the derivative of pollutant emissions over time. Particulate number intensity is the derivative of particulate number over time. Fuel consumption intensity is the derivative of fuel consumption (optionally: mass or volume) over time.

2.2. Test vehicle specifications and preparation

The research object was a passenger car equipped with a conventional four-stroke spark-ignition engine fueled with gasoline and had a rated power of 81 kW. The engine had four cylinders in-line, a total displacement volume of 999 cm^3, and featured turbocharging. It was compliant with the Euro 6 AP emission standard. The vehicle had an automatic transmission.

The test vehicle underwent preconditioning for WLTP, including a 6–36 h soak at 23 ± 3 °C. The tires were inflated to the pressure recommended by the manufacturer. Auxiliary loads (e.g., HVAC, auxiliary electrical consumers) were disabled unless required by the WLTP. Vehicle mass and loading were set to represent the WLTC Class 3b specification. Gear shifting was managed by the stock automatic transmission; no alternative shift strategies or manual overrides were employed.

2.3. Test facility and instrumentation

Experimental work was conducted at the BOSMAL Automotive Research and Development Institute Ltd. in a climate-controlled chassis dynamometer laboratory compliant with WLTP requirements defined in Global Technical Regulation No. 15 (GTR15) [18]. The dynamometer was an inertia-simulated system equipped with an automated road-load simulation module calibrated for coastdown vehicle. Exhaust gas samples were collected via a constant volume sampling (CVS) tunnel. A selection of four Venturi nozzles allowed the exhaust gas flow (and, therefore, the exhaust gas dilution rate) to be adjusted from 2 m^3/min to 20 m^3/min. Pollutant emission analyzers included non-dispersive infrared (NDIR) for carbon monoxide and carbon dioxide, a chemiluminescence detector (CLD) for nitrogen oxides, and a flame ionization detector (FID) for hydrocarbons, with the function of distinguishing between methane and non-methane hydrocarbons. Analyzers were calibrated (spans, zeros, and linearity checks) before each test according to the WLTP. The measurement accuracy of emission analyzers was ≤ 2% at the measuring point or ≤ 1% of full scale. A pre-test analyzer stabilization period and post-test drift checks ensured measurement reliability. Commercial gasoline available at a gas station was used in the tests, which is permitted by Commission Regulation (EU) 2017/1151 for scientific research or development purposes. Fuel mass consumption intensity was calculated using the carbon balance method.

2.4. Driving cycle and data acquisition

The WLTC Class 3b driving cycle was performed following the reference velocity course, consisting of four phases (low velocity, medium velocity, high velocity, extra high velocity). The dynamometer controller enforced the velocity trace within tolerance limits—namely, none of the logged deviations exceeded the WLTP allowable range of ± 2 km/h within ± 1 s of a given point in time. Data were sampled at high frequency (10 Hz) to capture transient states adequately, with all channels time-synchronized with the dynamometer. Recorded variables included instantaneous vehicle velocity (v), pollutant emission intensities (E_{CO}, E_{CO2}, E_{NOx}, E_{HC}, E_{CH4}, E_{NMHC}), and mass fuel consumption intensity (q).

2.5. Signal processing and filtration

Raw data were subjected to low-pass filtration using a fifth-order Savitzky–Golay filter [19]. Filter parameters (window length and polynomial order) were selected to attenuate high-frequency measurement noise while preserving the transient features characteristic of the WLTC. Sensitivity checks were performed by varying window lengths to confirm the robustness of statistical characteristics.

2.6. Statistical analysis

For each variable, statistical characteristics [20] were determined separately for the complete driving cycle and within each test phase. They included average value (AV), median (M), standard deviation (D), kurtosis (K), skewness (S), range (R), minimum value (Min), maximum value (Max), coefficient of variation (W), and the ratio of range to standard deviation (Z).

3. RESULTS AND DISCUSSION

Fig. 2 presents an overview of the basic parameters of each of the WLTC 3b phases as well as the total values determined for the entire cycle. Although it follows the definition of the WLTC in the regulations, it is worth mentioning their values to provide background to the detailed considerations presented later in this section. The time taken to complete each phase of the cycle was as follows: I – 586 s, II – 423 s, III – 458 s, IV – 333 s. Distance traveled in the subsequent phases of the cycle equated to the following values: I – 3.09 km, II – 4.76 km, III – 7.16 km, IV – 8.25 km. The average vehicle velocities in the phases of the WLTC were as follows: I – 19.01 km/h, II – 40.48 km/h, III – 56.29 km/h, IV – 89.22 km/h, and whole cycle – 46.53 km/h.

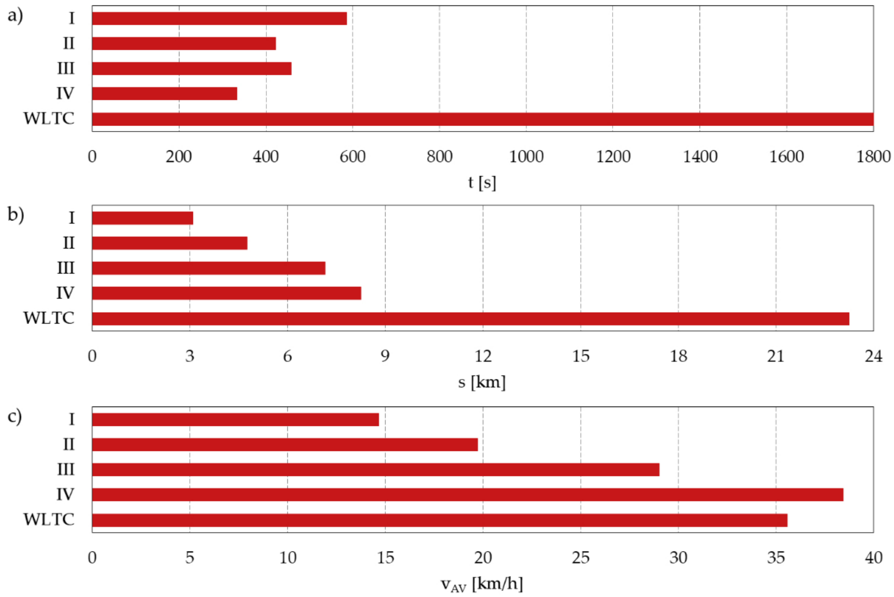

Fig. 2. Basic parameters of the WLTC 3b test and its individual phases: a) duration, t, distance travelled, s, and average vehicle velocity, v_{AV}

In order to allow for a comprehensive view that supports a qualitative evaluation of the examined processes, we also present the outcomes of empirical tests. Fig. 3 shows the filtered time courses of the recorded variables, which are as follows: a) carbon monoxide emission intensity, E_{CO}, b) carbon dioxide emission intensity, E_{CO2}, c) nitrogen oxides emission intensity, E_{NOx}, d) hydrocarbons emission intensity, E_{HC}, e) methane emission intensity, E_{CH4}, f) non-methane hydrocarbons emission intensity, E_{NMHC}, g) mass fuel consumption intensity, q, and h) vehicle velocity, v. The comparison of the time courses of the measured values in one figure shows which of them are interdependent or vehicle-velocity-dependent as well as the extent to which this is the case.

There is a strong correlation between carbon dioxide emission intensity (Fig. 3b) and mass fuel consumption intensity (Fig. 3g), as confirmed by the Pearson linear correlation coefficient of 0.98 at a 0.05 level of significance. Carbon dioxide is a component of exhaust gases that comes primarily from the complete combustion of fuel molecules. Its concentration is not subject to limitation by exhaust gas treatment systems and can only be reduced by reducing fuel consumption. It should be emphasized that the procedures for determining fuel consumption in the homologation regulations do not require the physical measurement of fuel flow but can instead be based on the carbon mass balance. This method takes into account the emission of exhaust gas components containing carbon, and carbon dioxide is decisive for quantitative reasons. Importantly, there is a clear dependence of carbon dioxide emission intensity (Fig. 3b) and mass fuel consumption intensity (Fig. 3g) on vehicle velocity (Fig. 3h), which is clearly related to the dependence of the engine load on the vehicle velocity.

A certain dependence of organic compounds emission intensity (Figs. 3d–f) on velocity (Fig. 3h) can also be observed. This refers to the entire group of organic compounds, including methane and non-methane hydrocarbons. In this case, two phenomena overlap. First, high hydrocarbon emissions are characteristic of high-engine-load conditions.

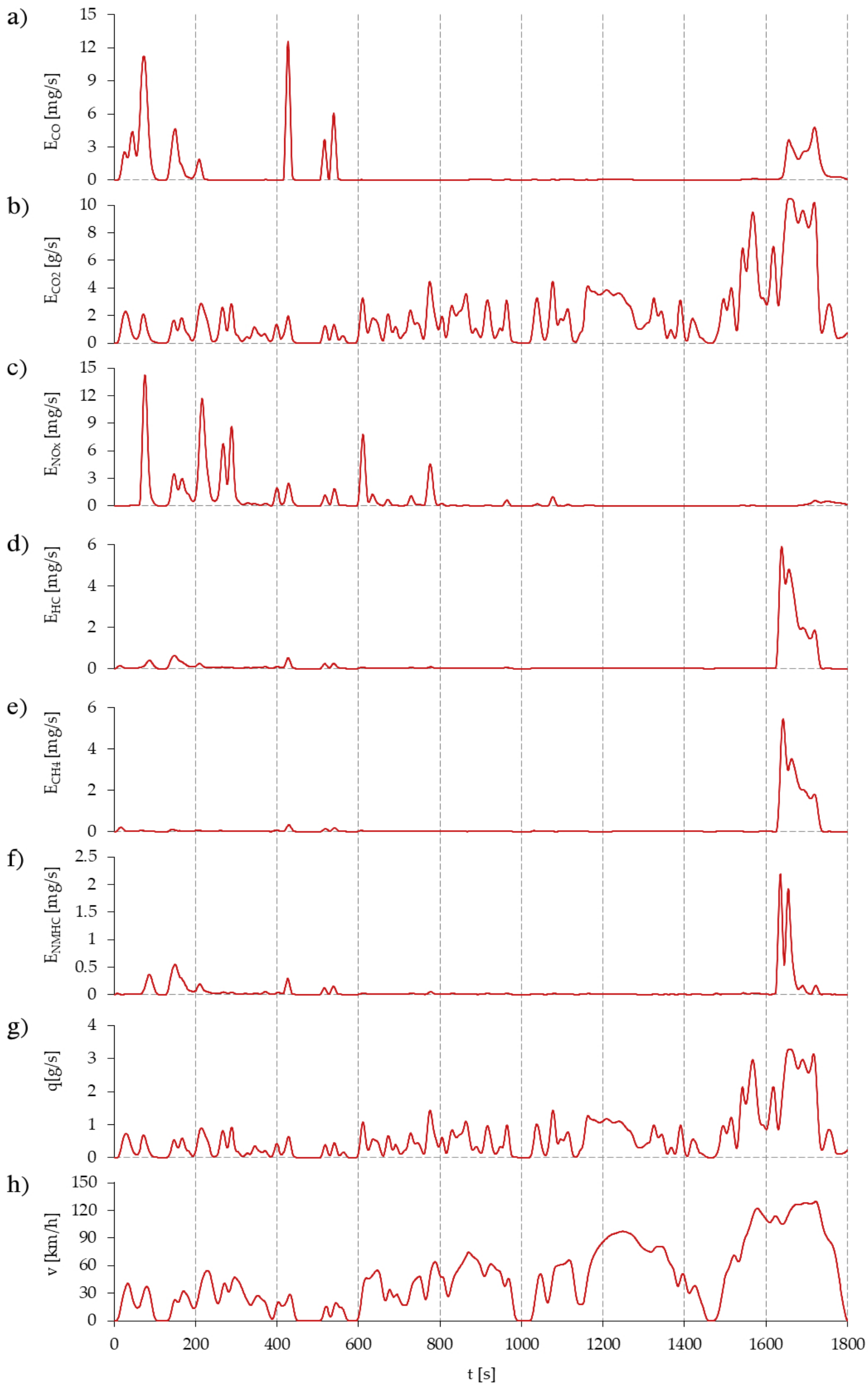

Fig. 3. Results of vehicle testing on a chassis dynamometer in the WLTC 3b test: a) carbon monoxide emission intensity, E_{CO}, b) carbon dioxide emission intensity, E_{CO2}, c) nitrogen oxides emission intensity, E_{NOx}, d) hydrocarbons emission intensity, E_{HC}, e) methane emission intensity, E_{CH4}, f) non-methane hydrocarbons emission intensity, E_{NMHC}, g) mass fuel consumption intensity, q, and h) vehicle velocity, v

This applies to the last phase of the WLTC cycle (i.e., extra high velocity). Second, trace amounts of hydrocarbons were recorded at the beginning of the cycle, in the low velocity phase. This second effect is related to the influence of the temperature of the catalytic converter on its efficiency. As mentioned in Section 2, the vehicle tests were performed with a cold engine start. This means that the catalytic converter needed time to warm up to reach the optimum operating temperature and, consequently, an increase in efficiency that allowed for the reduction of hydrocarbon emissions. This also applies to other chemical compounds (i.e., carbon monoxide (Fig. 3a) and nitrogen oxides (Fig. 3c)).

The statistical characteristics of the variables measured in the empirical tests for the WLTC and its four individual phases are presented in Tables 1-5.

Table 1

Statistical characteristics of the variables measured in Phase I of the WLTC 3b test

Characteristics	v [km/h]	E_{CO} [mg/s]	E_{CO2} [g/s]	E_{NOx} [mg/s]	E_{HC} [mg/s]	E_{CH4} [mg/s]	E_{NMHC} [mg/s]	q [g/s]
AV	19.01	1.274	0.765	1.524	0.112	0.049	0.070	0.244
M	18.52	0.019	0.532	0.316	0.067	0.037	0.027	0.168
D	14.68	2.439	0.743	2.652	0.131	0.053	0.110	0.238
K	-0.68	6.769	0.115	6.976	4.032	8.889	5.767	-0.001
S	0.37	2.579	0.996	2.620	2.044	2.653	2.420	0.969
R	54.64	12.602	2.861	14.279	0.636	0.347	0.557	0.920
Min	0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	54.64	12.602	2.861	14.279	0.636	0.347	0.557	0.920
W	0.772	1.914	0.970	1.740	1.168	1.089	1.563	0.977
Z	2.87	9.89	3.74	9.37	5.66	7.08	7.95	3.77

Table 2

Statistical characteristics of the variables measured in Phase II of the WLTC 3b test

Characteristics	v [km/h]	E_{CO} [mg/s]	E_{CO2} [g/s]	E_{NOx} [mg/s]	E_{HC} [mg/s]	E_{CH4} [mg/s]	E_{NMHC} [mg/s]	q [g/s]
AV	40.48	0.013	1.405	1.524	0.039	0.029	0.014	0.444
M	45.34	0.008	1.196	0.316	0.039	0.031	0.012	0.387
D	19.75	0.016	1.004	2.652	0.017	0.012	0.009	0.315
K	-0.53	0.823	-0.087	6.976	1.686	0.755	4.546	0.013
S	-0.50	1.348	0.672	2.620	-0.033	-0.533	1.644	0.687
R	74.49	0.061	4.478	14.279	0.095	0.065	0.056	1.427
Min	0.00	0.000	0.002	0.000	0.001	0.000	0.000	0.001
Max	74.49	0.061	4.479	14.279	0.095	0.066	0.056	1.428
W	0.488	1.212	0.714	1.740	0.427	0.415	0.674	0.711
Z	1.84	4.53	3.19	9.37	2.41	2.21	4.13	3.22

The investigation of the statistical characteristics of the analyzed processes is crucial, since their stochastic nature requires the application of dedicated statistical tools to ensure a reliable and objective assessment. Unlike in other phases, the median velocity of the vehicle was lower than the average velocity in Phase I and in the complete cycle. For pollutant emission intensity and mass fuel consumption intensity, such regularity does not occur. Kurtosis was negative for the vehicle velocity in all phases as well as in the complete cycle. It was also negative in Phase II for quantities concerning hydrocarbons and methane emission and in Phase III for a strongly correlated pair: carbon dioxide emission and fuel consumption. Finally, it was negative in Phase IV for nitrogen oxides, carbon dioxide, and fuel consumption.

Table 3

Statistical characteristics of the variables measured in Phase III of the WLTC 3b test

Characteristics	v [km/h]	E_{CO} [mg/s]	E_{CO2} [g/s]	E_{NOx} [mg/s]	E_{HC} [mg/s]	E_{CH4} [mg/s]	E_{NMHC} [mg/s]	q [g/s]
AV	56.29	0.025	1.892	0.045	0.028	0.022	0.009	0.584
M	60.43	0.018	1.719	0.004	0.029	0.022	0.008	0.542
D	29.07	0.023	1.298	0.138	0.009	0.010	0.005	0.396
K	-1.02	0.212	-1.299	27.922	1.342	0.939	0.852	-1.261
S	-0.33	1.062	0.181	5.109	-1.011	0.065	0.713	0.175
R	97.24	0.087	4.484	1.002	0.046	0.056	0.024	1.432
Min	0.00	0.000	0.005	0.000	0.000	0.000	0.000	0.003
Max	97.24	0.087	4.489	1.002	0.047	0.056	0.024	1.435
W	0.516	0.921	0.686	3.063	0.342	0.439	0.545	0.679
Z	1.73	3.45	2.37	22.22	1.68	2.53	2.84	2.45

Table 4

Statistical characteristics of the variables measured in Phase IV of the WLTC 3b test

Characteristics	v [km/h]	E_{CO} [mg/s]	E_{CO2} [g/s]	E_{NOx} [mg/s]	E_{HC} [mg/s]	E_{CH4} [mg/s]	E_{NMHC} [mg/s]	q [g/s]
AV	89.22	0.893	4.810	0.129	0.911	0.804	0.194	1.486
M	106.43	0.126	3.808	0.023	0.035	0.029	0.017	1.167
D	38.51	1.322	3.381	0.174	1.542	1.330	0.459	1.052
K	-0.20	0.522	-1.357	-0.216	1.771	1.776	7.471	-1.347
S	-0.97	1.364	0.271	1.142	1.704	1.624	2.894	0.277
R	129.71	4.788	10.455	0.571	5.915	5.461	2.185	3.274
Min	0.00	0.000	0.005	0.000	0.000	0.000	0.000	0.003
Max	129.71	4.788	10.461	0.571	5.915	5.461	2.185	3.278
W	0.432	1.480	0.703	1.353	1.692	1.654	2.370	0.708
Z	1.45	5.36	2.17	4.44	6.49	6.80	11.28	2.20

Therefore, these were platykurtic distributions. The distributions of other quantities were found to be leptokurtic. The skewness in most cases was positive with a large range of values. Therefore, in most cases, the measured variables had right-skewed distributions. Statistical studies showed that, in general, the distributions of the measured variables were significantly asymmetric. The statistical properties of the distributions of the quantities studied indicate the difficulty of unifying these properties. The values of the determined statistical characteristics were influenced by the operating characteristics of the driving cycle and the technical features of the vehicle engine.

Table 5

Statistical characteristics of the variables measured in the complete WLTC 3b test

Characteristics	v [km/h]	E_{CO} [mg/s]	E_{CO2} [g/s]	E_{NOx} [mg/s]	E_{HC} [mg/s]	E_{CH4} [mg/s]	E_{NMHC} [mg/s]	q [g/s]
AV	46.53	0.590	1.951	0.659	0.221	0.177	0.064	0.607
M	39.73	0.018	1.233	0.049	0.038	0.029	0.012	0.395
D	35.59	1.608	2.235	1.754	0.744	0.645	0.218	0.692
K	-0.49	19.961	4.281	21.807	27.663	27.493	48.768	4.335
S	0.64	4.090	2.067	4.366	5.127	5.051	6.605	2.071
R	129.71	12.602	10.460	14.279	5.915	5.461	2.185	3.278
Min	0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	129.71	12.602	10.461	14.279	5.915	5.461	2.185	3.278
W	0.765	2.727	1.146	2.664	3.360	3.643	3.397	1.139
Z	2.79	21.37	5.36	21.68	26.71	30.82	34.12	5.40

Fig. 4 presents the average values of pollutant emission intensity and mass fuel consumption intensity in the complete WLTC as well as in its four phases. The average emission intensity of carbon monoxide was the largest for the first phase of the driving cycle, when the engine load is the smallest and there is a large share of engine idling time. At this point, the issue of the efficiency of the catalytic converter is also revealed, as it does not operate to its full capacity at the beginning of the cycle, right after a cold engine start. The average emission intensity of carbon monoxide is also large in the fourth phase of the cycle due to the high engine load in this phase. The largest values of the average emission intensity of carbon dioxide and the average mass fuel consumption intensity were observed in the fourth phase of the driving cycle, in which the vehicle velocity–and, consequently, the engine load–is the largest. The largest average emission intensity of nitrogen oxides was observed in the first phase of the driving cycle, which again confirms the effect of an unheated catalytic converter. The largest average emission intensity of organic compounds was recorded in the fourth phase of the cycle, during which the working conditions of the engine are very demanding.

Fig. 5 shows the coefficient of variation in the complete WLTC as well as in its four phases for the variables of vehicle velocity, pollutant emission intensity, and mass fuel consumption intensity. The coefficients of variation of the measured variables can be used to assess their dynamic properties. It was found that the most dynamic properties were observed in methane emission intensity for the entire test (coefficient of variation = 3.64), and the least dynamic were found in Phases II and III, also for methane emission (coefficient of variation = 0.41). Moreover, regarding the ratio of the range and the mean value, the highest values were found for the whole WLTC test, especially for the examined organic compounds. The coefficients of variation in pollutant emission intensity were different in individual phases of the driving cycle, which varied in duration. Therefore, the coefficients of variation of the emission intensity of individual pollutants for the entire test may differ.

The study has several limitations that should be acknowledged. Firstly, the dataset was derived from a single vehicle and a single driving test cycle, which restricts the generalizability of the findings. However, as the primary objective was to model vehicle velocity as a stochastic process, emphasis was placed on analyzing a single realization of the velocity course as a representative sample rather than averaging parameters across multiple driving tests. While averaging results can yield estimates that more closely approximate real values, the process of averaging may obscure insights into the dynamic characteristics inherent in individual measurements.

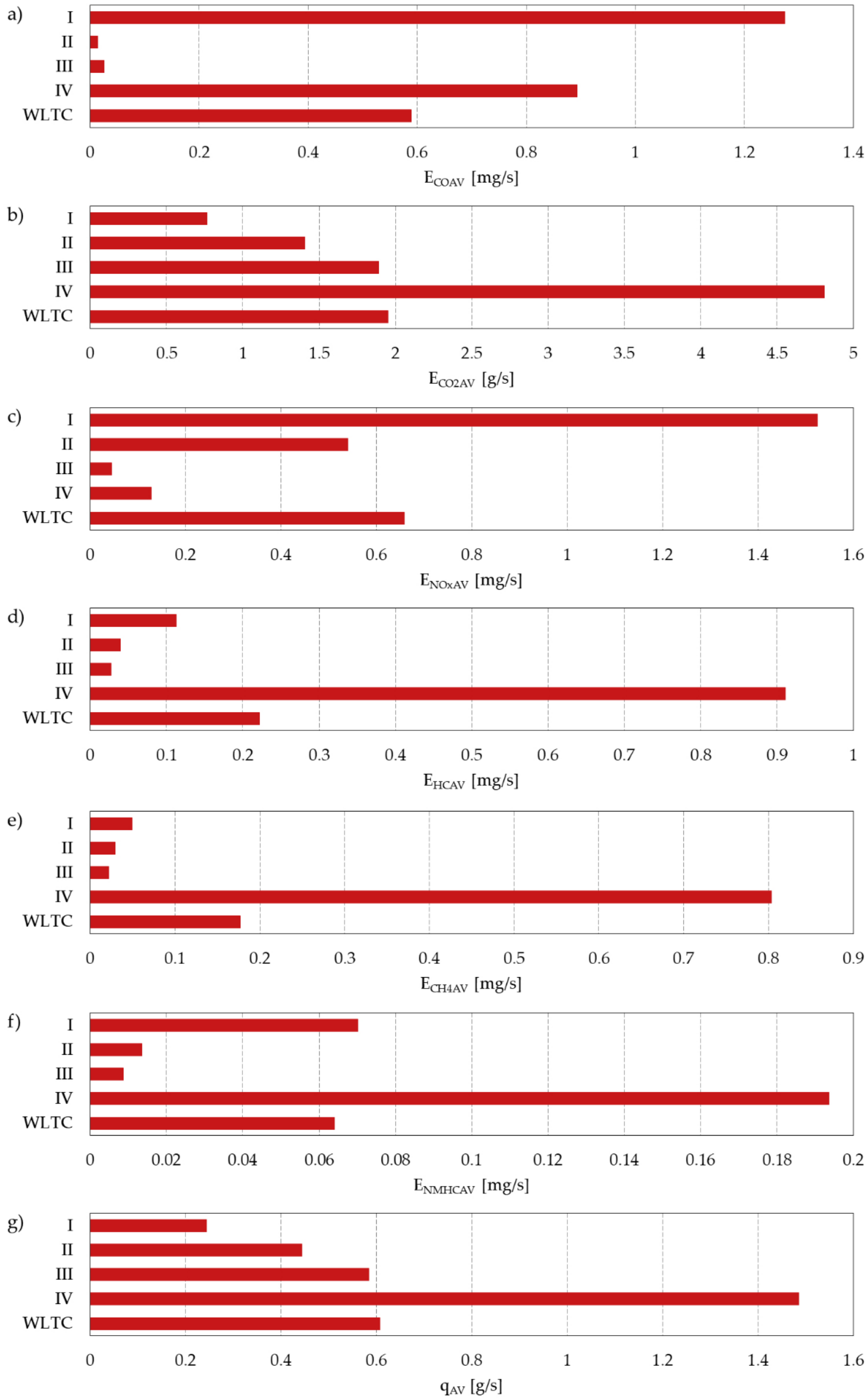

Fig. 4. Average values of the measured variables in the WLTC 3b test and its individual phases: a) average carbon monoxide emission intensity, E_{COAV}, b) average carbon dioxide emission intensity, E_{CO2AV}, c) average nitrogen oxides emission intensity, E_{NOxAV}, d) average hydrocarbons emission intensity, E_{HCAV}, e) average methane emission intensity, E_{CH4AV}, f) average non-methane hydrocarbons emission intensity, E_{NMHCAV}, and g) average mass fuel consumption intensity – q_{AV}

Additionally, the use of the coefficient of variation as a variability metric creates certain limitations due to its definition, particularly when average emission intensities approach zero, as observed in Phases II and III of the WLTC (e.g., for carbon monoxide). This can lead to disproportionately high coefficient of variation values and potentially misleading interpretations of data dynamics. Nonetheless, to maintain methodological consistency and enable comparability of results, this metric was retained in the present study, with the prospect of incorporating alternative variability measures, such as the interquartile range to median ratio or the median absolute deviation to median ratio, in future research.

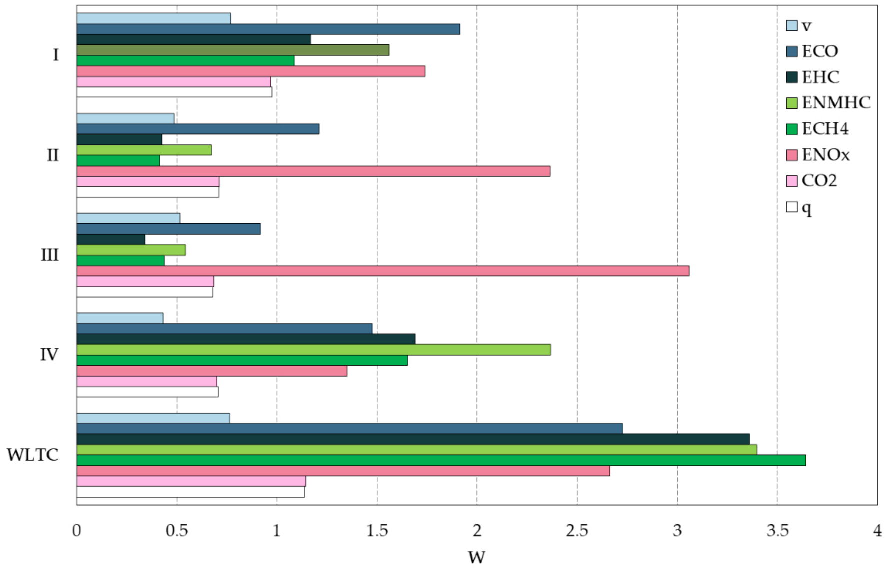

Fig. 5. Coefficient of variation, W, of the variables: vehicle velocity, v, carbon monoxide emission intensity, E_{CO}, hydrocarbons emission intensity, E_{HC}, methane emission intensity, E_{CH4}, non-methane hydrocarbons emission intensity, E_{NMHC}, nitrogen oxides emission intensity, E_{NOx}, carbon dioxide emission intensity, E_{CO2}, and mass fuel consumption intensity, q, in the WLTC 3b test and its individual phases

4. CONCLUSIONS

The most important conclusions drawn from the present research can be summarized as follows:

1. There was significant variation between the measured variables due to their statistical properties in the complete WLTC as well as in its four phases. In general, statistical research shows that the distributions of the variables measured are strongly asymmetric.
2. The median vehicle velocity in Phase I and in the complete driving cycle was lower than the average velocity, but this was not the case in the other phases. Moreover, this trend was not maintained for pollutant emission intensity and fuel consumption intensity.
3. Kurtosis was negative for vehicle velocity in the complete WLTC as well as in its four phases. It was also negative in Phase II for hydrocarbon and methane emission intensity, in Phases III and IV for carbon dioxide emission intensity and mass fuel consumption intensity, and in the last phase for nitrogen oxides.
4. Skewness was positive in most cases with a large range of values.
5. The greatest variability in measurement results was found for methane emission intensity over the entire WLTC test, covering four phases (coefficient of variation = 3.64); the lowest variability was found in Phases II and III, also in the case of methane (coefficient of variation = 0.41). The coefficients of variation of the measured variables indicate their dynamic properties. The ratio of range and standard deviation had the highest values for the entire WLTC test, especially for organic compounds.

The scope of the current research could be significantly expanded for variables related to exhaust emissions and fuel consumption in dynamic conditions determined by vehicle velocity in single tests. These options would be as follows:

1. Expanding the scope of work to include research in the following domains:
   - frequency (to determine the power spectral density of the variables)
   - variable values (to determine the probability density of the variables)

2. Expanding the scope of work to include research on the relationships between the variables related to exhaust emissions and fuel consumption, along with the variables related to the engine operating state.
3. Research for other vehicle velocity values, modeling different driving conditions. In particular, it would be advisable to treat vehicle velocity variables as stochastic processes and conduct research on their implementation [17].

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