**Keywords:** vehicle-to-truck collision modeling; components of the collision impulse

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# SELECTED PROBLEMS IN A VEHICLE-TO-TRUCK COLLISION MODELING

**Summary**. A selected case of a frontal, eccentric and oblique road vehicle-to-truck collision has been analyzed here as an example of the potential resultant motion by the motor vehicle with the truck performing a planar motion. This case is a pure problem of collision mechanics, so the main aim of the case presented here was to identify the selected parameters of such a collision with the three components of the collision impulse of only one vehicle taking part in it. The problem was analyzed with the use of a computer simulation performed in a software called V-SIM which is quite popular as a tool for accident reconstruction by e.g., forensic experts and others associated with road safety, especially in Poland. The results of all simulations were important in providing the basics of mathematical modeling regarding collision mechanics. The obtained observations may further be used for a dedicated software, should such be created. Not only a normal and a tangential but also a bi-normal direction versus the adopted plane of collision was considered.

## 1. INTRODUCTION

Various aspects of vehicle collisions have been considered in multiple works. From the point of view of modeling the problem has been analyzed, e.g., in [1] and [2]. These papers mostly concerned the mechanical dependencies between colliding vehicles. On the other hand, paper [3] presented the considerations related to identifying the main and crucial parameters of frontal collisions as a basis for a correct accident reconstruction.

The main aim of this paper was to analyze the selected problems of a frontal collision between a vehicle and a truck, which can be considered, e.g., when a forensic expert would like to use certain mechanics principles in a proper accident reconstruction with the inclusion of a resultant motion performed by the vehicle.

Collisions between motor vehicles could be modeled regardless of the type of vehicle (autonomous, electric, etc) and the decision-making machines and the pre-crash systems that could prevent vehicles from being involved in collisions. However, even if these systems would work without any faults, collisions are still possible and the considerations on the mechanics of this process seem important.

Systems enabling collision prevention or the so-called collision mitigation in road traffic are mostly up to date; these systems, according to some researchers, e.g., [4], require detection and self-learning. On the other hand, data on road crashes was used in broad scopes of road safety, for example in an overview of the reviews regarding the most common interventions undertaken to prevent road accidents [5]. The other use was an idea of accidents where the specialized systems enabled reporting of such

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incident to the dedicated databases or institutions [6] or even examination of the tendencies occurring in road accidents with the common behavior by the drivers included [7].

Research can be conducted in order to understand collisions, for example between autonomous vehicles with the use of simulations [8] and validate the mathematical models of a collision with the results of crash tests [9]. More advanced research has included analyses of collisions with the use of finite element methods [10]. On the other hand, road geometry has often been included as a cause of accidents [11].

Regardless of the improvement in road traffic safety, i.e., collision avoidance, pre-crash safety systems and autonomous vehicles, the use of mathematical modeling or simulations in road accident analysis might still be worth considering, and whether it regards electric, autonomous or conventional vehicle collision may still occur to a more or less extent in the coming years. Therefore, this paper is devoted to analyzing the mathematical modeling of a collision between a regular vehicle and a truck with the inclusion of a potential resultant motion. V-SIM, which is the only name of this software, was used in its 4.0.34 version to run the simulations of such a collision in order to obtain certain results and screenshots used in the mathematical modeling of this specific case. The V-SIM program is used in the analysis of vehicle driving in various terrain conditions, vehicle collisions and vehicle collisions with obstacles in an environment defined by the operator. Driving in various road conditions along with transporting the loads has been discussed in another paper [12]. This allows the recreation of the actual driving path of the vehicle before, during and after the collision. In the V-SIM software, the analysis of the movement of vehicles is carried out according to the principles of dynamics in three-dimensional space. The vehicle is a rigid body with 10 degrees of freedom, of which six are reserved for the vehicle's body and four are reserved for the wheels. The program has a kinematic model of suspension of wheels, tires, steering, brakes, engine and clutch. The operator can introduce additional tasks to the vehicle, e.g., acceleration, braking, turning the steering wheel, locking the wheel, tire deflating and other.

In the V-SIM simulation, a collision impulse model was used and the main question was whether it was possible to include the resultant motion in a mathematical model of such a collision for two scenarios and if such a phenomena might be included in this type of software.

## 2. SIMULATION OF A VEHICLE-TO-TRUCK COLLISION

This simulation aimed to illustrate a collision under potentially real circumstances between a passenger vehicle (no. 2 in Fig. 1) and a truck (no. 1 in Fig. 1) prepared such that the front wheels were unlocked during the impact. Such an approach enabled creating a less complex mathematical model due to the possible additional impulses coming from both the locked wheels and the excessive friction forces. These forces could produce additional impulses but in this case, they will be omitted, as it was assumed that have minor meaning. In the presented case, only impulses regarding collision forces are included in the collision model.

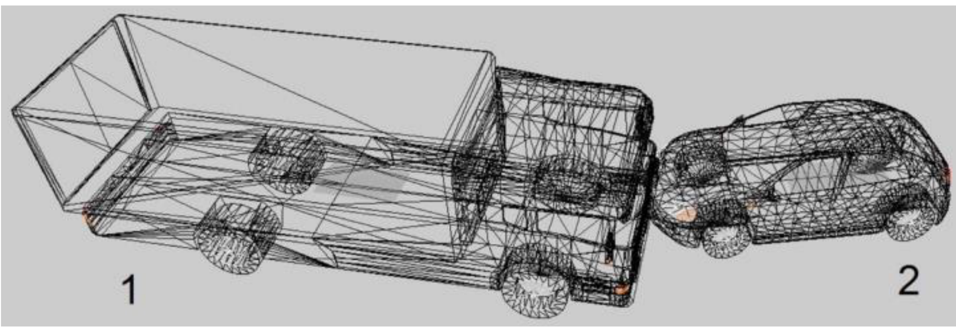


Fig. 1. The mutual positions of the vehicle and the truck at the start of the collision (own research based on the V-SIM program)

Before the simulation was prepared a few general assumptions were made based on [1], according to which both the vehicle and the truck were considered linear models (having linear suspension characteristics) and their masses and the moment of inertia were adopted from the V-SIM's vehicles database. In addition, a planar motion of both the vehicle and the truck was assumed on a dry road surface (coefficient of friction 0.8). However, the possibility of the resultant motion will be taken into account in a later part of this paper.

Before the collision a truck (vehicle no. 1 in Fig. 1) was moving at about 70 km/h (19.44 m/s), while vehicle no. 2 (Fig. 1) was moving at 90 km/h (about 25 m/s). The mass of the truck (vehicle no. 1) was 2895 kg, and the mass of vehicle no. 2 was 1125 kg. In both vehicles, the mass of the driver (75 kg) was included. Both masses and all moments of inertia were assumed to remain constant after the collision.

The coefficient of restitution was adopted during the simulation by V-SIM at R = 0.04, which seems fair considering the mutual penetration of the vehicles and a collision between a passenger car and a truck, in which the truck has a differently structured front and bumper but it stiffer than the car. However, due to the larger mass of the truck, the deformations of both cars could be different than in the case of vehicles with similar masses. Moreover, the simulated collisions were frontal and oblique and the vehicles primarily moved along a dry road plane (coefficient of friction = 0.8).

The so-called impact plane mentioned in a later part of this paper is understood as a vertical plane perpendicular to the road and tangential to the bodies of surfaces of both vehicles at the same time, of course at the point of initial contact.

In Table 1, some selected results of the simulation are presented, containing the translational and the angular velocities of the truck and the car before and after the collision. These velocities are crucial from the point of view of both the modeling and reconstruction of any road accident. In addition, the masses and moments of inertia of both the truck and the vehicle are presented because they are necessary for composing the mathematical model of the presented collision. We call Table 1 a collision protocol in later parts of this paper.

Table 1

Results of a simulation of a frontal and oblique collision between a truck and a vehicle without the locked front wheels (own research)

| Before the collision | | |
|---|---|---|
| Vehicle | Truck (1) | Passenger (2) |
| Mass | 2895 kg | 1125 kg |
| The moment of inertia in relation to the longitudinal, lateral and vertical axis $O_x$ / $O_y$ / $O_z$ intersecting the center of mass of the vehicle | 2325 kg·m$^2$<br>14636 kg·m$^2$<br>18119 kg·m$^2$ | 410 kg·m$^2$<br>1654 kg·m$^2$<br>1576 kg·m$^2$ |
| The coefficient of restitution | 0.04 | 0.04 |
| The forward velocity $v_x$ | 19.44 m/s | 25 m/s |
| The lateral and vertical velocity $v_y$, $v_z$ | 0 | 0 |
| The angular velocity around the longitudinal, lateral and vertical axis $O_x$, $O_y$, $O_z$ | 0 | 0 |
| The impulse of the collision | 35300 N | 35300 N |
| After the collision | | |
| The forward, lateral and vertical velocity $v_x$ / $v_y$ / $v_z$ | 7.64 m/s<br>-3.13 m/s<br>0.40 m/s | -6.33 m/s<br>1.09 m/s<br>-1.04 m/s |
| The angular velocity around the longitudinal, lateral and vertical axis $O_x$ / $O_y$ / $O_z$ | -1.10 rad/s<br>0.69 rad/s<br>0.27 rad/s | -0.26 rad/s<br>1.82 rad/s<br>4.00 rad/s |

Based on the results in Table 1, it seems necessary to decompose the after-collision velocities into two components in the case of the truck and three components for the vehicle. The initial planar motion can be transformed into the resultant motion for vehicle no. 2, as it had a lower mass and at least its rear wheels could momentarily lose contact with the road.

The discussed components would be determined along the normal and the tangential axes of the local $O_{ntb}$ coordinate frame for the truck (no. 1 in Fig. 1) and the passenger vehicle (no. 2 in Fig. 1). This frame will be discussed further, and O is the point of a geometric center of this collision, i.e., the point of primary contact between the vehicles.

Initially, in V-Sim, the velocities resulting from the simulation are presented in relation to the global, inertial $O'_{xyz}$ frame (Fig. 2). It seems necessary to switch to the natural coordinates frame $O_{ntb}$ in order to properly build a mathematical model of such an exemplary collision. Some geometric dependencies seem necessary, which will be presented in the next chapter. Fig. 2 presents a period of the simulation when the vehicles remained in contact to highlight both the translational and the rotational nature of their motion.

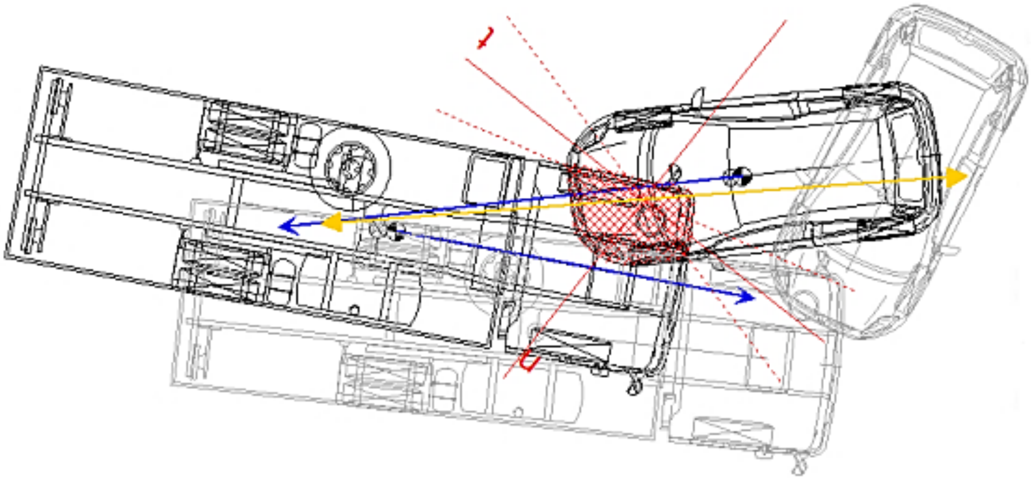

Fig. 2. The collision of a passenger vehicle and a truck without the front wheels locked (own research based on the V-SIM program)

One visible effect of the simulation without the front wheels locked can be observed in Fig. 3. The truck continued its course in the disturbed, yet straightforward direction and the vehicle rotated counterclockwise. Interestingly, if the front wheels were locked during the collision, maybe it would be more realistic, but it would reflect a situation in which both drivers pushed their brakes to gain a maximum braking force. Here, however, the simulation reflects a situation in which the drivers did not recognize the dangerous situation on the road and did not have time to push the brakes. A mathematical model of such a collision, i.e., without locked front wheels, will be adopted based on [1] and [3], among others.

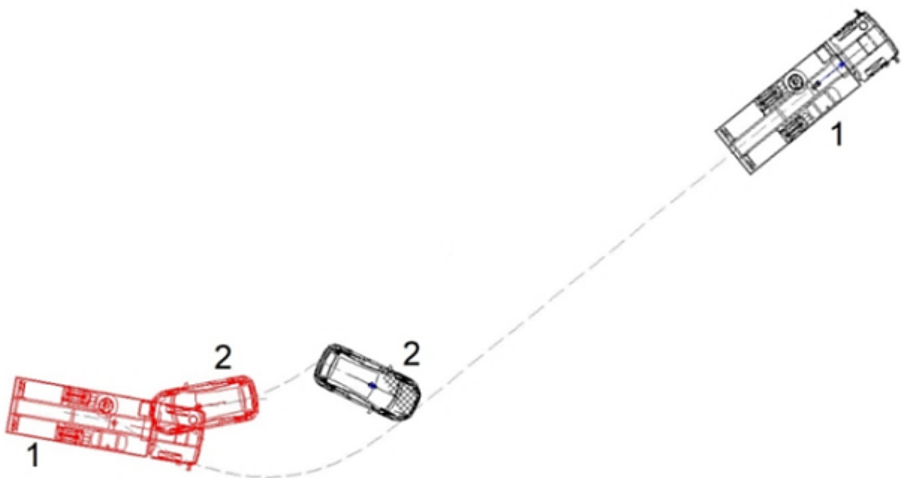


Fig. 3. Post-collision configuration of both the truck and the vehicle without the front wheels locked (own research based on the V-SIM program)

## 3. PARAMETERS NECESSARY FOR MATHEMATICAL MODELLING

Some simple trigonometric assumptions have to be made in order to properly model the discussed collision. Since the collision model is usually based on the local $O_{ntb}$ system, it seems obvious to make a transformation from the initially assumed global $O'_{xyz}$ system, according to which all velocities were determined in V-SIM, to the $O_{ntb}$ frame adopted for this case. It is important that O remains the center of the collision and the origin of the $O_{ntb}$ frame.

In Figs. 4 and 5, the mutual positions of both colliding vehicles are been presented in a plan (Fig. 4) and a side view (Fig. 5).

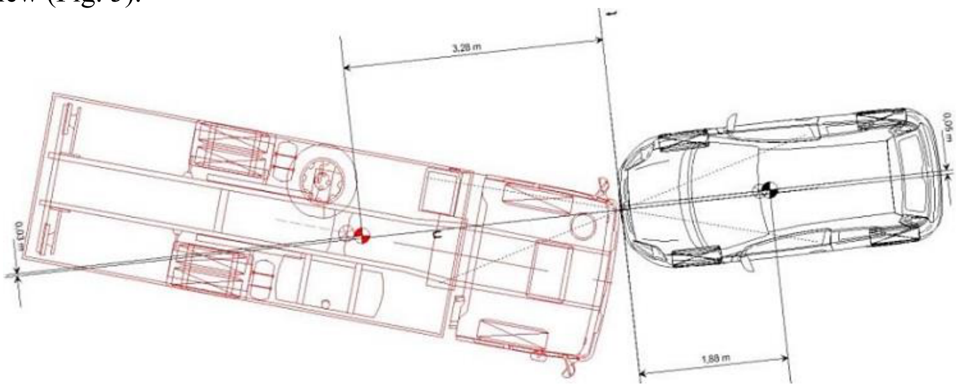

Fig. 4. The mutual location of both vehicles in a plan view (own research based on the V-SIM program)

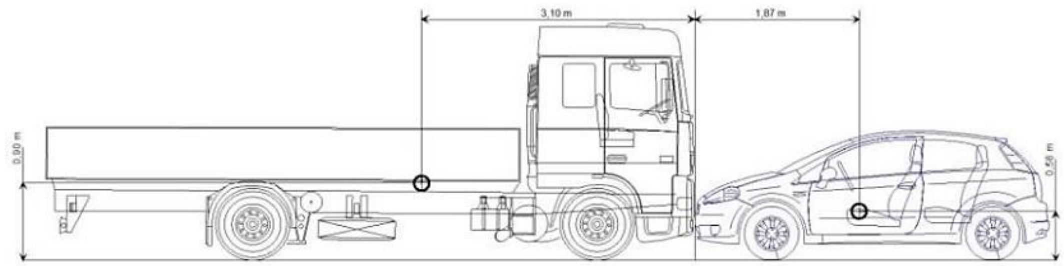

Fig. 5. The mutual location of both vehicles in a side view (own research based on the V-SIM program)

In such cases, the axes of the local frame $O_{ntb}$ could be variously oriented in relation to both vehicles. Usually, the appropriate software designed to reconstruct or simulate a collision adopts the point of primary contact between the colliding vehicles. This point is usually the origin of a local frame (let us call it $O_{nt}$ or $O_{ntb}$, depending on the planar or resultant motion considered during the collision), according to which the equations of motion before or after the collision are described.

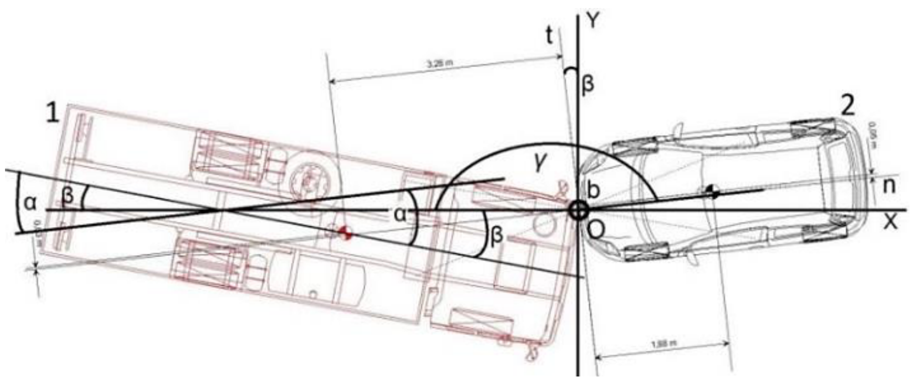


Fig. 6. The angles necessary to transform the $O'_{xyz}$ frame into the $O_{ntb}$ frame along with the numbers of vehicles (own research based on the V-SIM program)

In the presented example it was assumed that the $O_t$ axis would be located most conveniently, i.e., along the front of vehicle no. 2, in order to compose the mathematical model of this collision. Hence, the axis $O_n$ will lay perpendicularly to $O_t$. Moreover, the axis $O_b$ will be perpendicular to the plane composed of the two previous axes ($O_n$ and $O_t$). Hence, the adopted $O_{ntb}$ frame will be as in Fig. 6. Although the adoption of such a frame relates to the analyzed example, the presented methodology can be valid for other, even more complex examples of road collisions.

In Fig. 6, some angles necessary to transform the obtained velocities from the $O'_{xyz}$ frame to the $O_{ntb}$ frame are also presented.

It seems necessary to relate the adopted local $O_{ntb}$ frame to the global $O'_{xyz}$ in order to determine velocity components in the normal and the tangential direction versus the adopted plane of collision (here, the $O_t$ plane). This can be done with the use of some specific angles marked in Fig. 6 and described as follows:

- α – the angle between the speed of the truck (no. 1) and the normal axis ($O_n$) of the $O_{ntb}$ frame. This determines the division of the speed into a normal and a tangential component.
- $\beta$ – the angle between the speed of the truck and the x-axis of the global $O'_{xyz}$ frame, which determines its primary location in relation to this frame.
- γ – the angle between the speed of the vehicle (no. 2) and the x-axis of the global $O'_{xyz}$ frame, which determines its primary location versus this frame.

Taking into consideration the adopted angles, a quite simple formula can be used:

$$\alpha = (180° - \gamma) + \beta, \tag{1}$$

which enables the determination of $\alpha$. If $\alpha$ is known or can be determined, with β and γ regarded as the angles of the primary positions of the truck (no. 1) and the vehicle (no. 2), then further considerations can be continued.

In Figs. 7 and 8, the preliminary parameters necessary for completing the collision model are presented in relation to the primary location of the vehicles. Taking into account the example of a collision with the potential resultant motion of vehicle no. 2, a physical model of the collision requires a component of the impulse in the binormal direction (Fig. 8). In both Figs. 7 and 8, the positive directions in the translational and angular directions are marked. The initial and essential parameters contain the impulses for both vehicles ($S_{n1}$, $S_{t1}$, $S_{b1}$, $S_{n2}$, $S_{t2}$, and $S_{b2}$), although, considering that the truck will not move upwards during the collision, the binormal impulse will regard only vehicle no. 2.

All of these parameters are presented as scalars since they are components of the resultant impulses acting on the truck and the vehicle.

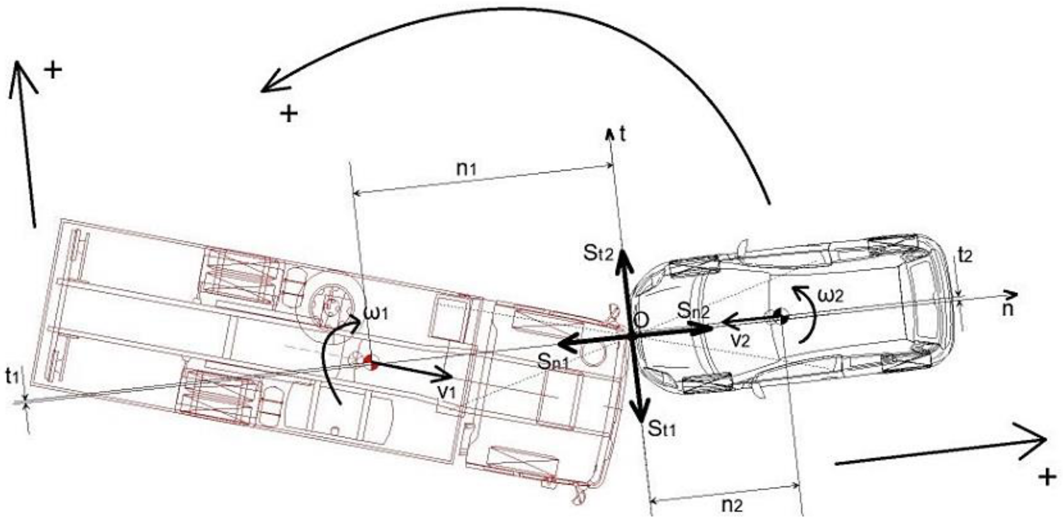


Fig. 7. The mutual positions of both vehicles at the start of the collision with the positive directions marked (own research based on the V-SIM program)

The necessary dimensions are also marked in Figs. 7 and 8, as is the location of each vehicle at the moment of an initial contact, which is the best configuration for adopting the origin of the above-mentioned $O_{ntb}$ frame and composing a mathematical model of this collision, i.e., the equations describing the kinematic state of both the truck and the vehicle, either at the start or at the end of the collision.

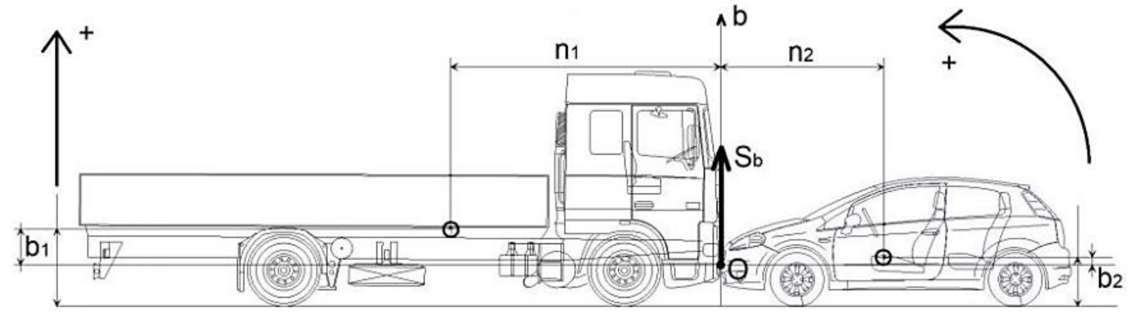


Fig. 8. The mutual positions of both vehicles at the start of the collision with the positive directions marked from a side view (own research based on the V-SIM program)

In Fig. 9, the angle α is marked to distribute the speed into tangential and the normal components ($v_{1t}$ and $v_{1n}$ respectively). As for the vertical velocity of vehicle no. 2 ($v_{2b}$), it was assumed that it is perpendicular to the road plane and regards only vehicle no. 2. In this case, the truck (no. 1) has a mass big enough not to perform the resultant motion. Therefore, the binormal velocity of the truck in Fig. 10 is 0. These components of velocity are used to describe the kinematic state of both the truck and the vehicle after the collision, according to a mathematical model.

The variable α is the angle between the speed of the truck (no. 1) and the $O_n$ axis (Fig. 9). As a reminder, the apostrophes above the parameters mark the magnitudes at the end of the collision, while the index '1' refers to the truck (no. 1) and '2' specifies the vehicle (no. 2). The indexes 'n' and 't' denote the normal and the tangential direction respectively, whereas 'b' denotes the binormal direction.

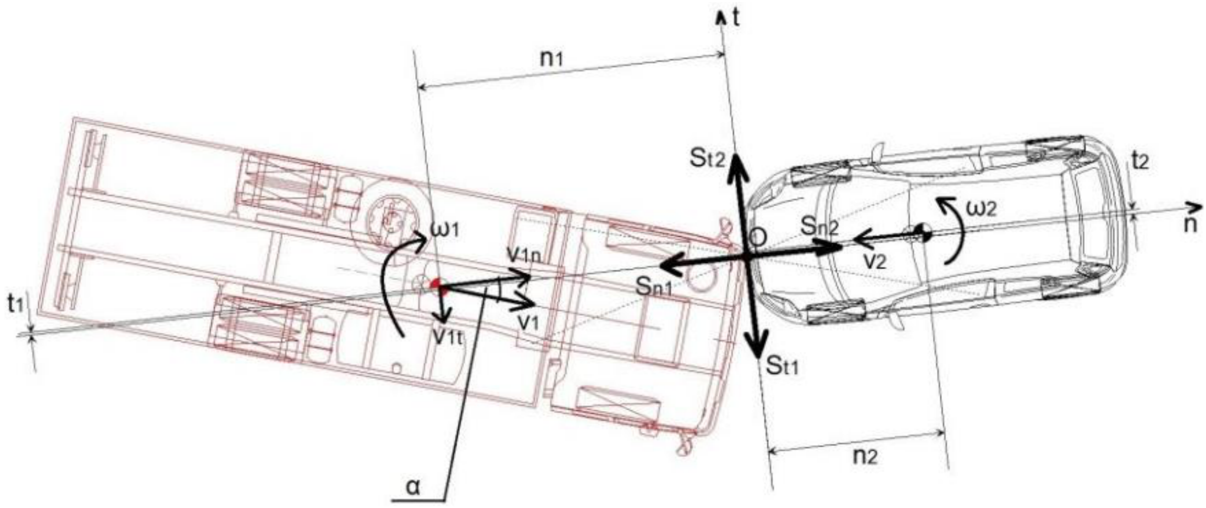


Fig. 9. The mutual location of each vehicle at the beginning of the collision with the angle and the components of the velocities marked – plane view (own research based on the V-SIM program)

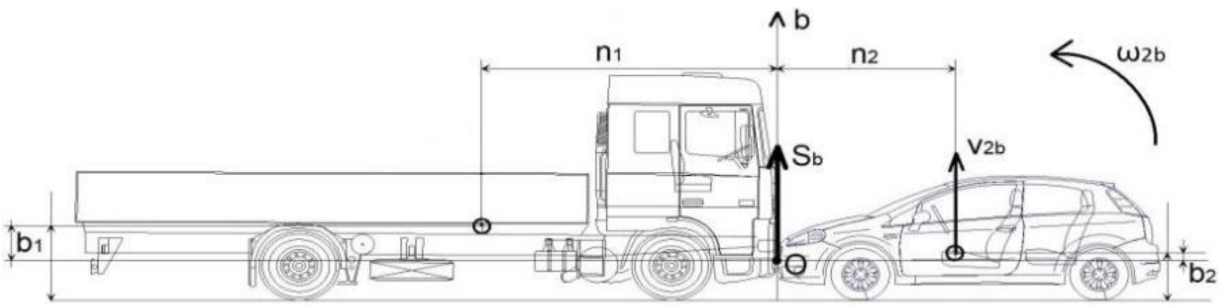


Fig. 10. The mutual location of each vehicle at the beginning of the collision with the angle and the components of the velocities marked – side view (own research based on V-SIM program)

Referring to Figs. 7 – 10 a typical free body diagram is not necessary when considering vehicle collisions because the forces such as those in contact between the wheels and the road constitute permanent loads and do not generate impulses unless there are significant differences in the type of road surface for different wheels. Moreover, these forces do not affect the change in speed after the collision, because it is a momentary phenomenon, lasting too short to change these forces in a noticeable way.

As for Fig. 10 the article assumes the hypothesis that only a passenger vehicle performs the resultant motion, because the mass of the truck prevents its wheels to detach from the road surface.

Since the necessary parameters are available based on, e.g., [1] and Figs. 9 and 10, and given the issue of a collision between vehicles with rough surfaces, it is possible to compose the mathematical model of the discussed example. It was assumed that the tangential velocities change as the bodies of the truck and the vehicle remain in contact for the duration of the collision.

Of course, the presented example presents an eccentric case, so the normal ($O_n$) axis does not intersect the center of mass of either the truck (no. 1) or the vehicle (no. 2). That is why it was assumed that apart from the normal and the tangential velocity change, the eccentric collision also causes changes in the angular velocities of both vehicles.

When considering this type of collision, along with the possibility of the resultant motion in the case of vehicle no. 2, the impulse can be divided into three components: the normal ($S_n$), the tangential ($S_t$) and the binormal ($S_b$), relative to the adopted collision plane (the vertical plane mutually tangent to both the truck and the vehicle at the moment of a first contact). Here, however, the binormal impulse regarded only vehicle no. 2 along with the binormal velocity (Fig. 10).

The following equations, which are based on Figs. 9 and 10, can be used in order to use the mathematical model of this collision:

- normally to the impact plane, i.e., along the *On* line:

$$m_1(v'_{1n} - v_{1n}) = -S_{n1}, \;\; m_2(v_{2n} - v'_{2n}) = S_{n2}, \tag{2}$$

- tangentially to the impact plane, i.e., along the *Ot* line:

$$m_1(v'_{1t} - v_{1t}) = -S_{t1}, \;\; m_2(v_{2t} - v'_{2t}) = S_{t2}, \tag{3}$$

- binormally to the impact plane, i.e., along the *Ob* line:

$$m_2(v'_{2b} - v_{2b}) = S_b, \tag{4}$$

- in the direction of rotation according to the adopted angular direction:

$$I_1(\omega_1 - \omega'_1) = S_{n1}t_1 - S_{t1}n_1, \;\; I_2(\omega'_2 - \omega_2) = -S_{n2}t_2 - S_{t2}n_2, \tag{5}$$

- in the rotation direction on a vertical plane perpendicular to the road according to the adopted angular direction:

$$I_{2t}(\omega'_{2b} - \omega_{2b}) = -S_b n_2, \tag{6}$$

As a result, such equations of motion, e.g., at the end of this collision, can be presented:

$$v'_{1n} = v_{1n} - \frac{S_{n1}}{m_1}, \;\; v'_{2n} = v_{2n} - \frac{S_{n2}}{m_2}, \tag{7}$$

$$v'_{1t} = v_{1t} - \frac{S_{t1}}{m_1}, \;\; v'_{2t} = v_{2t} - \frac{S_{t2}}{m_2}, \tag{8}$$

$$v'_{2b} = v_{2b} + \frac{S_b}{m_2}, \tag{9}$$

$$\omega'_1 = \omega_1 - \frac{S_{n1}t_1}{I_1} + \frac{S_{t1}n_1}{I_1}, \;\; \omega'_2 = \omega_2 - \frac{S_{n2}t_2}{I_2} - \frac{S_{t2}n_2}{I_2}, \tag{10}$$

$$\omega'_{2b} = \omega_{2b} - \frac{S_b n_2}{I_{2t}}, \tag{11}$$

In equations (2)–(11):

- $S_{n1}$, $S_{t1}$, $S_{n2}$, $S_{t2}$, $S_b$ – components of the impulses of the truck (no. 1) and the vehicle (no. 2),
- $v_{1n}$, $v_{2n}$ – the normal velocities of vehicles 1 and 2 before the collision,
- $v_{1t}$, $v_{2t}$ – the tangential velocities of vehicles 1 and 2 before the collision,
- $v_{2b}$ – the binormal velocity of vehicle 2 before the collision,
- $\omega_1$, $\omega_2$ – the angular velocities of vehicles 1 and 2 before the collision,
- $\omega_{2b}$ – the angular velocity of vehicle 2 on the vertical plane before the collision,
- $n_1$, $t_1$, $n_2$, $t_2$ – the coordinates of the centers of mass of vehicles 1 and 2 respectively versus the geometric center of the collision (point O),
- $I_1$, $I_2$ – the moments of inertia about the vertical axes of vehicles 1 and 2, respectively,
- $I_{2t}$ – the moment of inertia about the lateral axis of vehicle 2.

In this case, some additional parameters can usually be included in a collision protocol of each software designed to simulate the collisions, such as V-SIM. The moments of inertia, e.g., $I_{2t}$, as well as the longitudinal, lateral and angular velocities, can be found in the collision protocol. The only problem is regarding the vertical velocity ($v_{2b}$) of vehicle no. 2. In the case of mathematical modeling, it can be included as a factor allowing the analysis of the potential resultant motion of vehicle no. 2 during the collision. Such parameters could also be included in the algorithm of any software devoted to road collision analyses.

Equations (7)–(11) contain eight unknowns, i.e., the after-collision velocities $(v'_{1n}, v'_{1t}, v'_{2n}, v'_{2t}, v'_{2b}, \omega'_1, \omega'_2, \omega'_{2t})$, and five components of the impulse $(S_{n1}, S_{t1}, S_{n2}, S_{t2}, S_b)$. Therefore, it seems that the translational and the angular velocities require the impulse components for each vehicle. In practice, there are 13 unknowns, so we need 13 equations.

Simplifying this, we can assume that impulses $S_1$ and $S_2$ are equal since, according to the collision theory, the colliding vehicles are considered as one body during the collision. Of course, in more complex analyses, both impulses could be calculated as integrals of the colliding forces, which is difficult without specifically dedicated software other than, e.g., V-SIM. However, for practical reasons, a simplification regarding the equality of these impulses seems acceptable.

The main problem is related to dividing these impulses into three components (the normal, tangential and binormal). Of course, in this specific case, a simple hypothesis can be used:

$$S_1 = \sqrt{S_{n1}^2 + S_{t1}^2},\ S_2 = \sqrt{S_{n2}^2 + S_{t2}^2 + S_b^2}, \tag{12}$$

with the impulses $S_1$ and $S_2$ known from the collision protocol. In this simplified way, two additional equations can be obtained. This gives us 10 equations: (7)–(11) and (12) for 13 unknowns.

A hypothesis regarding the friction between the colliding vehicles can be used, for example, as in [2], in order to gain three more equations. If it is assumed that a collision causes friction between the surfaces of the vehicles, then an impulse ratio ($\theta$) could be used as a factor generating three additional equations:

$$S_{n1} = \theta S_{t1},\ S_{n2} = \theta S_{t2},\ S_b = \theta S_{t2}, \tag{13}$$

which could complete the previous equations. In that way, a simplified collision model containing 13 equations for 13 unknowns can be composed. Adopting the same impulse ratio in two directions (tangential and binormal) can be regarded as a far-reaching simplification, but a ratio specifying the phenomena between the vehicles when in contact could be averaged for each vehicle.

Of course, such an attempt is simplified yet correct enough to be applied, e.g., by an expert without any complicated mechanical calculations and additional factors such as a coefficient of restitution, which is even more difficult to determine. Of course, such a mathematical model would be valuable for, e.g., referring to the simulation results of analytical calculations, provided that the software includes a potential resultant motion.

## 4. CONCLUSIONS AND DISCUSSION

From the analysis presented above, it can be concluded that, under certain circumstances, e.g., high-speed collisions, some additional phenomena could be included in the mathematical modeling of road accidents as well as in a software used to improve, e.g., the forensic opinions.

The present attempt is simplified and easy enough to be used by any expert if some necessary information and parameters are provided, e.g., in a protocol of a collision generated by software. The mathematical approach could be used either to provide a specific area of verification for the data obtained from a crash simulation via computer software or to change the algorithm of such software to include more specific road events, such as the resultant motion of at least one of the colliding vehicles.

If the coefficient of restitution is unknown or difficult to specify, then an approach such as that presented here could be an alternative. Some additional remarks can also be made. The collision impulse has to be known to use the equations presented here and provided that some additional assumptions are made, the impulse should be easily divideable into two or three components, depending on the motion adopted for a collision model. In some more advanced considerations, the coefficient of restitution and the additional accelerations of the vehicles performing the resultant motion will also be provided. In the simple calculations presented here, the only factor that requires analysis is the vicarial impulse ratio ($\theta$) between the surfaces of the vehicles in the analyzed collision.

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