PROBLEMY TRANSPORTU DOI: 10.20858/tp.2018.13.4.13

Keywords: routing tasks; small-lot cargo; optimal route; optimization of cargo transportation

Viktor DANCHUK, Olena BAKULICH, Vitaliy SVATKO*
National Transport University
M. Omelianovycha-Pavlenka 1, Kyiv, Ukraine, 01010
**Corresponding author*. E-mail: vitaliy_svatko@ukr.net

IDENTIFYING WAREHOUSE LOCATION USING THE RADIATION THERAPY METHOD IN LOGISTIC DISTRIBUTION SYSTEM

Summary. The paper suggests a method for determining the optimal location of service points (warehouses) based on the method for optimal planning of radiation therapy of malignant tumors. This method enabled us to identify the location of the most optimal number of warehouses taking into account their capacity for the required volume of freight transportation and distance from warehouses to consumers. The results of the study coincide with the results obtained by using the method of ant algorithm. The proposed method of finding the optimal location of warehouses enables to significantly minimize the cost of delivering goods from a producer to a consumer.

1. INTRODUCTION

Currently, there is a lot of competition in transportation market. As a result, the optimization of freight transportation is of great importance. Today much of the road transportation accounts for small-lot freight when the size of the consignment sent or received is much smaller than load capacity of the vehicle. In this regard, for companies providing delivery services, the optimal route and optimal location of the warehouse are decisive items.

Finding the optimal location of infrastructure objects solves multiple problems at once. On the one hand, the density of large cities does not allow a large number of sites to be used for the construction of warehouses and freight transshipment points. On the other hand, the suppliers are interested in reducing the number of such warehouses through the optimal location of the existing ones. In addition, in determining the optimal location of the warehouse, customers' needs should also be taken into account. Therefore, among the conditions that must be considered when determining the optimal location of the warehouse are the volume of customers' needs, the capacity of warehouses and load capacity of vehicles engaged in distribution of goods. Existing methods do not always enable us to fully meet the challenge of transportation logistics.

Based on the aforementioned, the development and improvement of the current methods for the optimal location of warehouses with allowance made for their capacity, the number of consumers, the required volume of freight transportation and distances from warehouses to consumers are still topical.

Many scientific papers focus on the issues of finding the optimal location of infrastructure objects. A review of main methods for optimal location of objects is proposed by Drezner [1]. The paper deals with the classification of widely used methods for optimal location of objects on a network. Among them there are such methods as generalized Weiszfeld, “The big triangle, small triangle”, etc. The suggested methods allow us to determine both one and several objects of optimal location. It is achieved through applying iteration procedure. A brief description, the relationship between the methods, advantages and disadvantages, and the scope of application are given in the paper. The advantages of these methods are their focus on the specific group of tasks, which in its turn makes it possible to obtain the results of high accuracy. Among the disadvantages of the suggested methods are that none of them are universal and do not meet all the requirements and demands of the market. In addition, an extremely large number of factors that must be included in the calculations can be regarded as the disadvantage.

So in the study by Sun [2], the problem of finding the optimal location of objects on a network is solved with the help of Tabu search. This method of finding the optimal location of objects consists in the phased finding of an optimal solution and comparing it with a previously found one. The speed of calculations along with high accuracy of found results is one of the advantages of the method, whereas the impossibility of simultaneously finding the optimal location of several objects on a network is one of its disadvantages.

In the paper by Kazakov and Lempert [3], it is proposed to solve the problem of optimal location of objects using "wave" method, based on the analogy between the location of global extremum of integrated functional and distribution of light in an optical heterogeneous environment.

Solving the problem of the optimal location of objects of various natures is the subject of research in different fields. The papers by Wei et al and Griffin et al [4, 5] examine ways and methods of optimization for solving the problem of optimal location for companies providing preventive health services, the so-called community health centers. For instance, the paper by Wei et al [4] proposes a new methodology for finding the optimal location of the preventive health service center, employing the algorithm Interchange. Among the factors taken into account in calculations are distance from patients to the service center, the number of patients and medical care requests, their diseases, etc. The paper by Griffin et al [5] suggests the optimization model that is designed to determine the best location, the number of medical healthcare service centers and the capacity of these enterprises. Thus, when determining the location of such facilities, the authors minimize fixed and variable costs for upkeep and maintenance of centers. The location optimization model of centers proposed in Griffin et al [5] can reduce costs and increase service quality by 20% on average.

The problems of optimal location of objects and resources have features that must be taken in consideration when formalizing such tasks and developing algorithms for accomplishing them. The paper by Sonmez and Lim [6] presents an analysis of currently existing algorithms for solving problems of optimal location of objects on the transportation network as well as their classification and algorithm solution. The authors also proposed the model of optimal location of objects whose location can change when modifying set conditions. The model makes it possible to change the location of infrastructure in the future, abandoning previously defined and finding new optimal locations without increasing set costs.

The strategic planning of a supply network to customers is emphasized in the paper by Melo, Nickel and Saldanhada [7]. As a part of the proposed mathematical model, the task of planning the location of objects aimed at serving customers is accomplished. Besides, the possibility of changing previously found optimal solutions is provided through modifying the information on costumers' needs. The grave disadvantage of the mathematical model is that it enables us to solely find an optimal solution to the tasks of only small dimension.

The paper by Taji et al. [8] considers the problem of the optimal location of structures in the city, such as a railway station system. In the paper [8], the model is presented as a tree whose vertices are railway stations and whose edges are lines connecting respective stations. The problem is to find the railway station location and create a route that is minimal for passengers. The authors proposed a heuristic algorithm that is divided into two stages. At the first stage, it determines the initial position of objects, whereas at the second, it reveals the most optimal location. The algorithm is executed until the optimal location of objects is found.

The method of optimal location of the basic container terminal based on the method of dichotomy is proposed in the paper by Kazakov and Pospelov [9]. It is suggested to apply this method for the optimal location of a warehouse.

So, the results of the literature sources analysis show that there are currently no methods that enable us to fully solve the problem of multivariable process optimization of transport logistics. In this respect, it is still topical to develop optimization techniques that are based on the methods used to describe the processes of another physical nature. Hence, in our opinion, applying the method for planning of radiation therapy of malignant tumors might be promising [10, 11]. The study by Klepper

[10] solves the problem of optimal planning of radiation therapy of malignant tumors in which it is necessary to allocate a specified number of radiation sources within the damaged area so that the cumulative effect of sources is as homogeneous as possible. The modification of the model presented in the paper by Klepper [10] is proposed in the paper by Koriashkina [11] using the elements of the theory of continuous task theory of optimal set partitioning.

With the aim to further develop the methodology for optimizing the location of warehouses and routing processes of freight transportation, the paper suggests the intelligent method for solving such kind of problems. It is based on the method for optimal planning of radiation therapy of malignant tumors [10] and its modification [11].

2. EMPLOYING THE METHOD FOR OPTIMAL PLANNING OF RADIATION THERAPY OF MALIGNANT TUMORS FOR SOLVING THE PROBLEMS OF THE OPTIMAL LOCATION OF WAREHOUSES

2.1. A Background: the method for optimal planning of radiation therapy of malignant tumors

According to the study by Klepper [10], in the problem of optimal planning of radiation therapy of malignant tumors, it is necessary to allocate a specified number of radiation sources within the damaged area. In the interstitial radiation therapy to achieve full therapeutic effect (tumor disease-free recovery), it is essential to place radiation sources in the tumor in such a way that its dose field should be as homogeneous as possible. The point is that at low levels of radiation (in local minima field of influence), there is a relapse possibility, and vice versa, in the high-dose radiation, there might be radiation necrosis, which are treated with difficulty.

So, the problem will be interpreted as follows: "field service" is affected area of the body whose cells are "customers", and "service points" are the sources of radiation placed inside the affected area, creating therapeutic radiation field neutralizing lesions. Besides, we assume that lesions in different parts of the skin may be different. The problem is to place a specific number of radiation sources in such a way that dose field (cumulative effect of sources) is as homogeneous as possible.

Let G be a limited area of service in k-dimensional space R, points of which are labelled as x. Let us denote $\tau_i = (\tau_i^1, ..., \tau_i^n) \in G, i = \overline{1, N}$, the sources of influence on the environment, which should be placed. Let the influence of each i-source at the point $x \in G$ be described by the function $d_i(r_i) = d_i(\|x - \tau_i\|), i = \overline{1, N}$ where $\|.\|$ is the Euclidean norm. The combined effect of all sources $\tau_i, i = \overline{1, N}$ at the point $x \in G$ forms a field of action $D(\tau, x)$, described by the function [10]:

$$D(\tau, x) = \sum_{i=1}^{N} d_i(\|x - \tau_i\|) \quad . \tag{1}$$

It ($\tau_i, i = \overline{1, N}$) should be placed in such a way that it minimizes the level of field $D(\tau, x)$ in this area G as much as possible. In other words, it is necessary to place the sources so that the field throughout the area G is as homogeneous as possible. Mathematically, this requirement is written as follows:

$$\min_{x \in G} D(\tau, x) \to \max_{\tau \in G^N}. \tag{2}$$

Unlike the model [10], where the function of source influence $d_i(r_i)$ was chosen as a power function $d_i(r_i) = \frac{1}{r_i^{\theta}}$ when $\theta = 2$, the paper, according to Koriashkina [11], proposes to consider the following function:

$$d_i(r_i) = Q_i \exp(-\alpha_i r_i), \tag{3}$$

where Q_i is the intensity (power) of i-source; $\alpha_i > 0$ is the function parameter showing how "wide" the performance of the source τ_i is (it is selected experimentally); $r_i = \|x - \tau_i\|, i = \overline{1, N}$. A choice of the type of the function of source influence is based on the following considerations. Power functions of type $d(r) = \frac{1}{r^\theta}, \theta > 0$ meeting the condition $d_i(+0) = \infty$ have an unpleasant feature that requires the removal of these points when calculating the function (1) implementing the algorithm for solving the task [11]. This greatly complicates or makes almost impossible to use maximization of numerical methods that worked well even in solving problems of undifferentiated optimization, the convergence of which is theoretically proved. The function of the (3) type is devoid of such disadvantages.

2.2. Employing the method for optimal planning of radiation therapy of malignant tumors for solving the problems of the optimal location of warehouses

The proposed method in studies by Klepper and Koriashkina [10, 11] solves the problem of the optimal location of radiation sources of malignant tumors inside the damaged area so that the cumulative effect of sources is as homogeneous as possible. From another point of view, this method can be employed to solve the problem of the optimal location of infrastructure objects such as warehouses, shops, repair shops, and first-aid stations. In this context, the problem of the optimal location lies in bringing the objects of infrastructure nearer to a consumer. In our case, we will regard infrastructure objects as warehouses, and consumers as supermarkets. Limited area *G* is a service field whose points are supermarkets whereas infrastructure objects are warehouses which are located inside limited area *G*. The problem consists in locating warehouses closest to supermarkets.

Let the limited area of service *G* have limited *n* set of delivery points (warehouses) $(A_1, A_2, ..., A_i, ..., A_n)$, in which there is the same cargo in the quantity of $a_1, a_2, ..., a_i, ..., a_n$ units. The total volume of *n* warehouses *a* makes up

$$a = \sum_{i=1}^{n} a_i \quad . \tag{4}$$

On the other hand, we have a limited service area *G* with *m* limited set of customers (supermarkets) $(B_1, B_2, ..., B_j, ..., B_m)$ whose demand is $b_1, b_2, ..., b_j, ..., b_m$ units, respectively. However, the total volume of demand for goods is as follows:

$$b = \sum_{j=1}^{m} b_j \quad . \tag{5}$$

We believe that the transportation from each supply point to each consumption point is possible. C_j denotes the total transportation costs associated with shipping freight unit from the point of delivery A_i to consumers $B_j (i = 1, n; j = 1, m)$. The influence of each *i*-warehouse at the point $x \in G$ is described by function (3).

Each warehouse is characterized by the location with coordinates $(x_i; y_i)$ and the distance to each consumer (supermarket) as well as the maximum capacity (the amount of cargo that can be stored at this facility). Each buyer is characterized by the location coordinates $(x_j; y_j)$, the distances to each supplier and the size of demand (the volume of delivered product). The task lies in locating warehouses for a specific number of customers in such a way that they are situated most optimally to suppliers (i.e., as close as possible to them). That means that it is necessary to opt for warehouses from existing ones $(A_1, A_2, ..., A_i, ..., A_n)$ whose total transportation costs C_{ij} for goods delivery to all consumers will be minimal.

Initially, we find the optimally located (closest) to all consumers for existing *n* warehouses. At the same time we presume that the found warehouse is of such a capacity which can meet all the demands of consumers. The calculation uses such an exponent as Q_i - intensity (power) of *i*-source (see 2.1). In our case, the value of this exponent is determined by inversely proportional to the value of work performed during cargo transportation from a certain warehouse to all consumers. The size of transport work performed is determined by formula W

$$W = G_{fr} S_{fr} \quad , \qquad (6)$$

where G_{fr} - is cargo weight and S_{fr} - is mileage with freight.

Function parameter α_i, which shows how wide the effect of source τ_i is, is selected experimentally. In our case, the distances between warehouses and supermarkets do not exceed 30 km (see Table 2). This means that the action of the field of relevant source (warehouse) τ_i, which consists in the ability to deliver goods from the corresponding warehouse to any of the supermarkets, should extend to the area $G \in [0, 30]$ km. By experimentally searching for the value of the parameter of the function α in (3), at which the optimal convergence of the objective function D (τ, x) in (2) is observed for the optimal number of iterations, it was determined that α = 0.2. In this case, the number of iterations in all cases did not exceed 15, and the rate of convergence is 5 s.

So, using the algorithm for solving the optimization problem (1) - (6) allows us to find the optimal location of the warehouse, which completely meets the needs of all consumers. Let it be warehouse A_t whose capacity is a_t units.

However, we know that when planning the location of warehouses to meet the needs of consumers in a metropolis, there is often a situation when the capacity of one warehouse makes it impossible to fully meet the needs of all customers (supermarkets) in a limited area of service.

Then we take the case when the capacity of warehouse A_t does not satisfy the total demand for the goods of the volume *b*. We choose the minimal distance between two respective objects as a criterion, which will be used to find consumers served by optimally found warehouse A_t. Thus, this warehouse will serve only those consumers who are at the minimal distance from it, and its total capacity will not exceed the consumers' demand.

Let warehouse A_t completely meet the demand for goods of *k* consumers from the set $\{B_m\}_i$ with total value

$$Y = \sum_{l=1}^{k} y_l \quad , \qquad (7)$$

where $(y_1, y_2, ..., y_k)$ is the individual demand of *k* consumers. At the same time

$$a_t = Y, k < m \quad . \qquad (8)$$

As the optimally found warehouse with its capacity completely satisfies the condition (8), there is a need to find an additional warehouse to meet the demand of other consumers. Again the next optimal warehouse A_t for delivering goods to consumers is found through algorithm (1) – (6), and now, we consider the set $\{A_{n-1}\}$ in which warehouse A_t with the capacity a_t is not included and the set $\{B_{m-k}\}$ in which *k* consumers with the demand according to (7) is not included too.

If as a result of the optimization, the next found warehouse satisfies the conditions in (8) then the procedure (1) – (6) is repeated until the number of optimal warehouses *S* with total capacity YS is found, which satisfies the condition $YS = b$.

3. CASE STUDY: SOLVING THE PROBLEM OF THE OPTIMAL LOCATION OF WAREHOUSES USING THE METHOD OF RADIATION THERAPY

As the problem of optimizing road light freight transportation in megacities has been especially topical over the recent years, we deem that short shelf-life food delivery from a producer to a consumer is the most representative variant of such kind of shipment. The problem of road light freight transportation is typical of densely populated cities which, in its turn, are characterized by congested transport networks, high density of traffic and a large number of customers (supermarkets). A plot of land in Kyiv, which is shown in Fig. 1, was selected as an example for solving the problem of the optimization, using the proposed method.

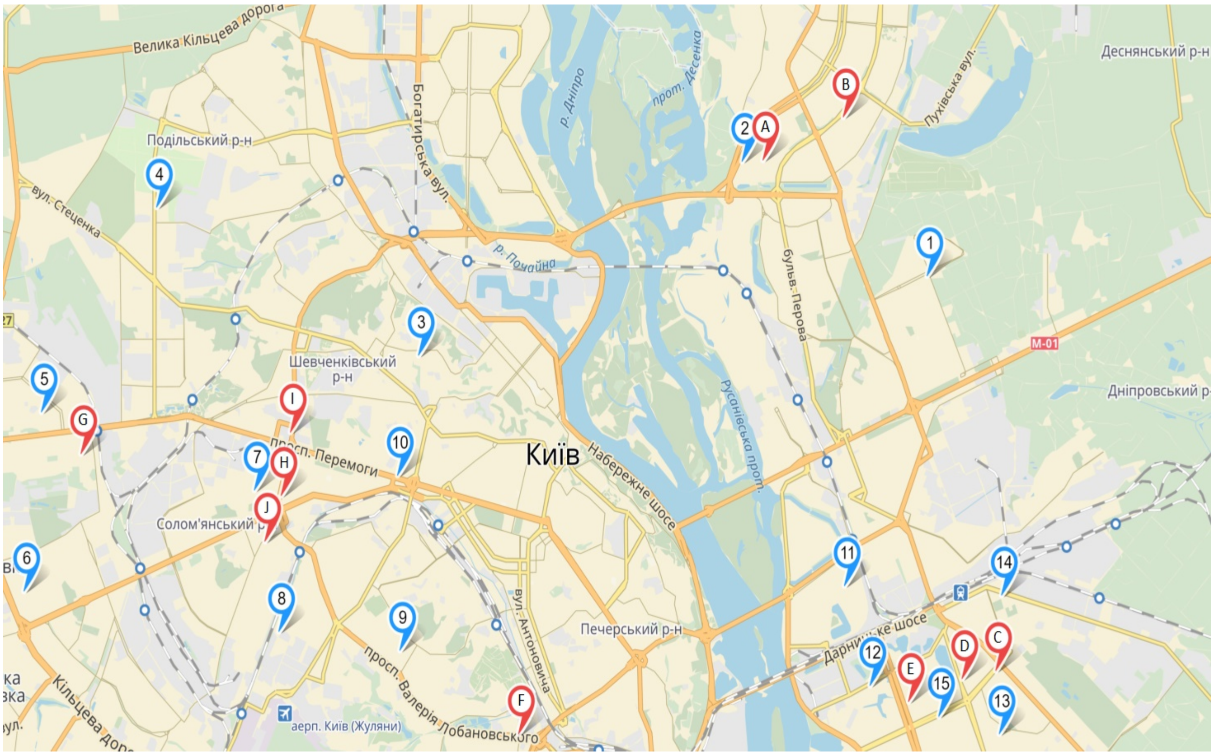

Fig. 1. The location of warehouses and supermarkets in Kyiv: 1-15 supermarkets, A-J warehouses

As our system includes the main manufacturer, distribution centers (warehouses) and end users (supermarkets), we can talk about the classic model of the logistic system. The schematic representation of the model of the logistic system is shown in Fig. 2.

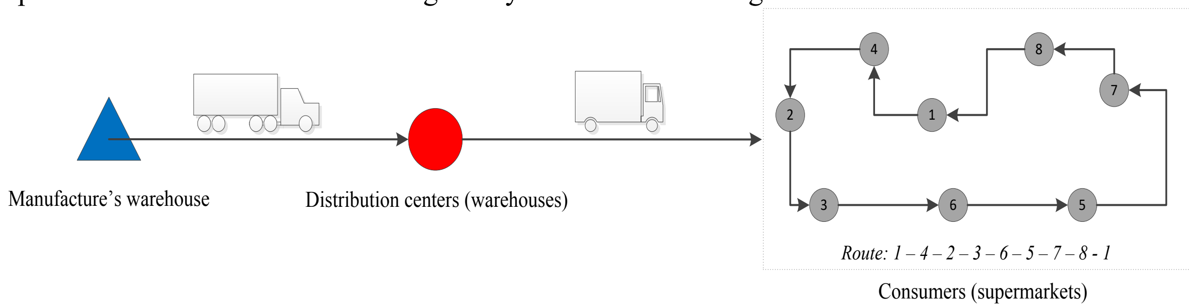

Fig. 2. The physical model of the distribution system will supply products with one distribution center

Let us have as output data in this area of the territory of 10 warehouses (distribution centers) and warehouse in the territory of the manufacturer of products, located in the city of Kiev. Each warehouse is characterized by location (coordinates), the distance to each customer (supermarket) and a maximum capacity (the amount of cargo that can be stored at this facility). In addition, there are 15 customers (in this case they are supermarkets). Data on consumers' demand (supermarkets) and the capacity of suppliers (warehouses) shown in Table 1. Delivery of goods is carried out according to the scheme of the main manufacturer - distribution centers - supermarkets.

The information about the distance between customers and warehouses as well as the weight of freight is shown in Table 2.

In determining the total cost of delivery of goods in the logistics system, the following groups of costs are taken into account: transport costs and expenses for driver's labor.

In determining the optimal location of the warehouse in the logistics distribution system, the total cost of delivering goods from the manufacturer to the consumer is divided into two parts: the cost of delivering goods from the manufacturer to distribution centers and the cost of delivering goods from distribution centers to end consumers (supermarkets). Considering the costs in this form, in our opinion, will enable the complex solution of the problem of finding the optimal location of the warehouse, for the entire system of distributed logistics.

Table 1

Customers' demand and suppliers' capacity

Marking of warehouses	**Address**	***Weight [kg]**	**Marking of supermarkets**	**Address**	***Weight [kg]**
MAIN	Brovarsky avenue, 16 km	45000	1	Lisoviy avenue, 28	500
A	Kashtanova street, 7	10000	2	Onore de Balzaka street, 6	750
B	Mayakovskiy avenue, 26	18000	3	Polovetska street, 14	1100
C	Kharkivs'ke shose street, 152a	17000	4	Marshala Hrechka street, 22	600
D	Revutskogo street, 15	22000	5	M. Krasnova street, 19	1200
E	Hryhorenka avenue, 5a	19000	6	Tuluzy street, 3B	500
F	V. Lobanovskoho avenue, 119	25000	7	Mashynobudivna street, 27	900
G	Verkhovynna street, 7	17000	8	Ushynskoho street, 27	1000
H	Borschahivska street, 154	13000	9	Preobrazhenska street, 27	400
I	O. Dovzhenka street, 1B	9000	10	Peremohy avenue, 18	1600
J	M. Holeho street, 7B	11000	11	P. Tychyny avenue, 15	1100
			12	Zdolbunivska street, 7	2200
			13	Dekabrystiv street, 2	900
			14	Sormovska street, 13	750
			15	A. Akhmatovoi street, 9/18B	2100
Total		**161000**	**Total**		**15600**

Note: *in the field Weight there is capacity for warehouse while there is freight demand for supermarkets

Using formulae (1) – (6), we found an optimally located warehouse among existing ones to consumers, that is, warehouse C, which is located at the address Kharkivs'ke shose street, 152a. The capacity of the determined warehouse C is 17000 kg, which is enough to cover the daily needs of consumers in the goods. The demand of all customers is 15600 kg.

After determining the optimal location and quantity of warehouses, it is necessary to create an optimal route for distributing goods from a warehouse to customers. To determine the optimal route of goods delivery from a warehouse to customers, the ant-colony method was used, as described in Dorigo, Maniezzo, and Colorni and Shtovba [12, 13]. As there was (1) – (8) 1 warehouse previously determined for the distribution of goods then there will be 1 delivery route.

Employing the ant-colony method given in Dorigo, Maniezzo, and Colorni and Shtovba [12, 13], the following optimum (minimum) delivery route from the distribution warehouse to the supermarkets was created (designation of the warehouse and supermarkets is shown in fig. 1 and in the table 1).

Table 2

Matrix of distance between warehouses and supermarkets, demand for freight weight and the volume of the work performed

	T	A		B		C		D		E		F		G		H		I		J	
		km	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**	**km**	**tkm**
1	0.5	5.7	2.85	5.2	2.6	9.3	4.65	10.3	5.15	10.6	5.3	15.7	7.85	23.4	11.7	18.8	9.4	17.4	8.7	20.6	10.3
2	0.75	1.1	0.825	3.6	2.7	12.2	9.15	13.3	9.975	13.5	10.12	19.3	14.48	18.9	14.18	14.3	10.73	12.9	9.675	16.1	12.08
3	1.1	12.8	14.08	14	15.4	18.5	20.35	18.5	20.35	17	18.7	12.1	13.31	10.7	11.77	5.7	6.27	4.3	4.73	7.5	8.25
4	0.6	15	9	16.2	9.72	28.5	17.1	24.7	14.82	23.1	13.86	15.2	9.12	6.6	3.96	7.9	4.74	6.6	3.96	9.8	5.88
5	1.2	19.6	23.52	20.3	24.36	24.1	28.92	26.9	32.28	24.6	29.52	14.5	17.4	2.4	2.88	7.2	8.64	7.6	9.12	9	10.8
6	0.5	22.8	11.4	22.2	11.1	28.2	14.1	26.7	13.35	24.4	12.2	13.6	6.8	4	2	6.2	3.1	7.3	3.65	8.2	4.1
7	0.9	17	15.3	18.2	16.38	23.2	20.88	22.4	20.16	20	18	9.3	8.37	6	5.4	1.8	1.62	1.6	1.44	3.9	3.51
8	1	18.1	18.1	19.4	19.4	19.3	19.3	19.6	19.6	17.3	17.3	6.5	6.5	10	10	3.6	3.6	4.4	4.4	3.2	3.2
9	0.4	19.6	7.84	20.8	8.32	16.1	6.44	16	6.4	14.5	5.8	3.7	1.48	11.5	4.6	5.1	2.04	6	2.4	4.7	1.88
10	1.6	14	22.4	15.3	24.48	16.1	25.76	16.2	25.92	14.7	23.52	8.2	13.12	9.1	14.56	4.8	7.68	5.1	8.16	5.4	8.64
11	1.1	9.8	10.78	10.7	11.77	6	6.6	6.5	7.15	5	5.5	9.6	10.56	20.5	22.55	16.3	17.93	17.7	19.47	16.4	18.04
12	2.2	12.5	27.5	12.5	27.5	3.5	7.7	2.8	6.16	1.3	2.86	10.5	23.1	21.5	47.3	17.8	39.16	18.6	40.92	17.3	38.06
13	0.9	13.5	12.15	13.6	12.24	2.5	2.25	3.3	2.97	3.7	3.33	12.2	10.98	25.9	23.31	19.5	17.55	20.3	18.27	19	17.1
14	0.75	12.4	9.3	12.5	9.375	1.7	1.275	2.6	1.95	4.3	3.225	13.3	9.975	24.3	18.23	20.6	15.45	21.4	16.05	20.1	15.08
15	2.1	13.8	28.98	13.9	29.19	4.9	10.29	4.2	8.82	1.6	3.36	11.2	23.52	25.2	52.92	18.8	39.48	19.6	41.16	18.4	38.64
To-tal	15.6	207.7	214.03	218	224.54	214.1	194.77	214	195.1	196	172.6	174.9	176.6	220	245.4	168	187.4	171	192.1	180	195.6

Note: * addresses of supermarkets are given in Table 1.

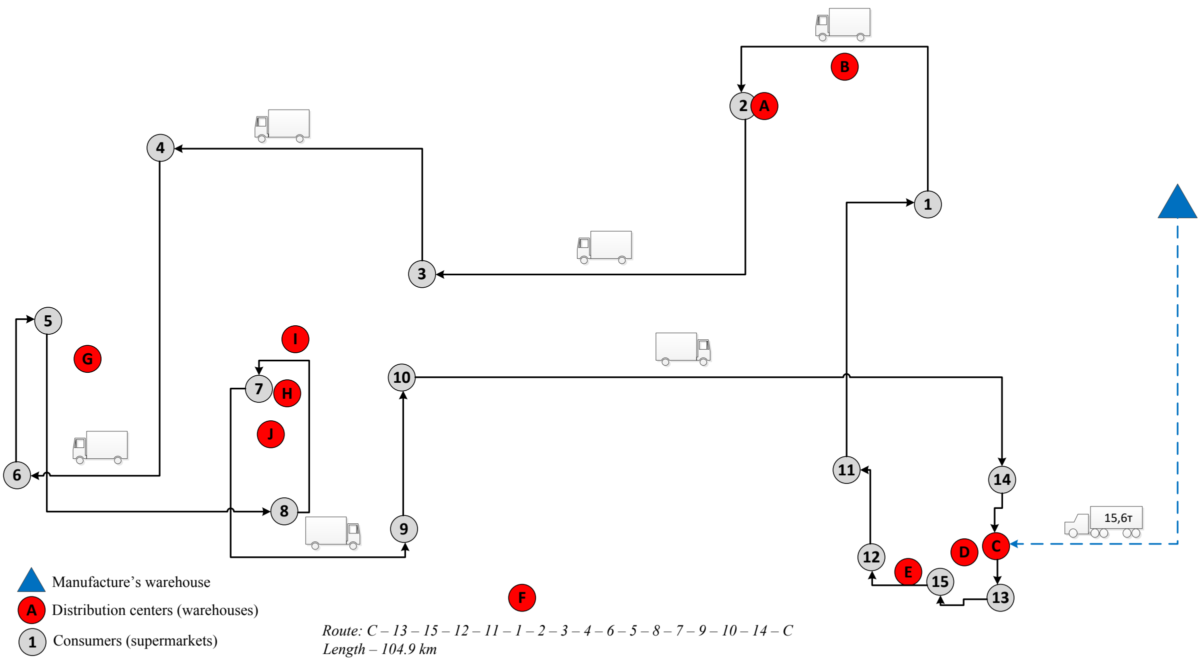

Fig. 3. The route of delivery of goods from the distributed center C to consumers is 1-15

Goods delivery route (Fig. 3): C – 13 – 15 – 12 – 11 – 1 – 2 – 3 – 4 – 6 – 5 – 8 – 7 – 9 – 10 – 14 – C (route length is 104,9 km).

Fig. 3 shows the schematic location of warehouses and supermarkets, and the route of delivery of goods from a specific warehouse to consumers.

After finding both the optimal location of the warehouses and their number by using the ant algorithm, the optimum (minimum) route of goods delivery from distribution centers (warehouses) to consumers was created.

4. RESULTS AND DISCUSSION

To check the accuracy of the applied method for optimizing the location of warehouses within transport logistics, the ant-colony method was employed [12, 13] which enabled us to create the optimal goods delivery route from a warehouse to a customer as a part of traveling salesman problem.

To do this, consider the same model problem as in section 3. In this task, there is a main warehouse located on the territory of the manufacturer of products, 10 distribution centers (warehouses) and 15 supermarkets. It is necessary to determine from those warehouses the total cost of delivery of goods from which to consumers will be minimal. Costs for the delivery of goods are divided into two groups: from the main warehouse of the manufacturer to the distributed centers and from the distributed centers to the supermarkets. The total cost of delivery of goods includes transport costs and driver's expenses. Thus, we consider the optimal location that composition and the cost of delivery of goods to all consumers will be minimal.

With the help of ant algorithm method for each of 10 warehouses, we find the shortest route of goods delivery to all 15 supermarkets. Using the data on the distance between the warehouses and supermarkets as well as the demand for cargo of each supermarket, shown in table 1 and 2, we calculate the volume of the transport work performed on each route.

As the weight of the load in the model problem is 15600 kg, for transportation vehicles models used MAN TGA (20000 kg) tractor. To do this, the fuel consumption data of the car were set by the company in accordance. The only time limits for the carriage of goods by road and lump-sum charges for the payment of drivers [14]. They are as follows: fuel consumption per unit distance –

$30 \times 10^{-8} m^3 / m$; fuel consumption per unit performance (transported tons of freight) – $1 \times 10^{-11} m^3 / kgm$, fuel consumption for the operation of the refrigerator $4 \times 10^{-11} m^3 / kgm$, type of fuel - "Diesel". The average cost of such fuel in the city of Kiev in October 2017 is $25.05 \times 10^3 UAH / m^3$.

Employing the data on fuel cost for the chosen model of transport vehicle and the average cost of fuel in the market, we calculate the total expenditure on the goods delivery from each warehouse to all the consumers.

In determining the optimal location of the distributed center, an important role in the total amount of expenses is played by the driver's labor costs. According to statistics [15] the average wage of a truck driver in the city of Kyiv in October 2017 is UAH 12,000. The standard of working hours for a five-day 40-hour working week [16] in October 2017 is 167 hours. The hourly wages of the driver are established at the enterprise that carries out the product outlet. The cost of 1 hour of the driver's work in October 2017 is 71.86 UAH. The duties of the driver include only the registration of documents, the dispatch of goods by destination and control over compliance with the rules of loading / unloading and acceptance / delivery of cargo. Thus, the time spent by the driver includes the time of loading or unloading the goods, time for document processing, time of delivery of goods (time for route passing between points) and maneuvering time. Normative data for finding the time for each type of work is given in Calculation of the norm of working time for 2017 Ukraine [16]. To calculate the time of delivery of goods, the average speed of traffic in the city of Kyiv was taken - 25 km/h. The results of the calculations are presented in Table 3.

It should be noted that in our model problem, only the driver's option is considered in the framework of his normal working day, which does not involve working out overnight, evening, night or holiday hours.

As can be seen from the calculation results, shown in table 3, the warehouse C, which is located at Kharkivs'ke shose street, 152a, is most optimally situated out of the 10 proposed ones. The cost of delivery from the aforementioned warehouse is minimal and equals 3227.58 UAH. The highest cost of goods delivery from the warehouse, which is located at M. Holeho street, 7B is 3838.35 UAH. In the paper, applying method of radiation therapy, we determined warehouse C which is at Kharkivs'ke shose street, 152a, as most optimally located to all the supermarkets. So, finding the optimal warehouse in a limited service area with the help of method of radiation therapy or the ant algorithm method gives the same results.

As the comparative analysis shows, finding optimally located warehouses through the method for optimal planning of radiation therapy of malignant tumors by Klepper [10] and the ant-colony method Dorigo [12, 13] produces the same result. That testifies the accuracy of the proposed method for finding optimally located service points, which is based on the method for optimal planning of radiation therapy of malignant tumors.

5. CONCLUSIONS

The ever-increasing demand for goods and the high intensity and density of vehicles in large cities (metropolises) force commodity producers to effectively plan logistics operations, taking into account these factors. In particular, the solution of warehousing issues in the logistic system to date enables the goods manufacturers to provide the necessary level of customer service at the least total costs. Particular attention is paid to the problems of finding the optimal size and location of warehouses.

In this paper, the method of determining the optimal location of service points (warehouses) is proposed based on the method of optimal planning of radiation therapy for malignant tumors. The developed method allows to determine from the available warehouses the most optimal number of warehouses taking into account their power and distance to consumers.

For approbation of the developed method, a fragment of the street and road network of the city of Kyiv was selected, where the delivery of small-piece cargo, namely, food products of short-term shelf life from producer to consumers (supermarkets), was selected.

Table 3

Costs of delivery of goods depending on the location and number of warehouses in the logistic system

Cost group	**Distribution centers**									
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Costs of delivery from the manufacturer to distribution centers										
Distance, km	14	14	15	15	16	19	31	27	26	27
Weight of cargo, t	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6
Work, tkm	218.4	218.4	234	234	249.6	296.4	483.6	421.2	405.6	421.2
Work of a refrigerator, hours	0.91	0.91	0.98	0.98	1.04	1.24	2.02	1.76	1.69	1.76
Fuel consumption mileage, l	8.4	8.4	9	9	9.6	11.4	18.6	16.2	15.6	16.2
Fuel expense work, l	2.4	2.4	2.57	2.57	2.75	3.26	5.32	4.63	4.46	4.63
Fuel consumption of a refrigerator, l	3.64	3.64	3.9	3.9	4.16	4.94	8.06	7.02	6.76	7.02
Total fuel, l	14.44	14.44	15.47	15.47	16.51	19.6	31.98	27.85	26.82	27.85
Loading / unloading time, min	80	80	80	80	80	80	80	80	80	80
Time of documents processing, min	55	55	59	59	62	74	121	105	101	105
Total time spent, hours	4.49	4.49	4.62	4.62	4.75	5.14	6.7	6.18	6.05	6.18
Fuel cost, UAH	361.78	361.78	387.62	387.62	413.47	490.99	801.09	697.72	671.88	697.72
Driver's cost, UAH	322.4	322.4	331.74	331.74	341.08	369.1	481.2	443.83	434.49	443.83
Total costs, UAH	*684.18*	*684.18*	*719.36*	*719.36*	*754.54*	*860.09*	*1282.29*	*1141.55*	*1106.37*	*1141.55*
Costs of delivery from distribution centers to supermarkets										
Distance, km	107	114	104.9	108	109.8	109.4	106.9	107.5	107.5	113.1
Weight of cargo, t	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6	15.6
Work, tkm	1669.2	1778.4	1636.44	1684.8	1712.88	1706.64	1667.64	1677	1677	1764.36
Work of a refrigerator, hours	6.96	7.41	6.82	7.02	7.14	7.11	6.95	6.99	6.99	7.35
Fuel consumption mileage, l	32.1	34.2	31.47	32.4	32.94	32.82	32.07	32.25	32.25	33.93
Fuel expense work, l	18.36	19.56	18	18.53	18.84	18.77	18.34	18.45	18.45	19.41
Fuel consumption of a refrigerator, l	27.82	29.64	27.27	28.08	28.55	28.44	27.79	27.95	27.95	29.41
Total fuel, l	78.28	83.4	76.74	79.01	80.33	80.04	78.21	78.65	78.65	82.74
Loading / unloading time, min	80	80	80	80	80	80	80	80	80	80
Time of documents processing, min	417	445	409	421	428	427	417	419	419	441
Total time spent, hours	8.29	8.74	8.15	8.35	8.47	8.44	8.28	8.32	8.32	8.68
Fuel cost, UAH	1960.94	2089.23	1922.46	1979.27	2012.26	2004.93	1959.11	1970.11	1970.11	2072.74
Driver's cost, UAH	595.57	628.26	585.76	600.24	608.65	606.78	595.10	597.9	597.9	624.06
Total costs, UAH	*2556.51*	*2717.49*	*2508.22*	*2579.51*	*2620.91*	*2611.71*	*2554.21*	*2568.01*	*2568.01*	*2696.8*
Total costs	3240.69	3401.67	**3227.58**	3298.87	3762.46	3471.80	3836.50	3709.57	3674.38	3838.35

The problem of road transport of small-tonnage cargoes is characteristic for large densely populated cities, which in turn are characterized by high traffic load of transport networks, high density of traffic flow and a large number of consumers (supermarkets). The task of determining from all available warehouses the most optimal number of warehouses, taking into account their capacity and distance to consumers, was solved for a given number of warehouses and supermarkets.

To verify the correctness of the developed method of searching the optimal arrangement of warehouses for consumers in the logistic system, the method of the modified ant algorithm was used. The results of the study with the proposed method coincide with the results obtained using the method of the modified ant algorithm.

The results of the research show that the proposed method of finding an optimal location of warehouses can significantly reduce the cost of delivering goods from the manufacturer to the consumer. This method can also be used to select the location of distribution centers or production facilities.

The paper proposes a new method for determining the optimal number and location of warehouses, which requires additional testing in the real activity of different enterprises. Therefore, there is a need for further research to determine the limits of the application of this method and the possibilities for its improvement. In addition, subsequent studies may be focused on taking into account more factors that influence the location of warehouses and reduce the total cost of delivering goods from manufacturer to consumer.

References

1. Drezner, T. A review of competitive facility location in the plane. *Logistics Research.* 2014. Vol. 7. No. 114. P. 1-12. Available at: https://link.springer.com/content/pdf/10.1007%2Fs12159-014-0114-z.pdf
2. Sun, M. Solving the incapacitated facility location problem using tabu search. *Computers & Operations Research.* 2006. Vol. 33. No. 9. P. 2563-2589.
3. Казаков, А.Л. & Лемперт, А.А. Об одном подходе к решению задач оптимизации, возникающих в транспортной логистике. *Автоматика и телемеханика.* 2011. Vol. 7. P. 50-57. [In Russian: Kazakov, A.L. & Lempert, A.A. On one approach to solving optimization problems arising in transport logistics. *Automation and Remote Control*].
4. Wei, Gu & Xin, Wang & McGregor, S.E. Optimization of preventive health care facility locations. *International Journal of Health Geographic's*. 2010. Vol. 9. No. 1. P. 9-17.
5. Griffin, P.M. & Scherrer, C.R. & Swann, J.L. Optimization of community health center location sand service off erring's with statistical need estimation. *IIE Transactions.* 2008. Vol. 40. No. 9. P. 880-892.
6. Sonmez, A.D. & Lim, G.J. A decomposition approach for facility location and relocation problem with uncertain number of future facilities. *European Journal of Operational Research*. 2012. Vol. 218. No. 2. P. 327-338.
7. Melo, M.T. & Nickel, S. & Saldanhada, F.G. Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. *Computers and Operations Research*. 2006. Vol. 33. P. 181-208.
8. Taji, T. & Tanigawa, S. & Kamiyama, N. & Katoh, N. & Takizawa, A. Finding an Optimal Location of Line Facility using Evolutionary Algorithm and Integer Program. 2008. *Journal of Computational Science and Technology*. Vol. 2. No. 3. P. 362-370.
9. Казаков, А.Л. & Поспелов, А.М. Определение оптимального местонахождения базового контейнерного терминала. *Транспорт Урала*. 2008. Vol. 2. No. 17. P. 57-64. [In Russian: Kazakov, A.L. & Pospelov, A.M. Determining the optimal location of the base container terminal. *Transport Urala*].
10. Клеппер, Л.Я. Оптимизация поля действия конечного числа источников в непрерывной среде. *Экономика и математические методы.* 2009. Vol. 2. No. 45. P. 113-119. [In Russian:

Klepper, L.Ya. Optimization of the field of action of a finite number of sources in a continuous medium. *Economics and mathematical methods*].

11. Коряшкина, Л.С. Об одной модели задачи оптимизации размещения и поля действия конечного числа источников в непрерывной среде. In: *Сборник материалов IX Международной научной конференции студентов и молодых ученых «Наука и образование 2014»*. Астана, 2014. P. 5823. [In Russian: On one model of the problem of optimizing the location and field of action of a finite number of sources in a continuous medium. *Collection of materials of the IX International Scientific Conference of Students and Young Scientists "Science and Education 2014"*].
12. Dorigo, M. & Maniezzo, V. & Colorni A. The Ant System: Optimization by a Colony of Cooperating Agents. *IEEE Trans. on Systems. Man and Cybernetics. Part B*. 1996. Vol. 1. No. 26. P. 29-41.
13. Штовба, С.Д. Муравьиные алгоритмы. *Exponenta Pro. Математика в приложениях.* 2003. Vol. 4. P. 70-75. [In Russian: Shtovba, S.D. Ant algorithm. *Exponenta Pro. Math in applications*].
14. *Єдині норми часу на перевезення вантажів автомобільним транспортом і відрядні розцінки для оплати праці водіїв.* Документ v0142400-87, поточна редакція. Прийняття від 13.03.1987. Available at: http://zakon3.rada.gov.ua/laws/show/v0142400-87. [In Ukrainian: *The only time limits for the carriage of goods by road and lump-sum charges for the payment of drivers*].
15. *Рівень середньої заробітної плати для водіїв у місті Києві*. Available at: http://kiev.trud.com/salary/12/76922.html. [In Ukrainian: *The average wage level for drivers in the city of Kiev*].
16. *Розрахунок норми тривалості робочого часу на 2017 рік в Україні*. Available at: http://www.msp.gov.ua/news/8632.html. [In Ukrainian: *Calculation of the norm of working time for 2017 Ukraine*].

Received 05.03.2017; accepted in revised form 05.12.2018