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MULTIGRAPH IS: Part 1. A FORMAL DESCRIPTION OF RAILWAY INFRASTRUCTURE FOR THE DIGITAL TWIN OF THE ETCS APPLICATION

Summary. The European Railway Agency has formulated assumptions for a target model of rail transport. Its important premise is digitalization to support the communication and transport services that the railways will make available to the public in the future. Part of the digitalization process is the digital description of the railway infrastructure in a formalized form to allow algorithmic processing. The formal description of infrastructure is not a new issue. However, attempts made so far have not resulted in a permanent definition of a generally accessible formalism allowing for a coherent representation of the physical railway infrastructure in a digital form. This paper presents the results of work carried out within the research project Digital Railway-The Digital Twin of the ETCS Application-Virtual Prototyping and Simulation of Operational Scenarios.

1. INTRODUCTION

The Railway Signaling Team of the Faculty of Transport at the Warsaw University of Technology has been implementing the project “Digital Railway. Digital Twin of ETCS Application – Virtual Prototyping and Simulation of Operational Scenarios.” Its assumptions envisage the development of a research workshop allowing the development of new technologies in the area of computer aided design and validation of the ERTMS/ETCS system (shortly ETCS). The basic element necessary for the new technologies being developed for a virtual prototyping concept is a mathematical representation of the railway infrastructure using graph theory. This element will be discussed further in the text.

The digital twin is a concept described by Grieves [1]. It is a digital representation of a real system and is also called a physical twin. In recent research, this concept has been used to map the real application of the ETCS system [2]. All the elements that make up the ETCS application and its environment were mapped. For each physical element, a digital object was created by mapping its configuration parameters (e.g., the set of packets in the fixed balise) and operational parameters (e.g., the train movement authority). The digital twin stores and provides information about the current configuration of the ETCS application described by the configuration parameters and the state of the ETCS application described by the operational parameters of its individual components. In simplified terms, the ETCS Application Digital Twin is a collection of models and algorithms. In the current literature, examples can be found of various applications of the digital twin concept for mapping a

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railway infrastructure component. In the research described in [3], the authors used the digital twin at a subway station to evaluate it at different stages of its lifecycle. Kampczyk and Dybeł in [4] used a digital twin to model the geometric properties of a railway point that changed under the influence of environmental parameters.

The ETCS Application Digital Twin requires a flexible and efficient data structure that allows a consistent description of various properties of the rail infrastructure. The authors of this article propose the use of the Multigraph IS for this purpose. The following sections of this article describe previous works on similar topics, such as the premise for the Multigraph IS and its construction. A graphical representation of a simple track layout is presented to visualize the nature of the structure of the Multigraph IS.

2. RELATED WORKS

The first relevant study at the Warsaw University of Technology's Faculty of Transport related to the formal description of railway infrastructure and, more specifically, the topology of the track layout and the functional infrastructure of signaling was written in the 1970s by Ostasz [5]. The formalism he developed was applicable to the automatization of designing signaling systems. The formal model organized infrastructure elements into sets of specific types of objects and indicated the relationships between them. The model shows a high degree of specialization toward the basic functions of signaling. The topology of the track system is modeled only by points and control area boundaries. In terms of the functional infrastructure of the traffic control system, there are isolated sections and signals, which have sets of attributes. The routes and their variants are also modeled.

Another work devoted to the formal description of signaling-infrastructure that is worth noting was written in 2008 [6]. Zabłocki focused on the modeling of station interlocking. The mathematical apparatus made use of sets, matrix-vector calculus, and the concept of automata. Relationships were defined between elements of sets and mapping, for example, functions of signaling ~~CCS~~. The scope of modeling was also limited to the elements of infrastructure relevant to the control system functions: signals, points, track controlled section, and interface to line block. In terms of modeling the topology of the track system, the author defined a neighborhood relationship. However, this relationship applies to the various types of signaling equipment mentioned and is specialized in the direction of the construction of track routes.

The most recent research in the field of formal description of railway infrastructure was carried out by Wontorski [7], together with Kochan [8]. This work was largely based on the model of Zabłocki. A new element applied was the use of the concept of a graph. There are several graphs in the model, and one of them is a model of the structure of the track layout and external signalling equipment: the TU graph. Its vertices are *ob* controlled areas, and its edges are *ut* elements of the track system connecting these areas. This approach has the advantage of using a graph structure that allows for a better understanding of the transition between the real system and the model.

The advanced use of graph theory to represent the topology of a track layout can also be seen in work conducted around the world. The approach presented in [9], which became the basis for the RailTopoModel standard published as the International Railway Solution issued by the UIC, was presented in [10].

Much work has also been devoted to rail infrastructure otology. A series of interesting issues related to this topic are described in [11-13], where one can find an interesting overview of this topic and a proposal for a railway knowledge graph, which is an example of the application of graph theory to build structures describing the railway infrastructure.

In a similar way, the possibility of unifying Ukrainian railway transport information systems is being explored using ontology support by Zhuchi [14]. This research has led to the development of a basic modular ontology framework model containing 12 components linked by logical definitions. It provides ontological support for technological processes. The application of the developed methods and tools makes it possible to achieve a greater decentralization of information systems and to standardize the representation of railway technological processes.

In [15], Magnien et al. compared different railway data models for use in a train management system. They analyzed the RSM multi-purpose rail system model, EULYNX DataPrep Signaling assets, IFC Railway infrastructure assets, TRANSMODEL Multimodal passenger traffic management and related assets, an X2RAIL-4 Data exchange model for operational purposes, incl. ATO X2RAIL-4 consortium JSON schema, Protobuf. The standardized description of these elements was provided using UML and JSON language.

From a railway safety point of view, the issue of creating an infrastructure description was addressed by Mahtani [16], who proposed a combination of multiple sources (both deterministic and not deterministic), which allowed the results to be interrelated, thus making them more reliable. A new methodology was described that adapts the infrastructure mapping system and points cloud analysis to the perception of railway tracks and traction elements to ensure the safety of autonomous trains.

Another take on the issue of describing railway infrastructure was presented in [17]. The authors dealt with a BIM methodology. Despite their focus on the modeling of building structures, they tried to develop libraries of linear infrastructure models to describe engineering structures, as well as linear infrastructure, in a consistent way. In collaboration with government organizations, they created appropriate extensions for programs such as Civil 3D, Revit, and AECOsim. This type of description is informal.

The essence of modeling the railway infrastructure and the train in ETCS applications was considered in [18]. The author pointed out the need to model such systems for high-speed rail transport in order to reduce the risk of errors in the implementation of ETCS applications. He applied modeling techniques belonging to model-based systems engineering and model-based safety analysis using the UML and SysML languages.

Love et al. [19] pointed out that rail infrastructure managers often encounter maintenance problems due to outdated as-built documentation. They supported their thesis with case study research. They indicated digital asset management as a method to improve this situation. This method uses a digital model and is common to many applications, and its consistency has been maintained through various means.

The authors of this article, using Multigraph IS, propose a much broader application of a graph structure. This is done to reflect on the physical railway infrastructure as accurately as possible to the extent that it will meet the needs of the digital twin of the ETCS application. Accurately mapping the route is important in ETCS Application Digital Twin ~~CBAE~~. It is necessary to accurately model distance, longitudinal gradient, adhesion, static speed profile, and other parameters (which will be discussed later) because they affect the train's behavior during specific phases of travel (e.g., braking). The accurate mapping of the route requires the topology of the track system to be mapped.

3. MULTIGRAPH OF RAILWAY INFRASTRUCTURE

3.1. Components

The mathematical model of railway infrastructure in the form of a multigraph (called the multigraph of railway infrastructure) will be designated as Multigraph IS (or *IS* for short):

$$IS = (V, E) \tag{1}$$

where:

- V – the set of vertices modeling the elements of the railway infrastructure, also denoted $V(IS)$
- E – the set of edges modeling the relationship between elements, also denoted $E(IS)$

Within these sets, subsets corresponding to different types of elements and relations, respectively, are distinguished. The subsets are designated using colors. The colors of relations (edges) determine the subgraphs of the IS. Subgraphs modeling selected properties of an infrastructure will be denoted by:

$$IS^k \tag{2}$$

where:

- k is the color designation of the subgraph.

Similarly, the subsets of vertices and edges will be labeled $V^k(IS)$ and $E^k(IS)$. While the same vertex and edge colors may have different meanings, they should not be equated.

The notations $V^k(IS)$ and $E^k(IS)$ are examples of the description of certain features specified on the whole *IS* model. This convention will also be applied to the substantive properties of the Multigraph IS application domain.

The concept of layer will be introduced in order to enable the substantive grouping of selected issues. A layer is a set of vertices and edges that are relevant to a substantive issue. From the point of view of the main subject of this research, the ETCS application layer is the most important. Formally, a layer is described by the following expression:

$$L^n(IS) = \{V^k, V^m, \dots, E^p, E^r, \dots\} \tag{3}$$

where *k* and *m* are the colors of vertices belonging to layers *n* and *p,* and *r* represents the colors of edges belonging to layer *n*. The vertices of the multigraph store the properties of model elements in the form of attributes organized into records or more complex structures. The formal notation of the attribute of any multigraph element has the following format:

$$\mathrm{Element.attribute} \tag{4}$$

The element symbol comes first, followed by an attribute name. For example, the notation

$$V_n^k.a \tag{5}$$

reads *attribute a of vertex n of color k.*

For a vertex, there may also be a function that determines, for example, the dynamic value of an attribute. We write it similarly to an attribute by adding round brackets. For example, the notation

$$V_n^k.f(\quad) \tag{6}$$

reads *function f of vertex n of color k.*

The resemblance to the notations used in the object-oriented design and programming approach from the computer sciences is provided here as intended as much as possible.

In the remainder of this article, relationships will be discussed. Their elements will be written in brackets. The round bracket convention will be used if the order of the elements is important. Where the order is not relevant, curly brackets will be used.

3.2. Meta elements of the Multigraph IS

In the Multigraph IS, the following meta elements are defined. They are used to model certain types of relationships between infrastructure elements. Such meta elements are relations:

- neighborhood – *S*
- succession – N
- affiliation – *P*
- functional link – *F*
- feature of the element – *CE*
- linear feature – *CL*
- area feature – *CO*

Neighborhood is a relationship indicating that two given pieces of railway infrastructure are adjacent to one another. This relationship is not directed. It is modeled as an undirected edge. If a vertex m of color o and a vertex k of color p are in a neighborhood relation, then we write it as follows:

$$S\{V_m^o, V_k^p\} = 1. \tag{7}$$

The use of curly brackets indicates an arbitrary order of the relation's arguments.

If such a relationship between the vertices m and k does not occur, we write it as follows:

$$S\{V_m^o, V_k^p\} = 0. \tag{8}$$

Succession is a stronger relationship than neighborhood. It is a relation indicating that two elements are adjacent to one another, with one being the successor of the other. It is modeled as a directed edge. If a vertex m of color o and a vertex k of color p are in a succession relation such that the vertex of the m of color o is the successor of the vertex k of color p, then we write it as follows:

$$N\left(V_k^p, V_m^o\right) = 1. \tag{9}$$

The use of round brackets means that the two vertices are of an ordered nature. Placing more vertices in parentheses means that the vertices are successively listed in pairs in a succession relationship.

If such a relationship between the vertices m and k does not occur, we write it as

$$N\left(V_m^o, V_k^p\right) = 0. \tag{10}$$

Affiliation is a relation that allows sets to be defined. That is, it can be indicated that an element is a component of another element. It is modeled in the form of a directed edge with a return from the constituent element to the element acting as a whole. If a vertex m of color o and the vertex k of color p are in an affiliation relationship, such that a vertex of k of color p belongs to the vertex of m of color o, then we write it as follows:

$$P\left(V_k^p, V_m^o\right) = 1. \tag{11}$$

If such a relationship between the vertices m and k does not occur, we record this as follows:

$$P\left(V_k^p, V_m^o\right) = 0. \tag{12}$$

A **functional link** is a relationship that allows cooperation between elements to be defined in the execution of a function. It is modeled in the form of an undirected edge. If a vertex m of color o and a vertex k of color p are in a functional linkage relationship, we write it as follows:

$$F\left\{V_k^p, V_m^o\right\} = 1. \tag{13}$$

If such a relationship between the vertices m and k does not occur, we record this as

$$F\left\{V_k^p, V_m^o\right\} = 0. \tag{14}$$

An **element attribute** is a relationship that allows the modeling of selected properties of model elements. As described in the previous section, the basic approach to modeling the properties of model elements are attributes of a certain type that describe the vertices of the multigraph. The element attribute relation is an information-equivalent solution that allows a given attribute to be modeled in the multigraph structure and to be taken into account in algorithms operating on this structure. This relation is directed. It is modeled as a directed edge from the vertex modeling the feature to the vertex modeling the element.

If the vertex m of color o is a feature of a vertex k of color p (i.e., m is in the relation of the feature of the element z), then we write it as follows:

$$CE\left(V_m^o, V_k^p\right) = 1. \tag{15}$$

If such a relationship between the vertices m and k does not occur, we write it as

$$CE\left(V_m^o, V_k^p\right) = 0\ . \tag{16}$$

3.3. Properties of the Multigraph IS

Due to the nature of the design principles of the signaling application (and, thus, of the ETCS application), it is possible to distinguish certain characteristic features of the relationship between infrastructure elements. Such features include the distance along the run path *D* and the inclination along the run path *G*. The distance along the driving path can be determined for the vertices of *IS* and the edges of *IS*.

In the case of vertices, this can only apply to vertices that have a length attribute. In this case, the attribute *D* is written as follows:

$$IS.D\left(V_k^p(IS)\right)\ . \tag{17}$$

In the case of edges, the distance feature is determined by the difference in the co-ordinates of the spot positions of the same positioning system between two vertices IS forming an edge. In this case, the feature *D* is written as follows:

$$IS.D\left(E_k^p(IS)\right) \quad . \tag{18}$$

For the *G* gradient feature, the inference is similar. Additional assumptions apply to the positioning system. It must provide location information in the height dimension. The slope along the path of travel is determined by analyzing the distance and absolute height difference within the same positioning system of two vertices. The feature *G* for the vertices is written as follows:

$$IS.G\left(V_k^p(IS)\right). \tag{19}$$

Meanwhile, in the case of edges, the feature takes the following form:

$$IS.G\left(E_k^p(IS)\right). \tag{20}$$

3.4. Topology model of the track system

The backbone of railway infrastructure elements is the graph of the track layout topology, just as it is in the physical twin. We speak of the track layout topology layer L^1. The track layout topology is modeled by track segments $V^2(IS)$, the neighborhood relations between them $S^1(IS)$, and the successions $N^2(IS)$, which yields $IS^1$and IS^2, respectively. Such an approach is consistent with the assumptions of the RailTopoModel [10]:

$$IS^{11} = \left(V^{16}(IS) \cup V^{17}(IS), E^{11}(IS)\right). \tag{21}$$

The topology can be mapped at different levels of detail. At a basic level, we adopt a representation where vertices model track segments whose neighborhoods are determined by the switch and crossing, while the ends are determined by the boundary point of the modeled area and the end of the track.

The perspective of the micro model is based on the level of detail of the topology described in the previous section.

Many railway infrastructure properties should be regarded from the point of view of routes throughout the railway network. The itinerary should be understood as a sequence of routes taken by the train in a modeled environment. The itinerary is modeled using the finitary relation $E^2(IS)$, which models the direction of the route between track sections. It is necessary to distinguish the initial track sections from which the routes will start in order to indicate the possible routes in the model. *IS* models this information in the form of vertices $V^{15}(IS)$, and the relationship *CE* is modeled by a set of $E^{115}(IS)$. That is, a track section V_m^2 is the starting element of the route if there is a relation CE^{115} between it and the beginning of the route V_k^{15}:

$$CE^{115}\left(V_k^{15}, V_m^2\right) = 1. \tag{22}$$

ISalso allows modeling at the level of macro detail [10]. For this purpose, the color 16 of the vertices introduced $V^{16}(IS)$ modeling operational points (e.g., stations, branch posts) and vertices of color 17 $V^{17}(IS)$ modeling the tracks connecting the operating points (the tracks can consist of one or more tracks). The relationship between these elements based on their physical proximity is modeled by the edges of color 11 $S^{11}(IS)$. Based on these elements, the macro topology model of the track layout will be defined as follows:

$$IS^{11} = \left(V^{16}(IS) \cup V^{17}(IS), E^{11}(IS)\right) . \tag{23}$$

IS contains subgraphs that aggregate mappings of the affiliations of various functional infrastructure elements to parent elements to form a structure hierarchy. Some of these, such as S^{12}, have been described above. Further relationships of this type are as follows:

The affinity of topology elements to macro model elements will be modeled by color edges 13 $E^{13}(IS)$:

$$IS^{13} = \left(V^2(IS) \cup V^{16}(IS) \cup V^{17}(IS), E^{13}(IS)\right). \tag{24}$$

The affiliation of macro model elements to a railway line will be modeled by edges of color 14 $E^{14}(IS)$, while the railway lines will be modeled by vertices of color 18 $V^{18}(IS)$, which leads to the following definition:

$$IS^{14} = \left(V^{17}(IS) \cup V^{16}(IS) \cup V^{18}(IS), E^{14}(IS)\right). \tag{25}$$

3.5. Functional infrastructure model

IS models the trackside signaling devices as functional elements of the infrastructure. These elements are placed along the elements of the topology. Different types of devices are modeled by vertices with different colors. Examples of elements of infrastructure are:

- $V^4(IS)$ – signals
- $V^6(IS)$ – unoccupied control sections
- $V^7(IS)$ – crossovers
- $V^8(IS)$ – derailments
- $V^9(IS)$ – points
- $V^{10}(IS)$ – shp resonators
- $V^{11}(IS)$ – balises
- $V^{15}(IS)$ – wheel sensors

The model for the distribution of trackside equipment along the track is implemented through neighborhood and succession relationships:

- $S^{10}(IS)$ – the proximity of infrastructure elements in accordance with the mileage
- $N^3(IS)$ – the succession of infrastructure elements for the normal direction of train movement
- $N^4(IS)$ – the succession of infrastructure elements for the reverse direction of train movement

It is also implemented through the relation of affiliation $P^{12}(IS)$ to the track sections $V^2(IS)$.

Using the listed *IS* elements, we define the following subgraphs:

$$IS^{12} = (V^4(IS) \cup V^5(IS) \cup V^6(IS) \cup V^7(IS) \cup V^8(IS) \cup V^9(IS) \cup V^{10}(IS) \cup V^{11}(IS)) \cup V^{12}(IS), E^{12}(IS)) . \tag{26}$$

$$IS^{12} = (V^4(IS) \cup V^5(IS) \cup V^6(IS) \cup V^7(IS) \cup V^8(IS) \cup V^9(IS) \cup V^{10}(IS) \cup V^{11}(IS)) \cup V^{12}(IS), E^{10}(IS)) . \tag{27}$$

$$IS^3 = (V^4(IS) \cup V^5(IS) \cup V^6(IS) \cup V^7(IS) \cup V^8(IS) \cup V^9(IS) \cup V^{10}(IS) \cup V^{11}(IS)) \cup V^{12}, E^3(IS)) . \tag{28}$$

$$IS^4 = (V^4(IS) \cup V^5(IS) \cup V^6(IS) \cup V^7(IS) \cup V^8(IS) \cup V^9(IS) \cup V^{10}(IS) \cup V^{11}(IS)) \cup V^{12}(IS), E^4(IS)). \tag{29}$$

IS^{12} models the assignment of functional infrastructure elements to topology elements. With its help, it is possible to unambiguously determine these topology elements' locations.

IS^{10} models the succession resulting from the distribution of functional infrastructure elements according to mileage. It makes it possible to search for relationships between all elements of a functional infrastructure.

Subgraphs IS^3 and IS^4 also model device sequences, but they take into account the direction of the train. Subgraph IS^3 (neighborhood in line with the principal direction) precisely models the infrastructure passed by a moving train in the normal direction. Subgraph IS^4 (neighborhood consistent with the opposite direction) precisely models the infrastructure passed by a moving train in the opposite direction to the principle direction.

Subgraphs IS^3 and IS^4 allow one to indicate the functional infrastructure elements relevant to the movement of the train. Note that the direction of the train is relevant only to selected types of elements. These include:

$$V^4(IS), V^5(IS), V^{11}(IS), V^{12}(IS)\ . \tag{30}$$

Meanwhile, they are not relevant to

$$V^6(IS), V^7(IS), V^8(IS), V^9(IS)\ . \tag{31}$$

Since a very important attribute of all trackside equipment is its location, it is necessary to analyze this issue in the context of its physical dimensions. For the following types, the physical dimensions are irrelevant to the functions performed, and their geometric center can be taken as an accurate point reflected in reality:

$$V^4(IS), V^5(IS), V^8(IS), V^{23}(IS)\ . \tag{32}$$

In the case of $V^{11}(IS)$, it should be assumed that this is the point determined by the so-called "reference remark" (the geometric co-ordinates of the balise are determined relative to this point). In the case of $V^{12}(IS)$, according to specification [20], the location of the first balise in a group is taken as the whole group location. The types of trackside equipment that need more attention are $V^9(IS)$ points and $V^6(IS)$ unoccupied control areas. These are devices whose dimensions are relevant to the functions performed. For any given point, the primary consideration is that, in the context of the topology model, the physical dimensions are located on three adjacent track sections (Fig. 1).

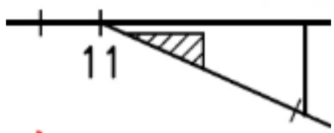

Fig. 1. Symbol of the point on the signaling plan (source: own development)

In the representation of a point considering the premise of possible automatic creation of a signaling plan based on the model of points, it is necessary to distinguish three components: the beginning of the point, the geometric center of the point, and the fouling point. In this situation, the geometric center of the point should be taken as the place of contact of adjacent track sections. Because multiple types of points are present in the railway network, the point model will have relations with all the track sections $V^2(IS)$, which are parts of the point design. The part of the track section connected to the beginning of the point will be highlighted. A similar solution will apply to railway junctions in the future.

4. CONCLUSIONS

The digitization of railways is placing increased demands on the accurate description of the rail infrastructure. Currently, in many commercial and academic centers, research is underway to standardize the description of a variety of its elements. However, the increase in accuracy results in a significant increase in complexity both due to the amount of data and the richer information content. This article points out that such a description can be formal. The author's idea of an IS multigraph is presented, which is a comprehensive form of infrastructure description that takes into account the different elements of the infrastructure and the important relationships that exist between them.

The elements presented in this paper are used for the topological model layer of the functional railway infrastructure. The formal shape of the model is ready for algorithmic processing. This effect gives hope for the development of a fast model based on algorithm verification (e.g., through a sketched virtual modeling process). It should be noted that the structure of the Multigraph IS proposed in this article is an effective structure of data describing the railway infrastructure, including the ETCS application. Owing to its polymorphic properties, it is a flexible concept that allows the consistent modeling of any device of the real system. Work on the development of this concept is being carried out in various directions. A continuation of this paper contains the verification of the correctness of ETCS applications, which was made by the authors. The verification was carried out by algorithmic processing of the Multigraph IS, and its main intention was to achieve the fulfillment of appropriate criteria, which will be formulated in the form of logical sentences.

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