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track alignment; power spectral density; track geometry quality; railway dynamics

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NUMERICAL SYNTHESIS OF THE TRACK ALIGNMENT AND APPLICATIONS. PART I: THE SYNTHESIS METHOD

Summary. This paper features a method to synthesize the track irregularities, by which the alignment may be analytically represented by a pseudo-stochastic function, as well as the implementation of such method in the numerical simulation of the dynamic behaviour of the railway vehicles. The method thus suggested allows a convenient formulation of the limits of the interval specific to the wavelengths of the track lateral irregularities, so that it will be representative for the frequency range of the vehicle lateral vibrations. The Part I of this paper demonstrates the method to synthesize the track alignment and its basic elements – the power spectral density of the track irregularities, as per ORE B176 and the values associated with the track quality levels, mentioned in the UIC 518 Leaflet. The Part II introduces the results of the numerical simulations regarding the dynamic behaviour of the railway vehicle during the circulation on a tangent track with lateral irregularities, synthesized as in the method herein.

LA SYNTHÈSE NUMERIQUE DU DRESSAGE DE LA VOIE ET APPLICATIONS. PARTIE I: LA MÉTHODE DE SYNTHÈSE

Sommaire. L'article présente une méthode pour la synthèse des irrégularités de la voie, avec laquelle le dressage peut être représenté analytiquement par une fonction pseudo aléatoire et aussi l'application de cette méthode dans la simulation numérique du comportement dynamique latérale des véhicules ferroviaires. La méthode proposée permet de choisir convenablement les limites d'intervalles spécifiques des longueurs d'onde des irrégularités latérales de la voie, de sort qu'elles sont représentatives pour le domaine de fréquence de la vibration latérale du véhicule. La Partie I de l'article, analyse la méthode de synthèse du dressage de la voie et donne ses éléments de base – la densité spectrale de puissance décrite conformant ORE B176 et les quantités associées de la voie précisées dans la Fiche UIC 518. La Partie II présente les résultats des simulations numériques du le comportement dynamique latéral du véhicule pendant la circulation sur une voie en alignement avec des irrégularités latérales synthétisées en utilisant la méthode présentée.

1. INTRODUCTION

The track geometry is affected by a series of irregularities, mainly due to the constructive imperfections, track exploitation, modification of the track infrastructure, as a result of the action in the environment factors or the ground movements. The track irregularities generate deviations from the ideal design geometry, which may be characterized by the lateral and vertical deviations of each rail from the nominal (ideal) position. Typically, these deviations are combined to give an alternative set of four independent irregularities: alignment, longitudinal level, cant variation and gauge variation [1, 2]. The track geometry variations represent an input system that influences the vibrating behaviour of the railway vehicle [3]. To achieve the simulations in the dynamics of the railway vehicles, often used for the study of the safety-related issues [4 - 7], ride quality [8, 9], ride comfort [4, 5, 10] or the track fatigue stress [11 - 13], it is necessary to have the absolute values of the track irregularities which are obtained with railway vehicles specially equipped to measure the track imperfections. Many times, this information is not available, hence the analytical description of the track irregularities can be used [14 - 16].

Since it is difficult to have such a detailed analytical description of the track geometry, a statistical representation is acceptable. This representation must contain information on the wavelengths, as well as the amplitudes of the track irregularities. A more complete description of the track geometry can be given by the power spectral densities of the measured track irregularities. A power spectral densities can be used either directly as an input for power spectra analysis or they can be retransformed into track irregularities as function of distance.

The paper features a method to synthesize the track alignment that can be applied in the numerical simulation of the dynamic behaviour of the railway vehicles. The method herein relies on the power spectral density of the track irregularities, as described by ORE B176 [17] and the specifications stipulated by the UIC 518 Leaflet [18] concerning the track geometry quality. On the one hand, the shape of the track lateral irregularities spectrum is being considered, as mentioned in the ORE report. On the other hand, there will be taken into account the standard deviations for alignment and the peak values of the isolated track errors, which are used in the UIC 518 Leaflet to define the track quality levels present in the testing and homologation of the railway vehicles from the perspective of the dynamic behaviour.

Part I of the paper is structured as follows. The Section 2 of the paper defines the variation of the track geometry due to the alignment. The current approaches in the review literature on the inclusion of the track irregularities in the vehicle computational models are described in Section 3. The Section 4 presents the quality track levels and the limit values of the standard deviations for alignment of the track, as well as the peak values of isolated track errors associated to them, as named in the UIC 518 Leaflet. The description itself of the suggested method for the numerical synthesis of the track alignment is in Section 5. The conclusions in the Section 6 summarize the achievements of the Part I of the paper.

2. ALIGNMENT-RELATED TRACK GEOMETRY VARIATION

The alignment is one of the major reasons for the lateral vibrations of the railway vehicle. This type of track irregularity can occur during its construction stage, during the maintenance procedures or as a result of the exploitation in the form of a consequence of the accumulated lateral track movements under traffic [19].

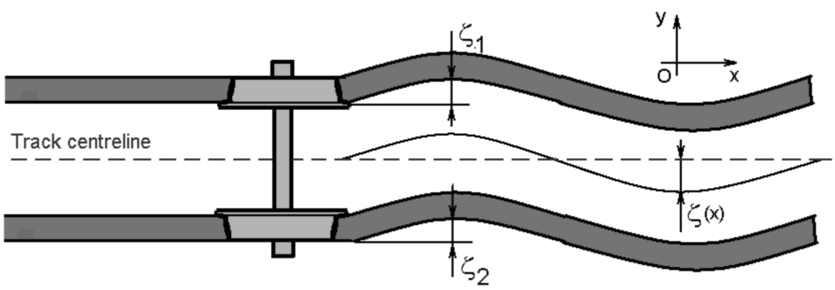

Fig. 1. Track alignment
Fig. 1. Dressage de la voie

A simple definition of the alignment is the deviations of the track from its ideal design geometry, characterized by the lateral displacement $\zeta_{1,2}$ of each rail, as shown in Fig. 1.

In accordance with the UIC 518 Leaflet [18], the track alignment is the geometrical error, in the traverse direction of the horizontal plane, represented by the difference between a point of the side of the rail, at a height of approximately 15 mm below the running plane, and the ideal mean line of the alignment.

3. THE ALIGNMENT REPRESENTATION IN COMPUTATIONAL MODELS

The track geometry irregularities represent the primary inputs in the numerical simulations for the dynamics of the railway vehicles. In the review literature, there are two distinct approaches to include the track irregularities in the computational models [1]. One of the approaches considers the track irregularities as absolute values, defined as distance functions, obtained by the measurement of the track geometry [1, 6, 20, 21]. Here, the alignment is usually defined on the track centreline, as a mean value of the alignment between left and right rails (see Fig. 1) [14, 19, 22], as in

$$\zeta(x) = \frac{\zeta_1 + \zeta_2}{2} \tag{1}$$

Another one, frequently used in the studies on the vehicle dynamics, is based on numerous measurements of the track geometry that have shown that the track irregularities represent a stationary stochastic process. In this context, the track irregularities may be introduced in the computational models as stochastic data characterized by the power spectral density [8, 14, 15, 23, 24].

While considering that the alignment $\zeta = \zeta(x)$, with $0 \le x \le \infty$, is a stochastic Gaussian ergodic process, defined by the mean value [14]

$$\overline{\zeta} = \lim_{L\to\infty} \frac{1}{L}\int_0^L \zeta(x)\mathrm{d}x \tag{2}$$

and the correlation function

$$R_\zeta(\xi) = \lim_{L\to\infty} \frac{1}{L}\int_0^L \zeta(x)\zeta(x-\xi)\mathrm{d}x \tag{3}$$

where: L represents the length of the track section where the measuring of the $\zeta(x)$ has been performed.

For $\xi = 0$, the mean square value may be derived from the relation (3)

$$\sigma_\zeta^2 = \lim_{L\to\infty} \frac{1}{L}\int_0^L \zeta^2(x)\mathrm{d}x \tag{4}$$

The Fourier Transform of the correlation function is the power spectral density, defined below in

$$S(\Omega) = \int_{-\infty}^{\infty} R_\zeta(\xi)\exp(-\mathrm{j}\Omega\xi)\mathrm{d}\xi \tag{5}$$

where: Ω is the wavenumber and it is expressed as a function of the wavelength Λ of the track irregularities, as in $\Omega = 2\pi/\Lambda$, and $\mathrm{j}^2 = -1$.

The power spectral density is used to also calculate the square mean value of the alignment

$$\sigma_\zeta^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\Omega)\mathrm{d}\Omega \tag{6}$$

The power spectral density is sometimes defined for $\Omega \geq 0$ only, and the relation (6) then becomes

$$\sigma_\zeta^2 = \frac{1}{2\pi}\int_0^\infty \Phi(\Omega)\mathrm{d}\Omega \qquad (7)$$

where: $\Phi(\Omega) = 2S(\Omega)$ is the one-side power spectral density.

Another important parameter to come across in this paper is the root mean square value of the alignment or the standard deviation, defined as below

$$\sigma_\zeta = \sqrt{\frac{1}{2\pi}\int_0^\infty \Phi(\Omega)\mathrm{d}\Omega} \qquad (8)$$

The statistical processing of the measured irregularities of the track allows the mathematical formalization of the power spectral density. For the average statistical properties of the European railways, as in ORE B176 [17], the below relation of the power spectral density of the lateral irregularities is regarded as representative

$$\Phi(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \qquad (9)$$

where the coefficients A, Ω_c and Ω_r are experimentally derived. For a high level quality, $A = 2.119 \cdot 10^{-7}$ radm, while for a low level quality, the coefficient A is $6.124 \cdot 10^{-7}$ radm. The coefficients Ω_c and Ω_r have constant values ($\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m).

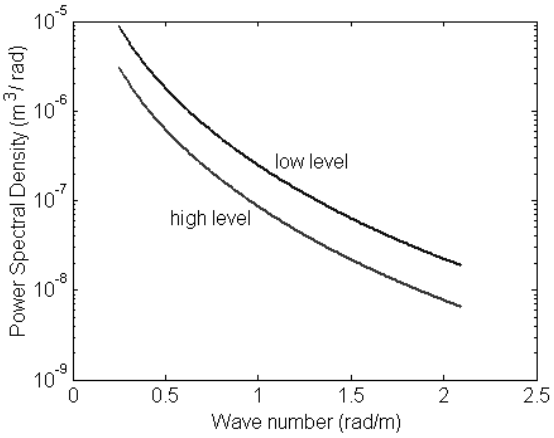

Fig. 2. Power spectral density
Fig. 2. Densité spectral de pouissance

The Fig. 2 shows the power spectral density of the alignment analytically obtained from the relation (9), for wavelengths between 3 and 25 meters, an interval that is correlated with the usual technique of measuring the track geometry [18]. While the vehicle is running at the speed of 200 km/h, the excitation frequency corresponding to the interval of wavelengths covers the 2.2 ... 18.5 Hz range. It should be mentioned that the inferior limit of this range is not small enough to include the lowest natural frequencies of the vehicle lateral movements.

4. THE TRACK GEOMETRIC QUALITY AS IN THE UIC 518 LEAFLET

In the context of the homologation of the railway vehicles in terms of the dynamic behaviour, the UIC 518 Leaflet [18] says that the track geometric quality is represented under three quality levels, namely QN1, QN2 and QN3. The quality levels QN1 and QN2 are defined, in dependence on a certain speed called the reference speed, for the standard deviation for longitudinal level and separately, for the ones in the alignment of the track. Likewise, for the two track quality levels, the peak values of isolated track errors are mentioned for information.

The quality level QN3 is defined as a function of the peak value of an isolated error corresponding to QN2, namely

$$QN3 = 1{,}3 \cdot QN2 \tag{10}$$

Should the quality of a track section correlates with the level QN3, then it will be excluded from the analysis, as not being representative in terms of the track standard geometry.

The paper herein aims to describe the track quality from the viewpoint of the limit values recommended for the alignment. Fig. 3 presents the standard deviations for alignment and the Fig. 4, the peak values of isolated errors, corresponding the quality levels QN1 and QN2, representative for the quality of the European network. It should be mentioned that the values for the alignment standard deviations correlate with a wavelengths interval, ranging between 3 and 25 meters.

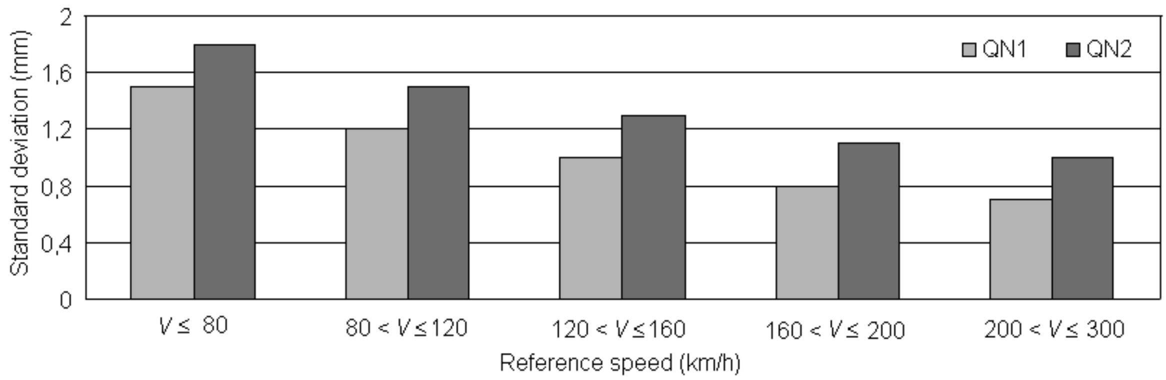

Fig. 3. Standard deviation for alignment for track quality levels QN1 and QN2
Fig. 3. Ecarts types du dressage pour les niveaux de qualité de voie QN1 et QN2

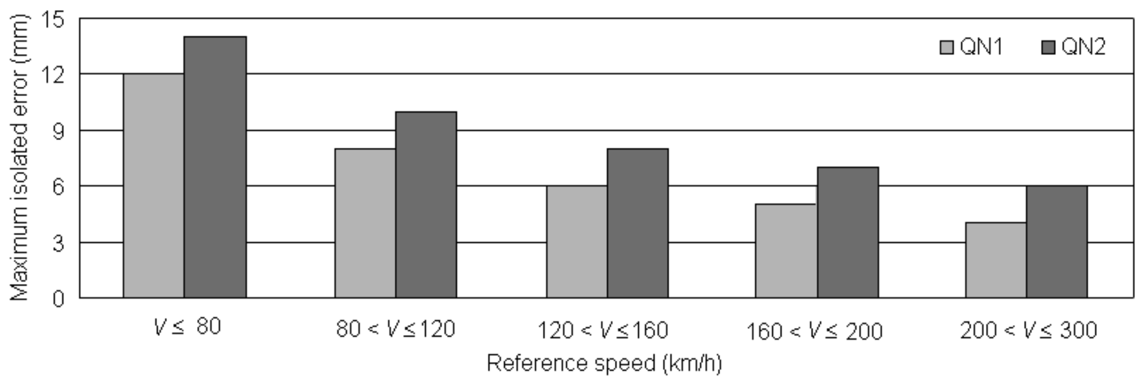

Fig. 4. The peak values for lateral alignment for track quality levels QN1 and QN2
Fig. 4. Les valeurs crêtes pour le dressage pour les niveaux de qualité de voie QN1 et QN2

5. DESCRIPTION OF THE METHOD FOR ALIGNMENT SYNTHESIS

The method to synthesize the track alignment is developed on the following elements:

- the power spectral density defined in the relation (9);
- the specifications in the UIC 518 Leaflet regarding the track geometric quality – on the one hand, the value of the standard deviation in the track alignment (see Fig. 3), and on the other hand, the compliance with the peak values of isolated errors (see Fig. 4).

The following relation of the power spectral density helps calculate the alignment synthesis

$$\Phi(\Omega) = \frac{A_{QN1,2}\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (11)$$

where the coefficient $A_{QN1,2}$ will be calculated so that the value of the alignment standard deviation due to the components with the wavelength between Λ_1 = 3 m and Λ_2 = 25 m should correspond to the stipulations in the UIC 518 Leaflet, according to the track quality level.

To this purpose, the alignment standard deviation between the wavelengths Λ_1 and Λ_2 needs to be calculated

$$\sigma_\zeta = \sqrt{\frac{1}{2\pi}\int_{\Omega_2}^{\Omega_1} \Phi(\Omega)\mathrm{d}\Omega} \quad (12)$$

where: $\Omega_{1,2} = 2\pi/\Lambda_{1,2}$.

From (11) and (12), we have

$$\sigma_\zeta = \sqrt{\frac{A_{QN1,2}\Omega_c^2}{2\pi} I_0} \quad (13)$$

where:

$$I_0 = \int_{\Omega_2}^{\Omega_1} \frac{\mathrm{d}\Omega}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (14)$$

Further on, the relation below is derived, based on which the track quality coefficient can be calculated,

$$A_{QN1,2} = 2\pi \frac{\sigma_\zeta^2}{\Omega_c^2 I_0} \quad (15)$$

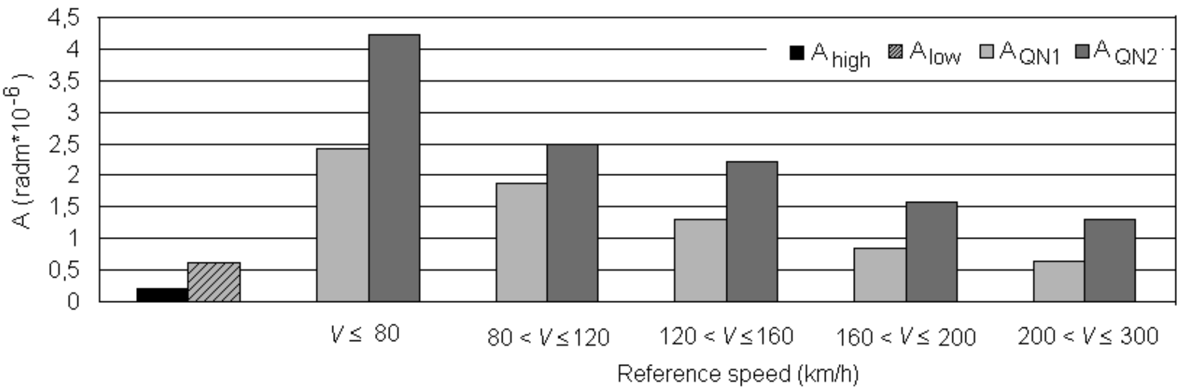

Fig. 5. The track quality coefficient
Fig. 5. Le coefficient de qualité de la voie

The Fig. 5 shows the values of $A_{QN1,2}$ calculated via relation (15) where σ_ζ has been assigned various values corresponding to a QN1 and to a QN2 quality tracks, as per in the UIC 518 Leaflet (see Fig. 3). The same figure includes the values of the coefficient A for high level and for a low level quality tracks. It can be noted that the values of the coefficient A, as in ORE B176 are much lower than the ones derived from the stipulations in the UIC 518 Leaflet; hence, their use in the numerical simulation of the dynamics in the railway vehicles is not quite pertinent.

The next step is the alignment synthesis, starting from the power spectral density in the equation (11). This operation underlies on the transformation of the alignment power spectral density $\Phi(\Omega)$, which is a continuous spectrum, with an infinity of spectral components, into the spectrum of the amplitudes $U(\Omega)$, a discrete spectrum with a finite number of spectral components (see Fig. 6).

The transformation criterion consists in the conservation of power of the two spectra in the wavelength range under study.

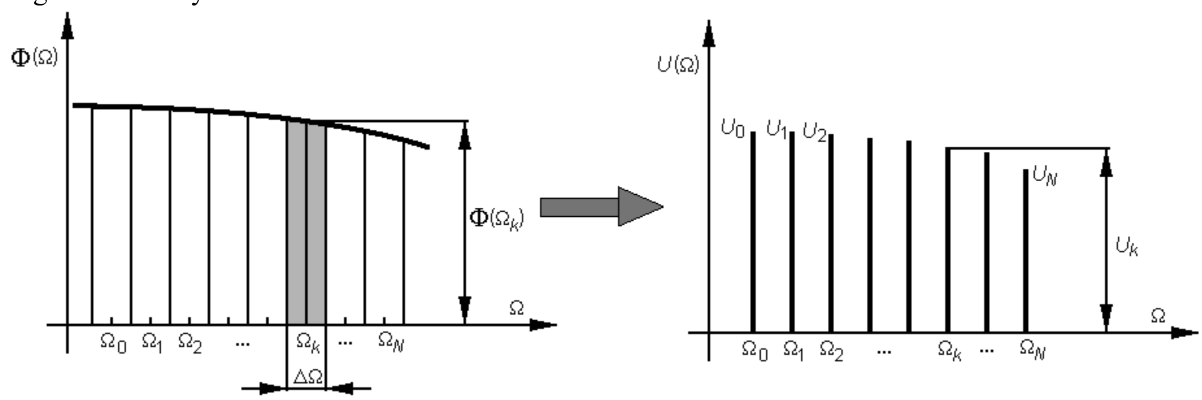

Fig. 6. Discretization of the power spectral density
Fig. 6. Discrétisation du spectre de la densité de puissance

A certain interval of wavelengths of the track lateral irregularities will be considered, ranging from the minimum wavelength Λ_{min} and the maximum one Λ_{max}. For this interval, a constant step partition of the wavenumber $\Delta\Omega$ is considered.

$$\Omega_k = \Omega_0 + k\Delta\Omega,\ k = 0, 1, 2, ..., N$$

where: $N + 1$ is the number of spectral components.

The following correlations are satisfied

$$\Omega_0 = \frac{2\pi}{\Lambda_{max}},\ \Omega_N = \frac{2\pi}{\Lambda_{min}} \tag{16}$$

The amplitude of each spectral component is obtained, if considering that the alignment standard deviation comes from relation (12) for the interval between $\Omega_k - \Delta\Omega/2$ and $\Omega_k + \Delta\Omega/2$, while having the spectral density constant for this interval

$$\sigma_\zeta = \sqrt{\frac{1}{2\pi}\int_{\Omega_k - \frac{\Delta\Omega}{2}}^{\Omega_k + \frac{\Delta\Omega}{2}} \Phi(\Omega)\mathrm{d}\Omega} \approx \sqrt{\frac{1}{2\pi}\Phi(\Omega_k)\Delta\Omega} \tag{17}$$

The amplitude of the spectral component corresponding to the wavenumber Ω_k is as below

$$U_k = \sqrt{\frac{1}{\pi}\Phi(\Omega_k)\Delta\Omega}\text{, where } k = 0, 1, 2, ...\ N \tag{18}$$

Now, the alignment can be assigned an analytical form, namely

$$\zeta(x) = \sum_{k=0}^{N} U_k \sin(\Omega_k x + \varphi_k), \tag{19}$$

where φ_k is the phase of the spectral component ‚k’. To give the track irregularities a stochastic nature, a uniform stochastic repartition will be selected for φ_k, with values between $-\pi$ and $+\pi$.

Following the gross synthesis of the alignment, which helped maintain the form of the spectral density as in equation (11) and the values of the alignment standard value in the UIC 518 Leaflet, there comes the adjustment of the form in the relation (19) so that the peak values of isolated errors keep within the stipulated limits (see Fig. 4). This adjustment is done via the scaling of the track lateral irregularities, with a coefficient equalling a ratio between the value of the admitted isolated error ζ_{adm} and the absolute maximum value of the lateral irregularities ($\max|\zeta(x)|$).

$$K_{\zeta} = \frac{\zeta_{\mathrm{adm}}}{\max|\zeta(x)|} \tag{20}$$

The following analytical form of the alignment thus results

$$\zeta(x) = K_{\zeta} \sum_{k=0}^{N} U_k \cos(\Omega_k x + \varphi_k) \tag{21}$$

To connect the alignment to the track without irregularities, the following smooth function is applied

$$f(x) = \left[6\left(\frac{x}{L_0}\right)^5 - 15\left(\frac{x}{L_0}\right)^4 + 10\left(\frac{x}{L_0}\right)^3 \right] H(L_0 - x) + H(x - L_0), \tag{22}$$

where: L_0 is the connection length, and $H(.)$ is Heaviside step function.

The smooth function meets the following requirements

$$f(0) = f'(0) = f''(0) = 0;\ f(L_0) = 1;\ f'(L_0) = f''(L_0) = 0. \tag{23}$$

Finally, the track alignment is described in the relation

$$\zeta(x) = K_{\zeta} f(x) \sum_{k=0}^{N} U_k \cos(\Omega_k x + \varphi_k). \tag{24}$$

Upon implementing the above method, the track alignment has been synthesized on a 2-km distance for a QN1 quality level track, as well as for a QN2 one (Fig. 7). To this purpose, the values of the alignment standard deviations and the peak values of isolated errors corresponding to the reference speed of 200 km/h have been taken into account (see Fig. 3 and Fig. 4).

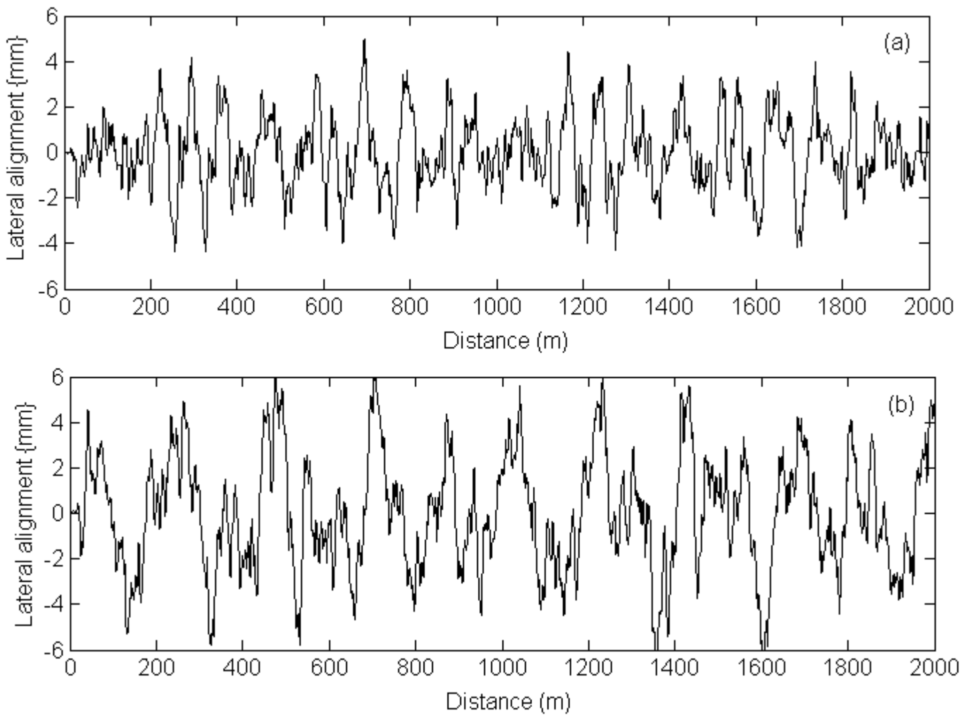

Fig. 7. The track alignment synthesis: (a) track quality level QN1; (b) track quality level QN2
Fig. 7. Synthèse du dressage de la voie: (a) niveau de qualité QN1; (b) niveau de qualité QN2

For the synthesis of the track geometry, the contribution of a number of 300 spectral components (k = 300) has been regarded, with wavelengths between 3 and 180 m. The maximum value of the wavelength has been established by considering that, for a speed of 200 km/h, the minimum frequency of the excitation due to the track lateral irregularities correspond to the frequency of 0.3 Hz, which is small enough to include the lowest natural frequencies of the vehicle lateral movements. Under the present conditions, the resulted value for the alignment standard deviation is 1.637 mm for QN1 quality level track, while the value for the QN2 quality level track is 2.543 mm.

6. CONCLUSIONS

The Part I of this paper introduces a method to synthesize the track alignment, so as to include it in the simulation codes for the dynamic behaviour in the railway vehicles. The method underlies on the power spectral density function in the ORE B176 report, where the track quality coefficient is modified to satisfy the specifications included in the UIC 518 Leaflet regarding the value of the alignment standard deviations and the peak values of isolated errors. Finally, the implementation of this method allows describing the track alignment in an analytical manner via a pseudo-stochastic function which the track quality level and the maximum velocity depend on. Another important benefit is that the method thus suggested allows a convenient formulation of the limits of the specific interval to the wavelengths of the track lateral irregularities, so that it will be representative for the frequency range of the vehicle lateral vibrations. The method may easily adjust to any other form of the power spectral density that describes the track irregularities.

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