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Phridon GOGIASHVILI[1]*, David KBILASHVILI[2], Jumber CHOGOVADZE[3], Gocha LEKVEISHVILI[4], Besik SIRBILADZE[5]

STUDYING THE DYNAMIC SYSTEM "TIRE-SUSPENSION-BODY" BY THE RESONANCE METHOD

Summary. By comparing the results of a pilot study of the damping coefficient of a pneumatic tire carried out on a test bench, this study justifies the possibility of using the less time-consuming experimental resonance curve method instead of the dynamic loop method. A mathematical model of the vertical oscillations of avehicle'sdynamic system "pneumatic wheel-suspension-sprung weight"was created on the test-bench, and an image of the amplitude-frequency characteristic of the sprung weight oscillation was obtained. The approaches and results of the pilot and analytical studies presented in the article aid the selection of the tire with the best damping properties. The use of the experimental resonance curve method to determine the damping characteristic of a vehicle's tire is limited in the external frequency range of the resonant frequencies. In this case, we have to use the dynamic loop method.

1. INTRODUCTION

During the movement of a vehicle, the interaction of its wheels with road surface irregularities causes mechanical oscillations of the vehicle's body and the driver's seat. These irregularitiesare accompanied by the impact of harmful vibration loads on the vehicle mechanisms and the driver's body [1]. Therefore, when developing new equipment, significant attention is paid to reducing vibration loads, which can be achieved through the use of suspension designs with the best vibration protective properties. Numerous scientific papers have dealt with the study of methods for regulating the parameters of the elastic-damping elements of the suspension and the adapted systems of dampers [2].

Analyses of mechanical vibrations in the vehicle suspension show that the formation of oscillating processes is predominantly influenced by the vertical dynamic loads generated in the contact zone of the pneumatic wheel with road irregularities [3]. Therefore, it is important to evaluate the impact of the damping characteristic of tires on the reduction of vertical oscillations in the vehicle's suspension. It has been found that the impact of the damping effect of tires on the reduction of vertical oscillations of the sprung and unsprung weights of the vehicle is particularly evident in the resonant modes of oscillations of these weights and is relatively less effective outside the resonant oscillation frequency

[1] Akaki Tsereteli State University; 59 Tamar Mepe, Kutaisi, 4600, Georgia; email: pridon.gogiashvili@atsu.edu.ge; orcid.org/0000-0002-1905-3618
[2] Akaki Tsereteli State University; 59 Tamar Mepe, Kutaisi, 4600, Georgia; email: david.kbilashvili@atsu.edu.ge; orcid.org/0000-0002-0516-8510
[3] Akaki Tsereteli State University; 59 Tamar Mepe, Kutaisi, 4600, Georgia; email: jumber.chogovadze@atsu.edu.ge; orcid.org/0000-0001-8460-0481
[4] Akaki Tsereteli State University; 59 Tamar Mepe, Kutaisi, 4600, Georgia; email: gocha.lekveishvili@atsu.edu.ge; orcid.org/0000-0002-7801-4547
[5] Akaki Tsereteli State University; 59 Tamar Mepe, Kutaisi, 4600, Georgia; email: doqtor2014@gmail.com; orcid.org/0000-0002-3521-1540
* Corresponding author. E-mail: pridon.gogiashvili@atsu.edu.ge

ranges [4]. Therefore, we believe it is possible to study the damping characteristics of the pneumatic tires of vehicles under test-bench conditions in resonant frequency ranges following the method of the so-called resonance curve. Subsequently, analytical and experimental evaluations of the impact of tire dampers on the oscillation processes occurring in the suspension can be carried out [5].

It is known that analytical studies to determinethe elastic-damping characteristics of tiresare complicated due to their design features because, at present, it is necessary to determine the values of the parameters of tires, which is feasible only by using a special test-bench [6]. Therefore, studies are often conducted using such a test-bench; however, in this case, the reliability of the experimental results depends significantly on the extent to which the test-bench design can simulatethe implementation of real load modes on apneumatic wheel [7].

The study of the vertical (radial) elastic-damping characteristics of a vehicle's pneumatic tire was conducted on a test-bench by static and dynamic methods. When using the first method, the vertical static load (G) wasgradually changed on the pneumatic wheel, and the deflection of the tire (h) was recorded for each fixed load. After this, the G (h) diagram, the so-called static hysteresis loop, was constructed at a certain scale [8]. When using the dynamic method, a vertical sinusoidal load (F) of a certain magnitude (A) and frequency (ω) was applied to the wheel under a fixed static load (G), and the axis displacement (h) was measured on the test-bench. With these two parameters, the F (h) dependence, or the so-called dynamic loop, was constructed at a certain scale [9, 10]. Then, using the above method, the values of the dynamic stiffness of a tire and the damping coefficients could be determined. Numerous experiments were required to determine the dynamic amplitude and frequency dependence of these parameters, which are associated with certain difficulties.

The pilot studies were carried out on atest-bench device developed by the research team, which allows for the creation of a vertical sinusoidal oscillating in a contact zone with the support sledge surface on the pneumatic wheel under the fixed static load. This allowed us to establish resonant modes of vertical oscillations of the sprung and unsprung weights in a dynamic system "pneumatic wheel-suspension-sprungweight"simulated on the test bench.

The aim of this study is to assess the impact of change in the tiredamping coefficient on resonant oscillations generated in a dynamic system "pneumatic wheel-suspension-sprung weight".

2. RESEARCH SUBJECT AND METHODS

The current research analyses the resonant oscillations generated in the vehicle's dynamic system "pneumatic wheel-suspension-sprung weight" and assesses the impact of changes in the values of tire damping coefficients.

Based on the above, the resonance curve method was selected under bench-scale conditions to determine the values of the tire damping coefficient.

The essence of this method is that the experimental resonance curve should be constructed by changing the sinusoidal disturbance impact on the pneumatic wheel. Further, through its use, the damping coefficient should be determined. In our case, the source of the variable external turbulent force by sinusoidal law on the pneumatic tire is an inertial vibrator on the drive shaft,of which the desired magnitude of the external exciting force can be realized by varying the values $P = P_0 \sin \omega t$ of the rotation frequency ω. Theweight of the vibrator's eccentrics m_0, where $P_0$and $\omega$are the amplitude and frequency of the Pforce, respectively. The total mass M of the sprung weight, suspension weight, and the wheel weight on the test-bench P is affected by vertical sinusoidal oscillations, whose equation of motion is as follows:

$$M\ddot{z} + k\dot{z} + cz = P_0 \sin \omega t, \tag{1}$$

where z is the vertical displacement of mass M.

After transforming equation (1), we obtained:

$$\ddot{z} + 2h\dot{z} + p^2 z = \frac{p_0}{m} \sin \omega t, \tag{2}$$

where $2h = \frac{k}{m}$represents the damping coefficient of the tire, $p^2 = \frac{c}{M}$ is the natural oscillation frequency of mass M, and$c$is the vertical rigidity of the tire.

The amplitude of forced sinusoidal oscillations D is determined using the formula below:

$$D = \frac{\mathrm{P/m}}{\sqrt{(\mathrm{p}^2-\omega^2)^2+4\mathrm{h}^2\omega^2}}. \tag{3}$$

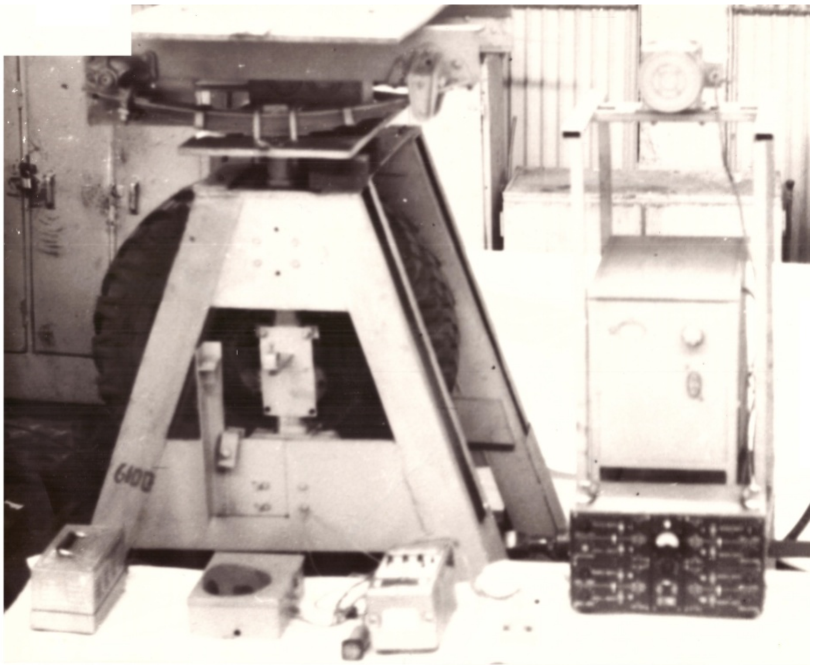

Fig. 1. Test bench for the pneumatic wheel

We will now introduce the notations $\gamma_d = \frac{1}{\sqrt{(\mathrm{p}^2-\omega^2)^2+4\mathrm{h}^2\omega^2}}$ and $\gamma_s = \frac{P_0}{c}$, where γ_d and γ_s are the system's static and dynamic factors, respectively. The dynamic factor shows the oscillation amplification ratio at resonance ($\omega = p$). As can be seen from equation (3),

$$\gamma_{\max} = \frac{\mathrm{p}}{2\mathrm{h}}.$$

The dynamic factor is a function of the frequency of the disturbing action, and a resonance curve of this relationship takes the form shown in Fig.2.

$$\gamma_d = a\gamma_{max}. \tag{4}$$

The following values are recommended for the factor a: $\alpha = \frac{1}{2}$ or $\alpha = \frac{\sqrt{2}}{2}$. When $\alpha = \frac{1}{2}$, then, per equation (4), two magnitudes of the frequency ω can be obtained:

$$\omega_{1,2} = (p^2 - 2h^2) \pm 2h\sqrt{h^2 + 3p^2}. \tag{5}$$

Since the natural oscillation frequency of a system is $p \gg 2h$ and the relationship $\gamma_d(\omega)$ is symmetric with respect to the line ω = p, the following approximate relationships are determined:

$$\omega_{1,2}^2 = p^2 \pm 2hp\sqrt{3}, \tag{6}$$

$$\omega_1 + \omega_2 = 2p.$$

If we denote $\omega_2 - \omega_1 = \Delta\omega$, we obtain

$$2h\left(\alpha = \frac{1}{2}\right) = \frac{\Delta\omega}{\sqrt{3}}. \tag{7}$$

Analogously, when $\alpha = \frac{\sqrt{2}}{2}$,

$$2h\left(\alpha = \frac{\sqrt{2}}{2}\right) = \Delta\omega. \tag{8}$$

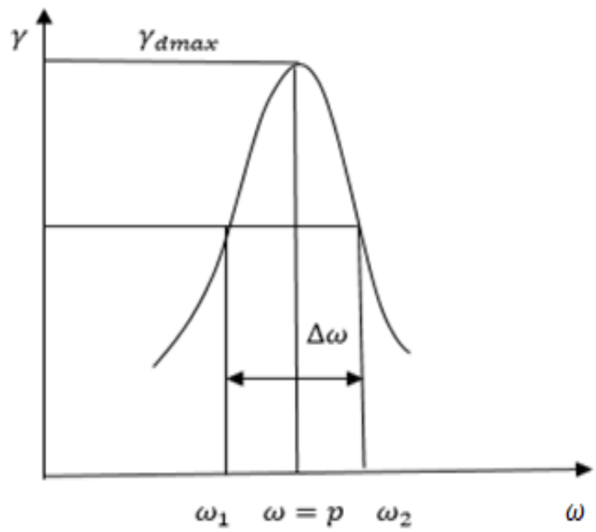

Fig. 2. Resonance curve

Hence, after constructing a curve $\gamma_d(\omega)$ (Fig.2), the values of ω_1 and ω_2, and the value of $\Delta\omega$ (determined by the magnitude of the given factors a), the values of the coefficient $2h$ are determined using equations (7) and (8). Finally, the tire damping coefficient is determined by the relationship $k_w = 2hm$. The oscillation damping coefficient below can also be used for this purpose:

$$\psi = \frac{2\pi\omega k_w}{c_w}. \quad (9)$$

The procedure for determining the damping coefficient of a tire when using a resonant curve is less time-consuming than when using the dynamic loop method. In addition, the difference between the two testsdid not exceed 10%, which is acceptable for the accuracy required for engineering calculations.

3. MATHEMATICAL MODELING OF THE VERTICAL OSCILLATIONS OF A PNEUMATIC WHEEL AND ANALYSIS OF THE RESULTS OF CALCULATIVE-PILOT STUDY

One of the best approaches to researchingthe effect of change in the magnitude of the tire damping coefficient on vertical oscillations of the sprungweight of the test system is to construct an amplitude-frequency characteristic and then conduct a qualitative and quantitative analysis of the reduction of oscillations. Numerous studies have shown that the amplitude-frequency characteristic provides a clear picture of the mechanism that generates dangerous resonant oscillations in the vehicle suspension and the impact of changes in the parameters of the elastic-damping elements on them [11, 12].

The design of the test-bench allowed us to carry out the pilot studies of the characteristics of vertical oscillations in the real dynamic system of vehicle “pneumatic wheel-suspension-sprung weight” [13]. Also, by designing a mathematical model of this system (Fig. 3) and applying the analytical expression of the amplitude-frequency characteristic of the sprung weight obtained through its use, we can assess the impact of changes in the tire damping coefficient on the vertical resonant oscillations of the sprung weight. This approach allows us to conduct a comparative analysis of the results of the pilot and analytical studies within a single experiment. It also supports the level of adequacy of the model.

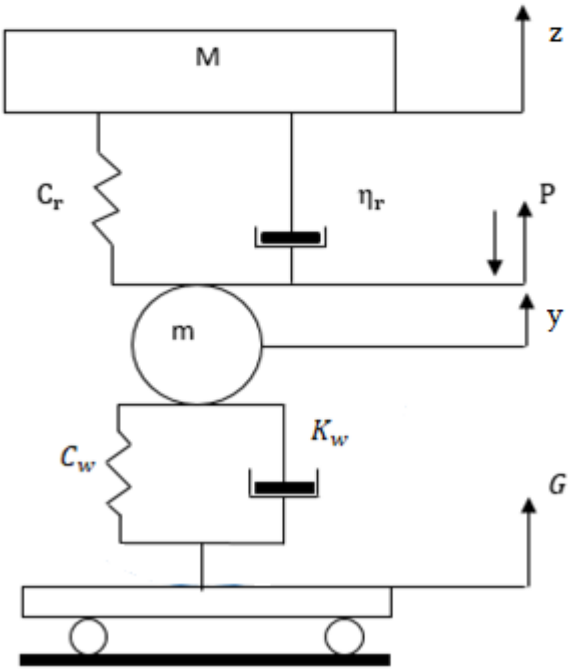

Fig. 3. A dynamic scheme of the equivalent oscillating system

As mentioned above, resonant oscillations can be established during an experiment by varying the exciting sinusoidal force created by the inertial vibrator. In this case, the vertical displacements of both the pneumatic wheel axis and the sprung weight are recorded at the fixed frequencies of forced sinusoidal oscillationusing a side-wire gage and the recording equipment required for the oscillating process. Experiments were performed for different values of the sprung weight M and the internal air pressure p_w in the tire.

The expression of the amplitude-frequency characteristic of oscillation of the sprung weight was derived by designing a mathematical model of oscillation of a dynamic system on the test-bench and performing the corresponding analytical transformations. For this purpose, the scheme of an equivalent oscillation system was first drawn, and the differential equations of weight motion were then composed.

The system of differential equations of weight motion is written as

$$\begin{gathered} \ddot{z} + 2h_r(\dot{z} - \dot{y}) + \theta^2(z - y) = 0 \\ \ddot{y} + 2h_w\dot{y} + p^2 y - \frac{2h_r}{\mu}(\dot{z} - \dot{y}) - \frac{\theta^2}{\mu}(z - y) = \frac{P_0}{M} sin\omega t \\ \mu Mg + mg + c_w y + k_w\dot{y} = G, \end{gathered} \tag{10}$$

where z is the vertical displacement of the sprung weight; y is the vertical displacement of the pneumatic wheel axis; $\theta^2 = \frac{c_r}{M}$is thenatural oscillation frequency of the sprung weight$M$; $p^2 = \frac{c_w}{m}$–is thenatural oscillation frequency of the pneumatic wheel mass$m$; $2h_r = \frac{\eta_r}{M}$is the damping coefficient of the suspension's sprung weight; $2h_w = \frac{k_w}{m}$is the damping coefficient ofthe tire; and μ is the coefficient of redistribution of the sprung weight on the vehicle's axles.

The vertical reaction force G of the pneumatic wheel when in contact with the surface of the support sledge of the test-bench is equal to the sum of the sprung weight Mg, the weight of the wheelmg, the elastic resistance force of the tire $c_w y$, and the non-elastic resistance force $k_w\dot{y}$. If the vertical deformation of the elastic element of the suspension is $X = X_0\ (\omega t - \varphi)$ and the vertical deformation of tire is $f = y$, then the solution to the equation system can be found using the trigonometric functions $X = X_0(\omega t - \varphi)$and $f = f_0\ (\omega t - \psi)$, where$\varphi$ andψ are the phase shifts of the forced oscillations of the wheel axis and sprung weight.

Finally, for the oscillation of the sprung weight on the test-bench, we obtained the following amplitude-frequency characteristic using the following formula:

$$X = \omega^2 \frac{P_0}{m\,p^2} \sqrt{\frac{p^4+4h_w^2}{(a+2k_r\omega 2h_w)^2+[b-(\vartheta^2-\omega^2)2h_w]^2}}, \tag{11}$$

whose coefficients are as follows:

$$a = \omega^2(p^2-\omega^2) - \theta^2\left[p^2 - \left(1+\frac{1}{\mu}\right)\omega^2\right],$$

$$b = -2h_r\omega\left[p^2 - (1+\tfrac{1}{\mu})\omega^2\right].$$

Thus, equation (11) allowed us to draw a diagram of the amplitude-frequency characteristic of the sprung weight oscillation to a certain extent. It also allowed usto assess the impact of changes in the magnitude of the damping coefficient of the tire on the value of the damping of resonant oscillations.

3.1. Analysis of the results of a pilot study of damping characteristics of tire

The experiments were conducted on the tire radial designs of a 5.5-ton capacity truck: 370/R80-508, models HP-54 and 380/R80-508, model HP-56. The damping coefficients of thetires was determined by constructing experimental resonance curves (Fig.4) following the above methodology.

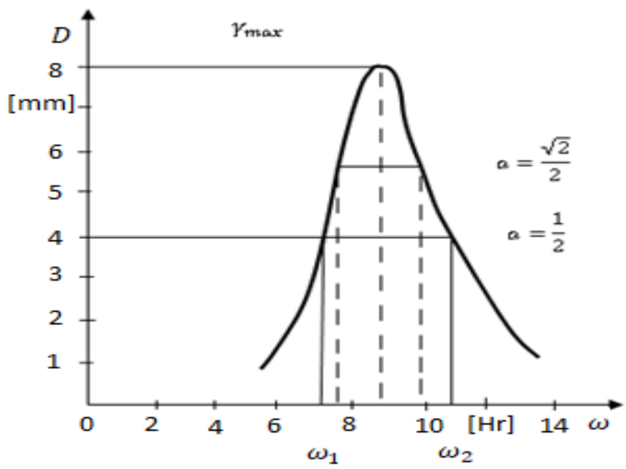

Fig. 4. An experimental resonance curve

Fig. 5 presents the results of the analysis of the experimental curve of the relationship between the damping coefficient and the normal load applied to a pneumatic wheel. For low values of normal load ($G_n = 10 \dots 20$ [kN]) applied to the wheel, the values of the tire damping coefficient $\psi(G_n)$increase slightly. This is because, in resonant oscillations, a smaller weight of tire participates in contact with the supporting surface of a wheel than at high values of normal load ($G_n = 25 \dots 30$ [kN]). Therefore, in this case, during the deformation of the tire, a relatively small amount of inelastic force is generated by internal friction.

When the vertical load applied to the wheel increases from $G_w = 15$ [kN] to $G_w = 30$ [kN], the damping coefficient of the tire increases, on average, by 1.25 times. However, this difference becomesmore pronounced asthe internal air pressure in the tirep_w decreases. For example, when $G_n = 25$ [kN], the value of the tire damping coefficient increases by an average of 30 ... 32% as the pressure decreases from $p_w = 0.5$ [MPa] to $p_w = 0.3$ [MPa]. These results were taken into account when analyzing resonant oscillations of the sprung weight.

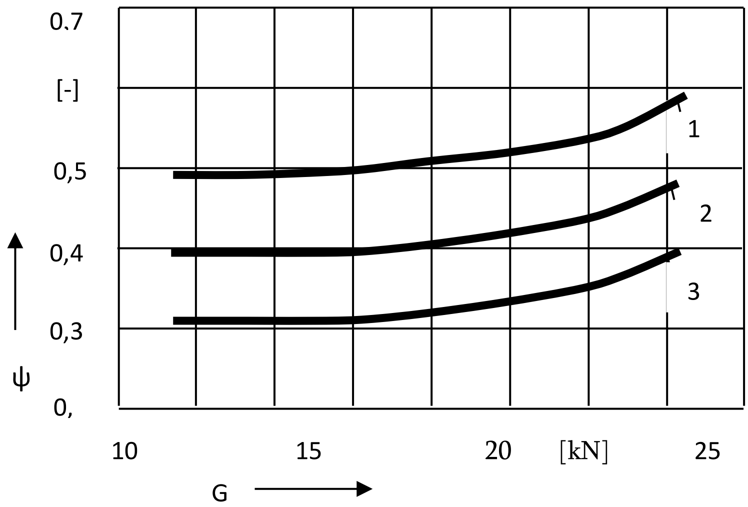

Fig. 5. A diagram of the relationship between the damping coefficient $\psi$of tire 380/R80-508, model HP-56 on a vertical load of a pneumatic wheel G. Tire internal pressures: 1-$p_w = 0.3$ [MPa]; 2-$p_w = 0.4$ [MPa]; 3-$p_w = 0.5$ [MPa]

3.2. Assessing the impact of the damping characteristics of a tire on resonant oscillations of the sprung weight

Diagrams of the amplitude-frequency characteristics of oscillations of the sprung weightwere constructed (Fig. 6). These diagrams were used to solve the problem of assessing the influence of tire dumping characteristics considering the abovementioned magnitudes of the normal load of the pneumatic wheel$G_n = 25.5$ kN, the internal air pressure in the tire, the values of tire damping coefficients determined by the bench-scale tests, and the results of calculations offormula (11).

As can be seen from the diagrams in Fig. 5, changing the magnitude of the pneumatic tire damping coefficient has a significant impact on the magnitude of the resonant oscillation amplitude of the displaced weight. For example, when the air pressure in the salt decreases from 0.5 MPa to 0.3 MPa, increasing the magnitude of the salt dampers coefficient by 30 ... 32% decreases the amplitude of the resonant oscillation of the sprung weight by an average of 18 ... 20%.

Furthermore, the results are not qualitatively different when using similar 370/R80-508 tire designs and dimensions, models HP-54 and 380/R80-508, model HP-56. The difference lies only in quantitative terms.In particular, the impact of tire 370/R80-508, model HP-54 on the reduction of the resonant amplitude values is lower (almost 12 ... 15%) than that of tire 380 / R80-508, model HP-56.

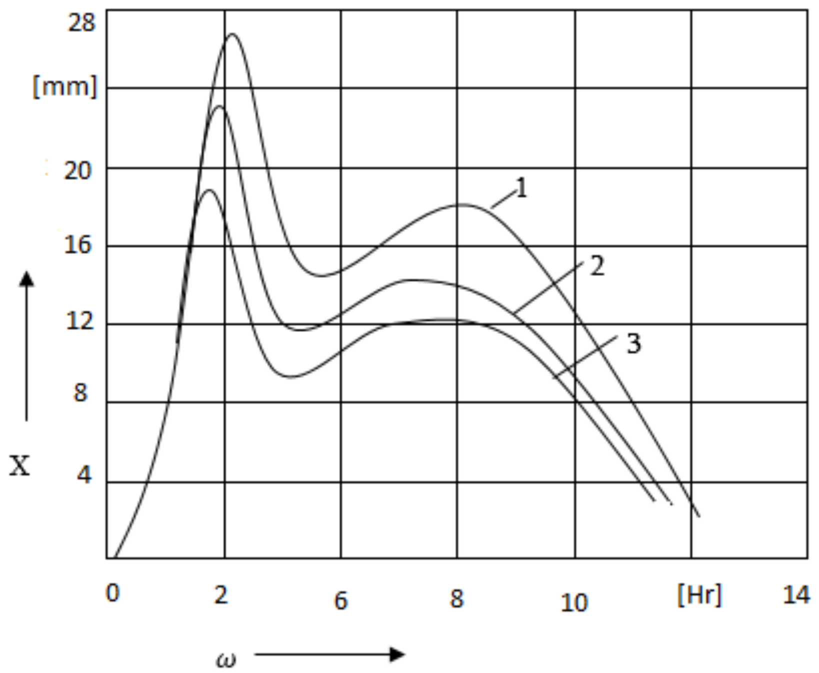

Fig. 6. The amplitude-frequency characteristic of the sprung weight of tire 380/R80-508, model HP-56. Tire internal pressures: 1-$p_w = 0.5$[MPa]; 2- $p_w = 0.4$ [MPa]; 3- $p_w = 0.3$ [MPa]

The experimental and analytical research approaches and results presented in the article allow us to select the tire with the best damper properties.

4. CONCLUSIONS

Based on the results of this study, the following conclusions can be drawn:

1. Acomparison of the results of a pilot study obtained by the methods of resonance curve and dynamic loop used todetermine the characteristic of tire damping coefficients in the vertical direction showed that the difference between them does not exceed 10%. Thus, a less time-consuming resonance curve method can be used for pilot studies.
2. The results of studies using the experimental resonance curve method show that the values of the tire damping coefficient vary widely, mainly depending on the values of the normal load on the wheel and the internal pressure of the tire. For example, when the internal air pressure in the tirewas reduced by two times, the ratio of dampers increased by 30 ... 32%. Similarly, in the case of a two-fold increase in the normal load on the pneumatic wheel, the tire damping coefficient increased by an average of 1.25 times.
3. The impact of changes in the magnitude of the tire damping coefficient on the resonant oscillations of the wheel and the sprung weight can be assessed using the mathematical model of a dynamic system “pneumatic wheel-suspension-sprung weight” on atest-bench.
4. The results of computational studies established that the impact of changes in the magnitude of the tire damping coefficient on the reduction of resonant oscillations of the sprung weight is significant. For example, when the internal air pressure in a tire decreases from $p_w = 0.5$ MPa to$p_w = 0.3$ MPa, the increase in the magnitude of damping coefficient by 30 ... 30% leads to a decrease in the amplitude of resonant oscillation of the sprungweight by an average of 18 ... 20%. Therefore, it would be a mistake to assume that the influence of the damping properties of tire is negligible in the suppression of the resonant oscillations of the sprungweight.

5. The pilot and analytical studies presented in the article allowed us to select the tire with the best damping properties.
6. The use of the experimental resonance curve method to determine the damping characteristic of a vehicle’s tire is limited in the external frequency range of the resonant frequencies. In this case, we have to use the dynamic loop method [14, 15].

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