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MATHEMATICAL MODEL OF OSCILLATIONS OF BEARING BODY FRAME OF EMERGENCY AND REPAIR RAILCARS

Summary. Nowadays, the importance of maintenance and effective use of available railcars in the railway transport is growing, and researchers and technical experts are working to address this issue with the use of various techniques. The authors address the use of analytical technique, which includes mathematical solutions for flexural and longitudinal fluctuations of the bearing framework of a railcar body frame. The calculation is performed in connection with the modernization of the body frame of emergency and repair rail service car, taking into account the variability in section, mass, longitudinal stiffness, and bending stiffness. It allows for extension of the useful life of their operation, with special focus on vehicles owned by Joint-Stock Company "Uzbekistan Railways".

The simulation of equivalent bearing body frame of emergency and repair rail service car was carried out using an elastic rod with variable parameters including stiffness and mass. The difference between the proposed model and the existing ones is due to the variability in cross section, mass, and the longitudinal and bending stiffness along the length of equivalent beam, which corresponds to the actual conditions of operation and data of the experimental studies conducted by the authors on the bearing frames of electric locomotives' variable sections.

The frequency analysis that was carried out with the use of the Mathcad 14 programming showed that the frequencies of natural oscillations change on *n* harmonicas = 1, 2, 3 … 5. As regards longitudinal oscillations of system, in case of introduction of the damping subfloor, the frequency of natural oscillations of the upgraded rail car frame λ_{1mn} increases on comparing with standard λ_{1n} (for example, in case of n = 5 the frequency is 0.587 and 0.602 Hz/m, respectively).

1. INTRODUCTION

In conditions of global financial and economic crisis, it is becoming important to address the issues on increasing the reliability of railway equipment by upgrading the individual structural assemblies during capital repair. Accordingly, it will allow extension of its useful life and performance. Thus, according to the norms of depot repair, cracks continue to develop and grow in size, weakening the most dangerous sections. It is obvious that the general state of body frame, spring suspension, and running gear of the rolling stock with tension will significantly depend on the initial bending of neutral axis and permanent dynamic forces. These factors cause a decrease of 1.2-1.5 times of total life of railcars (rail service car).

In modern academic and scientific literature, the challenges in increasing the reliability and strength of the frames, load-bearing body structures, and components of rail vehicles during their design, operation, and modernization are extensively studied [1, 2]. This study introduces an analytical–numerical method based on the dynamic strength of the bearing body frame of emergency and repair rail service car. The method assumes a beam-type pattern of its fluctuations with elastic fixing of its ends under harmonic load as it moves along the rail track with periodic roughness.

The main objectives for development of new designs of mechanical body components, running gear, and spring suspension systems of rail service cars and for modernization of existing ones are to increase reliability, strength, and durability. For example, with the advent of motor cars or railway transport vehicles, issues related to research and improvement of bearing body of the railcars and their spring suspension arose consequently. An analysis of research studies in the field of optimization of dynamic characteristics of a special self-propelled rolling stock, and development of methods of mathematical modeling and numerical analysis revealed that, since the 1960s, a great number of works dealing with dynamics of the rolling stock have been carried out, and papers have been published on spatial oscillations of locomotives and railcars that were driven along straight and curved sections of the rail track [3, 4].

The mathematical models of running parts of railway vehicles have also been discussed in the study by Kamaev [5], including wheel and rail track interaction. The studies also covered a comparative analysis of algorithms and optimization methods including ways of reducing computer time to solve an optimization problem. Analytical optimization methods are used only for quasi-linear systems for which an integral expression of optimization criterion could be explicitly written [6]. In general, the principles of optimization are not widespread and rarely used in the construction of running parts of rail service cars. This greatly explains the lack of sophisticated optimization programs for comparatively simple mathematical models and the lack of methodological principles of optimization with the use of complex mathematical and physical models.

A significant number of works are devoted to the theoretical aspects of optimization methods. The analysis of these methods allows choosing the Hook-Jeeves, Nelder-Mead, and the Powell method of directions among zero-order methods and the Devidon-Fletcher-Powell (DFP) method for their reliability, accuracy, and speed of convergence among the first-order methods [4, 7-8]. It is known that these methods of nonlinear programming are the core of the method of local minimum search. Obviously, in most cases, while solving problems of optimization of mechanical systems, the researchers are interested in optimum solution for the system.

Turning to previous studies in this sphere, similar studies with focus on mathematical modeling for repair of defective rail wheels were conducted by researchers from Vilnius Gediminas Technical University, Marijonas Bogdevicius, Rasa Zygiene, Bureika Gintautas, and Rimantas Subačius. The research conducted by the first two researchers allowed to construct mathematical models for assessing the impact of the uneven railroads and other elements on the structures of the railcar, especially wheels [9]. However, Bureika and Subačius concentrated on mathematical models for calculating bending tensions identified in various components of the rail car [10]. Moreover, numerical modeling by Ioan Sebesan and Dan Baiasu covered the impact of oscillations on body, bogie, and wheel elements, and this model allowed passenger car to be used regularly at the speed of 160 km per hour [11]. As a continuum of these movements in academic area, this study provides a similar approach with focus on mathematical modeling of fluctuations in the main bearing frame of the railcar rather than wheels.

As was noted in previous paragraph, most of the research studies covered the wheel–rail track contact dynamics and deterioration of wheels in those interactions. Studies conducted by Anakwo *et al.*[6], Manashkin and Myamlin [12], and Žygiene *et al.* [13] concentrated on the modeling of tensions between rail wheels and track and ways to optimize maintenance of wheels using developed mathematical models based on theories such as Kalker-x linear theory. The models allowed for effective and practical measurement of wear and tear of rail wheels and related parts for maintenance and repair purposes.

This article provides summary of a calculation algorithm for the simulation of load-bearing body frame of emergency and repair rail service car in tension. It presents the results of numerical studies on the stress–strain state of bearing body frame structure, taking into account the variability in section, mass, longitudinal stiffness, and bending stiffness along its length. Moreover, it outlines rationale for the choice of diagnostic parameters for the evaluation of dynamic strength, reliability, and predictable service life of bearing body frame structure of emergency and repair rail service cars.

The simulation of the work of equivalent bearing body frame of emergency and repair rail service car was conducted by using an elastic rod with variable cross section, mass, bending stiffness, and longitudinal stiffness. The difference between the proposed model and the existing ones [14, 15] is in the variability of cross section, mass, and the longitudinal and bending stiffness along the length of equivalent beam, which corresponds to the actual conditions of operation [16, 8]. In existing methods of calculation, a beam of uniform strength is considered for the simplification, or an approximate calculation is carried out on the model with taken parameters, excluding elasticity. These approximate models in dynamics may create an error up to 150 - 200% of the real strains and stresses [14, 15]. Therefore, in practice, pilot studies are always performed and dynamic correction coefficients are introduced into the calculations of strength and stability.

In contemporary literature, the theory of oscillations and reliability of body frame, spring suspension, and chassis of special self-propelled rolling stock is not a sufficiently studied area where optimization of dynamic characteristics, as well as methods for their rational design and modernization are taken into account. In connection with the massive failure of the railcars of JSC "Uzbekistan Temir Yollari", there is a need for development of a new way of modernization of special self-propelled rolling stock (WHSV) during overhaul in order to improve dynamic performance and the strength and reliability (specifically for railcars such as ADM (ADM - 1) and MPT (MPT-4), operated by "UTY" JSC).

2. ASSUMPTIONS IN DEVELOPMENT MODEL

This paper contains the following materials:

- Validation of assumptions and derivation of vibration equations conducted for dynamic calculation of load-bearing frame of the body of emergency and repair rail service car under combined longitudinal and bending vibrations of the system is presented.
- An analytical technique for solving bending and longitudinal vibrations of bearing framework of the body frame of rail service cars is given in the form of an elastic rod with variable cross section, mass, and bending and longitudinal rigidity, taking into account section variability, weight, bending rigidity, and longitudinal rigidity.
- A calculation algorithm and simulation program of the stress–strain state of a bearing frame structure of emergency and repair rail service cars are presented.
- The results of numerical study of the stress–strain state of bearing frame structure of emergency and repair rail service cars are presented, taking into account the variability in sections, weight, and longitudinal and bending rigidity along its length.

3. MATHEMATIC MODEL OF OSCILLATIONS

The following variable functions cover the parameters of the equivalent load-bearing body frame of the locomotive as a part of the proposed model of this study:

- The mass (m_K) per unit length of the body frame of emergency and repair rail service car (kg/m):

$$m_K(X) = m_O * (a_o + a_1 X + a_2 X^2), \tag{1}$$

- the area of cross section (F):

$$F(X) = F_O * (d_o + d_1 X + d_2 X^2), \tag{2}$$

the length of the main bearing body frame of emergency and repair rail service car is 12.96 meters and the X coordinate varies in the range of $0 \le X \le 12{,}96$ m;

- the reduced moment of inertia of frame section on the axis X_C - I_X (cm^4):

$$I_X(X) = I_O * (b_O + b_1 X + b_2 X^2), \tag{3}$$

where the coefficients a_o, a_1, a_2, d_0, d_1, d_2, b_0, b_1, and b_2 are obtained by approximation with the use of the spline functions [3] based on real data on the linear mass m_K (X), the cross-sectional area F (X), and the inertia I_X (X).

- the reduced bending stiffness is calculated as follows:

$$J_I(X) = E * I_X(X), \tag{4}$$

where $I_X (X)$ is calculated using the formula (3) and E is the elasticity of the frame material.

Fig. 1 shows the general overview of the elements of the railcar with details of the impact of forces, dimensions, and location of the units mentioned in the proposed model.

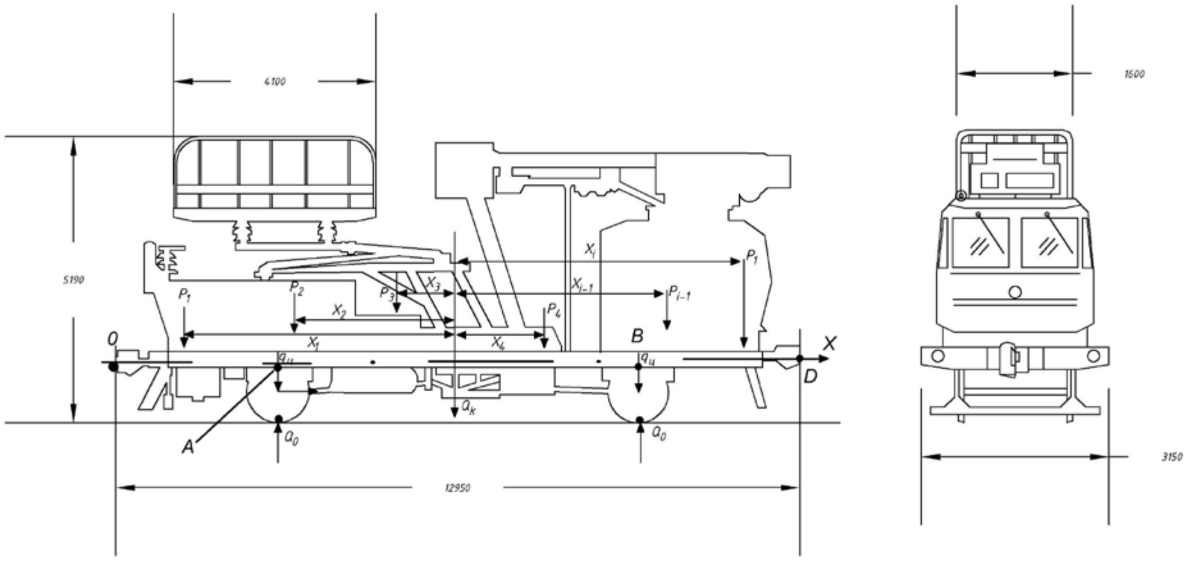

Fig. 1. Design scheme of the rail car for the equivalent load-bearing frame of the body frame

An assumption is made that the body frame of rail service car is represented in the form of an elastic rod (beam) with constant modulus of material elasticity ***E = const*** and the density ***ρ = const,*** and it has some static initial radius of deflection ***R***. The equations of bending and longitudinal oscillations for this model are similar to those used in [14, 15].

To analyze the stress–strain state of the equivalent frame of bearing structure of emergency and repair rail service car, differential equations of bending and longitudinal oscillations of straight rods of variable section are used (considering that torsional oscillations are relatively small compared with other components), similar to those used in [14, 15].

$$\begin{aligned} m_K(X)\frac{\partial^2 U(X,t)}{\partial t^2} - E\frac{\partial F(X)}{\partial X}\cdot\frac{\partial U(X,t)}{\partial X} - EF(X)\frac{\partial^2 U(X,t)}{\partial X^2} = \\ = N_D(X,t) + E\frac{\partial I_X(X)}{\partial X}\cdot\frac{1}{R^2} + 2EI_X(X)\frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3} \end{aligned} ; \tag{5}$$

where $W(X,t)$ is shear mixing centers of gravity

$$\begin{aligned} m_K(X)\frac{\partial^2 W(X,t)}{\partial t^2} + EI_X(X)\frac{\partial^4 W(X,t)}{\partial X^4} + E\frac{\partial^2 I_X(X)}{\partial X^2}\cdot\frac{\partial^2 W(X,t)}{\partial X^2} = \\ = P_D(X,t) + \frac{E}{R}\left[\frac{\partial^2 I_X(X)}{\partial X^2} + 2I_X(X)\cdot\frac{\partial^3 U(X,t)}{\partial X^3}\right] \end{aligned} . \tag{6}$$

After substituting the equations (1) ÷ (4) and their derivatives in the system of differential equations (5) ÷ (6), we obtain the nonlinear equations in the following form:

$$\left[m_0 \cdot \left(a_0 + a_1 X + a_2 X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial t^2} - E\left[F_0 \cdot \left(d_1 + 2d_2 X\right)\right]\cdot\frac{\partial U(X,t)}{\partial X}$$
$$- E\left[F_0 \cdot \left(d_0 + d_1 X + d_2 X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial X^2} = N_D(X,t) + \quad ; \quad (7)$$
$$+ E \cdot \left[I_0 \cdot \left(b_1 + 2b_2 X\right)\right]\cdot\frac{1}{R^2} + 2E \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot\frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3}$$

$$\left[m_0 \cdot \left(a_0 + a_1 X + a_2 X^2\right)\right]\cdot\frac{\partial^2 W(X,t)}{\partial t^2} + E \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot\frac{\partial^4 W(X,t)}{\partial X^4} +$$
$$+ E \cdot 2b_2 \cdot I_0 \frac{\partial^2 W(X,t)}{\partial X^2} = P_D(X,t) + \frac{E}{R}\cdot\left[2b_2 I_0 + 2\cdot\left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot\frac{\partial^3 U(X,t)}{\partial X^3}\right]. \quad (8)$$

By dividing each equation of the system (7)÷(8) by m_K (X), the entire frame of the body is divided into 120 points (X coordinate varies in the range of $0 \le X \le 12{,}96$ m). For each of the given K-section, the coefficients in the equations of the system (7)÷(8) are constant and they could be introduced using an iteration method (piecewise linear approximation) into computer solution in the procedure similar to the ones in [14, 15, 17, 18, 19,20].

After the introduction of systems, we obtain following equations:

$$\frac{\partial^2 U(X,t)}{\partial t^2} - A_{K1}(X)\cdot\frac{\partial U(X,t)}{\partial X} - B_{K1}(X)\frac{\partial^2 U(X.t)}{\partial X^2} = C_{K1}(X)\cdot\sin n\omega t +$$
$$+ D_{K1}(X) + E_{K1}(X)\cdot\frac{\partial^3 W(X,t)}{\partial X^3} \quad ; \quad (9)$$

$$\frac{\partial^2 W(X,t)}{\partial t^2} + A_{K2}(X)\cdot\frac{\partial^4 W(X,t)}{\partial X^4} + B_{K2}(X)\cdot\frac{\partial^2 W(X,t)}{\partial X^2} =$$
$$= C_{K2}(X)\cdot\cos n\omega t + D_{K2}(X) + E_{K2}(X)\cdot\frac{\partial^3 U(X,t)}{\partial X^3} \quad , \quad (10)$$

where the following functions are introduced:

- for longitudinal oscillations of the body frame of rail service car – an equation (9)

$$A_{K1}(X) = \frac{EF_0(d_1 + 2d_2 X)}{m_K(X)} \quad ; \quad B_{K1}(X) = \frac{EF_0\left(d_0 + d_1 X + d_2 X^2\right)}{m_K(X)}$$

$$C_{K1}(X) = \frac{N_{DK}(X)}{m_K(X)} \ , \ N_{DK}(X) = N_{D\cdot n}\sin\frac{(2n-1)\cdot\pi\cdot X}{2\ell_0} ,$$

The horizontal external dynamic load takes the following form:

$$N_{DK}(X,t) = N_{D\cdot n}\sin\frac{(2n-1)\cdot\pi\cdot X}{2\ell_0}\cdot\sin n\omega t \quad (11)$$

where n = 1,2,3…5 – is a number of harmonics, N_{Dp} – is taken according to experimental data obtained, depending on different modes of loading:

$$D_{K1}(X) = \frac{E\cdot\left(I_0\cdot\left(b_1 + 2b_2 X\right)\right)}{m_K(X)}\cdot\frac{1}{R^2} \ ; \ E_{K1}(X) = \frac{2E\cdot\left(I_0\cdot\left(b_0 + b_1 X + b_2 X^2\right)\right)}{m_K(X)}\cdot\frac{1}{R}$$

- for bending (transverse) oscillations of the body frame of rail service car – an equation (10)

$$A_{K2}(X)=\frac{E\cdot\left(I_0\cdot\left(b_0+b_1X+b_2X^2\right)\right)}{m_K(X)}\ ;\qquad B_{K2}(X)=\frac{2E\cdot\left(I_0\cdot b_2\right)}{m_K(X)}\ ;$$

$$C_{K2}(X)=\frac{P_{\ddot{A}K}(X)}{m_K(X)}\ ,\ \ P_{DK}(X)=P_{D\cdot n}\sin\frac{\pi\cdot n\cdot X}{\ell_0},$$

The vertical external dynamic load takes the following form:

$$P_{DK}(X,t)=P_{D\cdot n}\sin\frac{n\cdot\pi\cdot X}{\ell_0}\cdot\cos n\omega t \tag{12}$$

where: n = 1,2,3…5 – is a number of harmonics, P_{Dp}– is taken according to experimental data obtained, depending on different modes of loading.

$$D_{K2}(X)=\frac{E\cdot\left(2I_0\cdot b_2\right)}{R\cdot m_K(X)}\ ;\quad E_{K2}(X)=\frac{2E\cdot\left(I_0\cdot\left(b_0+b_1X+b_2X^2\right)\right)}{m_K(X)}\cdot\frac{1}{R}$$

The solution of the systems (7)÷(8) is performed with the linearization using Simpson's method, and then the Fourier method is applied to the differential equations with constant coefficients with further application of operational Laplace method. Moreover, numerical studies are carried out using the methods of piecewise linear approximation and boundary elements method, similar to the procedures given in [3,5,14, 15, 17, 18, 21] with the use of ***Mathcad 14*** programming environment. Initial conditions are taken as zero, and the boundary conditions in the form of elastic fixing of the ends.

Thus, it is possible to find a general solution of differential equations of bending and longitudinal oscillations of the body frame of emergency and repair rail service car (9) and (10) in the following format:

$$\begin{aligned}W(X,t)=&\sum_{k=1}^{\infty}W(X)*\{\frac{C_{K2}}{W(X)}\cdot\frac{\cos n\omega t-\cos\lambda_{2n}t}{\lambda_{2n}^2-(n\omega)^2}+W_0\cdot\cos\lambda_{2n}t+\\&+[\frac{D_{K2}}{W(X)}+V_{\hat{I}P}\ ]*\frac{1}{\lambda_{2n}}\cdot\sin\lambda_{2n}t\}\end{aligned}\ ; \tag{13}$$

where V is speed of the railcar, ω is frequency of oscillations (vibrations) on loading, and λ is vibrations

$$\begin{aligned}U(t)=&\frac{C_{K1}}{U(X)}\cdot\frac{n\omega\cdot\sin\lambda_{1n}t-\lambda_{1n}\sin n\omega t}{n\omega\cdot\lambda_{1n}\cdot\left(\lambda_{1n}^2-(n\omega)^2\right)}+U_0\cdot\cos\lambda_{1n}t+\\&+[\frac{D_{K1}}{U(X)}+V_{\hat{I}}\ ]*\frac{1}{\lambda_{1n}}\cdot\sin\lambda_{1n}t+\frac{H(X)}{U(X)}*\{\frac{C_{K2}}{W(X)}*W_1(t)+\\&+W_0\cdot\frac{\cos\lambda_{2n}t-\cos\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}+\frac{D_{K2}}{W(X)}*\frac{\sin\lambda_{2n}t-\sin\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}+\\&+V_{\hat{I}P}\ \cdot\frac{\sin\lambda_{2n}t-\sin\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}\}\end{aligned}\ , \tag{14}$$

$$H(X)=\left[E_1ch\omega_KX+E_2sh\omega_{KX}-E_3\cos\omega_BX+E_4\sin\omega_{BX}\right]$$

where

$$\omega_K=\sqrt{\frac{-\alpha^2}{2}+\sqrt{-\left(\frac{\alpha^2}{2}\right)^2+(\lambda_{2n})^2}}\qquad\omega_B=\sqrt{\frac{\alpha^2}{2}+\sqrt{-\left(\frac{\alpha^2}{2}\right)^2+(\lambda_{2n})^2}}$$

where

$$\alpha = \sqrt[4]{\frac{\lambda_{2n}^2}{A_{K_2}}} \qquad \beta = \sqrt{\frac{B_K}{A_K}}$$

where

$$W_1(t) = \frac{\cos \lambda_{1n} t}{\left(\lambda_{1n}^2 - (n\omega)^2\right)\cdot\left(\lambda_{2n}^2 - \lambda_{1n}^2\right)} - \frac{\cos n\omega t}{\left(\lambda_{1n}^2 - (n\omega)^2\right)\cdot\left(\lambda_{2n}^2 - (n\omega)^2\right)} - \frac{\cos \lambda_{2n} t}{\left(\lambda_{2n}^2 - (n\omega)^2\right)\cdot\left(\lambda_{2n}^2 - \lambda_{1n}^2\right)}. \quad (15)$$

Thus, as a result of using the method of iterations and piecewise linear approximation, we have managed to obtain an analytical and numerical solution for the analysis of joint bending and longitudinal oscillations of the bearing body frame of emergency and repair rail service car in the form of a model of an elastic rod with variable cross section, mass, bending stiffness, and longitudinal stiffness as it moves along the track with periodic joint roughness.

In order to better understand, perform thorough analysis, and come to conclusions, simulation of the mathematical model was carried out using testing railcars at simulation workplace. The idea behind the experiment was to install the so-called damping subfloor element in the frame control unit. The results of the simulation experiment are summarized in Table 1.

Table 1

Results of simulation experimental data

Checkpoint measurements of vibrations and stresses	The low-frequency component of the acceleration, Hz		The maximum amplitude of vibration acceleration, m/s²		The longitudinal tension (in the center of the frame), MPa		Bending stress (in the center), MPa	
	Experiment	Theory	Experiment	Theory	Experiment	Theory	Experiment	Theory
Frame body control (including damping subfloor)	2,59	2,64	14,06	-	3,2	3.1	28	29.1
Frame body control (standard design)	2,07	2.17	15.2	-	3.3	3.2	31	30,7

On the basis of the results in Table 1, experimental data received from simulation are to the greatest extent in accordance with the calculated mathematical model and have very small deviation. Accordingly, the total stress–strain state, with the introduction of the damping subfloor in the frame body structure of railcars, decreased by about 11-15% depending on the loading condition that facilitates the operation of the extension of useful life. The total dynamic voltage did not exceed the tensile strength in the experiment and ranged from 15.3 MPa to 41.23 MPa.

Furthermore, frequency analysis performed using the program in *Mathcad 14* showed that the frequencies of natural vibrations vary in harmonics $n = 1, 2, 3\ldots5$ as follows (see Figs. 1, 2):

- under longitudinal vibrations of the system with the introduction of damping bottom covering, the frequency of natural vibrations of modernized frame of electric locomotive $\lambda 1m_n$ increases compared with a standard one $\lambda 1_n$ (for example, at $n = 5$ the frequency is 0.587 and 0.602 Hz/m, respectively) (Table 2, Fig. 2).

Table 2

Change in natural vibrations of locomotive frame in harmonics
(with or without damping bottom covering) under longitudinal vibrations

n	a_n	λ_{1n}	λ_{1mn}
1	0.087	0.065	0.067
2	0.262	0.196	0.201
3	0.436	0.326	0.334
4	0.611	0.457	0.468
5	0.785	0.587	0.602

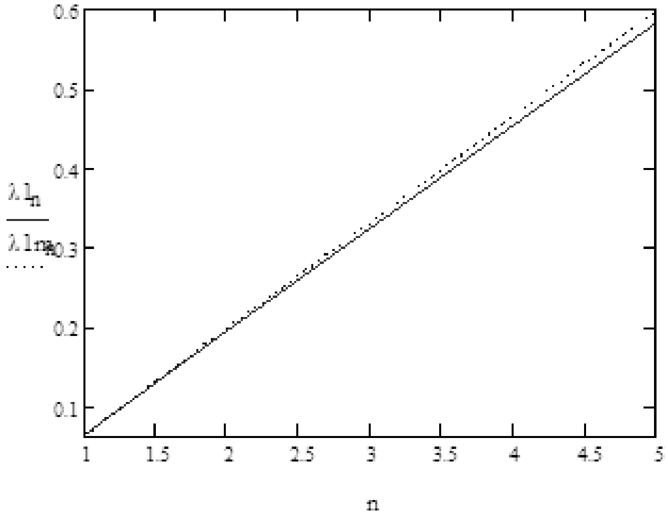

Fig. 2. Diagram of the change in eigenfrequency under longitudinal vibrations (in harmonics) for a standard frame $\lambda 1_n$ and a modernized one (with damping bottom covering) $\lambda 1m_n$.

- under bending vibrations of the system with the introduction of damping bottom covering, the frequency of natural vibrations of modernized frame of electric locomotive λ_{2n} decreased compared with a standard one λ_{20n} (for example, at $n = 5$, it is reduced from 1,321 Hz to 1,253 Hz, respectively) (see Fig. 3, Table 3).

Table 3

Change in natural vibrations of locomotive frame in harmonics
(with or without damping bottom covering) under bending vibrations

N	$\boldsymbol{\lambda_{20n}}$	$\boldsymbol{\lambda_{2n}}$	$\boldsymbol{\lambda_{1mn}}$
1	0.44	0.418	0.067
2	0.661	0.627	0.201
3	0.881	0.835	0.334
4	1.101	1.044	0.468
5	1.321	1.253	0.602

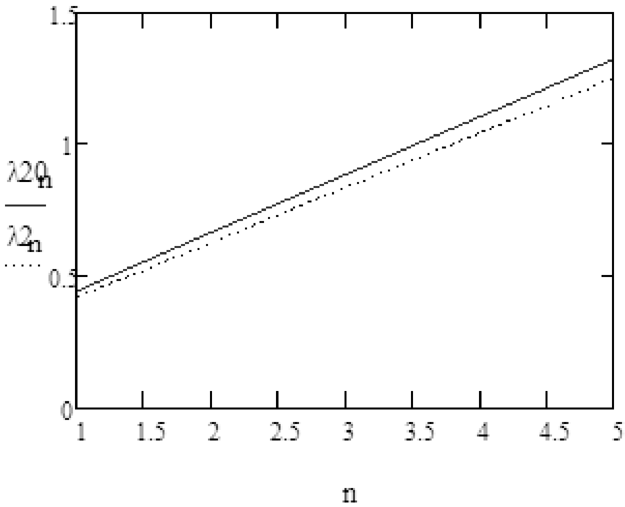

Fig. 3. Diagram of changes in Eigenfrequency under bending vibrations (in harmonics) for a standard frame λ20n and a modernized one (with damping bottom covering) λ2n for the first and the third section of the frame of emergency and repair rail service car

Hence, the results of both mathematical models based on experiment (simulation) are in line with proposed improvements for the railcars.

4. CONCLUSION

On the basis of numerical studies and comparative analysis with experiment (simulation) we have stated the following conclusions:

(1) Bending stresses appearing in the center of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, do not exceed the ultimate strength of the material, and in average range from 15 to 40 MPa depending on loading modes (the rate of motion).

(2) Longitudinal stresses appearing in the center of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, are about 20 ÷ 25% of the bending stresses (from 3 to 10.4 MPa). They reach their maximum values at breakaway and braking modes.

(3) The introduction of damping subfloor in frame design emergency replacement railcar reduces bending stresses in the frame for 10-12%, depending on the speed (from 31 MPa to 28 MPa at a speed of 40 km / h - 11.07%).

Accordingly, the use of mathematical modeling in the modernization and extension of useful life of railcars is highly applicable given the importance of low cost maintenance and use of railway resources effectively. The results of the simulation and mathematical modeling will be implemented in real-life conditions and will be compared with data on mathematical calculations and simulation.

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