Keywords: correlation-extreme navigation system; optimal sampling; random spatial fields; correlation function of spatial field; unmanned aircraft system

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MODEL OF GEOPHYSICAL FIELDS REPRESENTATION IN PROBLEMS OF COMPLEX CORRELATION-EXTREME NAVIGATION

Summary. A model of the optimal representation of spatial data for the task of complex correlation-extreme navigation is developed based on the criterion of minimum deviation of the correlation functions of the original and the resulting fields. Calculations are presented for one-dimensional case using the approximation of the correlation function by Fourier series. It is shown that in the presence of different geophysical map data fields their representation is possible by single template with optimal sampling without distorting the form of the correlation functions.

МОДЕЛЬ ПРЕДСТАВЛЕНИЯ ЭТАЛОННЫХ ГЕОФИЗИЧЕСКИХ ПОЛЕЙ В ЗАДАЧАХ КОМПЛЕКСНОЙ КОРРЕЛЯЦИОННО-ЭКСТРЕМАЛЬНОЙ НАВИГАЦИИ

Аннотация. Разработана модель оптимального представления пространственных данных для задач комплексной корреляционно-экстремальной навигации на основе критерия минимального отклонения корреляционных функций исходного и полученного полей. Представлены расчеты для одномерного случая с использованием аппроксимации корреляционной функции рядом Фурье. Показано, что при наличии картографических данных разных геофизических полей возможно их представление единым эталоном с оптимальной дискретизацией без искажения вида корреляционных функций.

1. INTRODUCTION

Navigation in the wide sense is performed using physical fields: magnetic heading determination is carried out by measuring the horizontal component of the Earth's magnetic field, inertial navigation uses components of the gravitational and inertial fields, satellite navigation creates artificial radio navigation field. But the given examples of navigation systems use the normal or regular components of the physical field, in contrast to the concept of correlation- extreme navigation, where it is necessary to have an abnormal, high-frequency spatial-time field.

Correlation-extreme navigation [1] is based on such property of the anomalous fields as unique correspondence of field parameters distribution to the particular region of the ground surface.

Correlation-extreme navigation system (CENS) is a system of processing the information provided in the form of random functions (fields) to determine the coordinates of motion. The name of this type of system is determined by the fact that the principle of their operation is to find the correlation between the realizations of random functions, and to determine the position or velocity of object by finding the extremum of correlation function that correspond to most probable region on the map.

Among the geophysical fields used in CENS, there are the following: field of relief, anomalous magnetic field of the Earth, anomalous gravity field, field of optical contrast of ground surface, the field of radar contrast, etc. One of the most widely used variant of CENS is terrain-aided navigation systems which work on the relief field. The most frequently referred algorithms for terrain navigation are TERCOM (terrain contour matching) and SITAN (Sandia inertial terrain-aided navigation). TERCOM is batch oriented and correlates gathered terrain elevation profiles with the map periodically [2].

Another variant of CENS is geophysical navigation systems based on magnetic/gravitational fields described in a number of works [3-4], etc. The exploitation of geomagnetic or gravitational anomalies as a source of information for the navigation has been proposed many years ago but the concept still requires practical demonstration. The idea is the same: having the map data of anomalous magnetic/gravitational fields the system compares the current and map data in terms of finding the correlation between them, then finds the matches as the extremum of correlation function and then provides the localization of vehicle.

There is no unique term for the proposed systems: terms like geophysical navigation system, gravity-aided navigation system, geomagnetism aided inertial navigation system in [5]. Also to the same class of systems the visual aided navigation systems can be related [6], since they work with field of optical contrast of ground surface. The only difference is in the form of information gathering: in each moment of time the frame of field is available unlike the previously mentioned fields where the single measurement (point) is done in time moment. Obviously, it causes the use of different methods of correlation but the whole principle is the same.

Anyway, all mentioned classes of systems can be unified to the single term - correlation-extreme navigation systems which completely describe its main principle of operation.

Actually the geophysical fields are deterministic because they are generated by objective physical processes. However, their description usually [1] uses a probabilistic approach due to the fact that a variety of factors affecting the value of the field at any time moment cannot be taken into account. Therefore let us consider the geophysical field of any nature as a spatially distributed random field.

The model of this field observed in the space R^3, is written in the form of a model of its realization

$$s(t,\mathbf{r}) = S(t,\mathbf{r},\xi(t,\mathbf{r})) + n(t,\mathbf{r}), \qquad (1)$$

representing by mixture of Gaussian noise $n(t, \mathbf{r})$ and the valid signal $S(t,\mathbf{r},\xi)$ that depends on the estimated random field $\xi(t,\mathbf{r})$ or generally is some functional from it. The valid signal may not necessarily contain all the components of vector ξ, and can only depend on some of them. Noise field $n(t, \mathbf{r})$ is considered to be delta-correlated in time, i.e. it has the correlation function of the following form

$$B_n(\mathbf{r},\mathbf{r}')\delta(t-t') \qquad (2)$$

With mapping the values of continuous random geophysical field values are obtained at the nodal points $i, j, k, i = 1,...,L_x, j = 1,...,L_y, k = 1,...,L_z$, where the parameters L_x, L_y, L_z determine the area of mapping space R^3. Thus the information of field, enclosed between nodal points, is considered missing, or it can be restored by various methods of interpolation.

With field sampling the problems arise associated with the choice of the spatial frequencies, for reasons of finiteness of such frequencies and in the general case of infinite interval of field existence. It is also must be noted that due to features of CENS the main information component of the field data is contained in high frequencies that characterize so called anomalous field components or features. Keeping all map data with fine sampling interval requires significant amount of memory and also computing resources for their processing, particularly in real-time mode, which is important for navigation tasks of unmanned aircraft systems (UAS) in different application fields [7]. Therefore, the primary problem is compact representation of map data of geophysical fields of different nature in the form of uniform templates.

2. PROBLEM STATEMENT

Let’s consider the possibility of creating such a template on the example of the use of several fields, each of which will henceforth be denoted by index *i*. Typically, the process of mapping includes the measurement of the field component at constant altitude, and if necessary, conversion to different levels (e.g., for the geomagnetic field, which is spatial). Thus, it is possible to set the reference value of the field as two-dimensional array of data given to the same resolution.

The most informative field is the optical one; moreover, its feature is the fact that at fixed time moments more than single value of the field (frame or field image) can be obtained onboard. For majority of other fields at a time moment the data is read at single point of space, and reliable navigation is possible only with the accumulation of sufficient amount of data. Redundancy of optical field can be reduced by providing an image as a set of its anomalous or feature points. They can be locally detectable features, such as the point of intersection of lines at angles close to 90 - Harris detector; points with significant differences of vicinity intensity (SIFT detectors (Scale-invariant feature transform) and its modification adapted to real time problems SURF - Speeded Up Robust Features), and others.

The regular grid with field sampling is usually a good choice because the nodal points are selected during mapping. However, with the observation of local contrast targets on the ground surface it is hardly possible to assume a situation in which all of them will be equally spaced from each other. Positions of feature points are random, and their distribution can vary significantly depending on the observed region (high concentration for areas with rich variety of textures - industrial regions, crossing of communication lines; and low concentration of homogeneous areas like forest plantations, desert, etc.). A set of feature points will be represented as a series:

$$S = \sum_{-N/2}^{N/2} s(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}_k + n), \tag{3}$$

where n - random variable with zero mean and the probability density function $p(n)$; s - value of the original signal at discrete $\mathbf{r} - \mathbf{r}_k$. According to given approach [1], the sampling intervals are chosen non-uniformly so that the probability densities in the points $k\Delta\mathbf{r}$ form the system of not overlapping functions. Let's consider the use of uniform samples, standardized for several fields.

3. ANALYSIS OF EXISTING APPROACHES TO THE REPRESENTATION OF SPATIAL DATA

Let's consider the existing methods for resampling of discrete data. Comparison of existing methods for digital satellite images of ground surface is given in [8]. Among the investigated methods there are weighted average method, bicubic interpolation method, the method of bilinear interpolation, cubic convolution method, nearest neighbor method, etc. Using these methods, it is necessary to take into account their accuracy (defined as a measure of similarity of two data before and after sampling by the correlation coefficient), computing resources for their realization, as well as the specifics of the data application. For the problems of correlation-extreme navigation it is appropriate to use the method of bilinear interpolation, since it requires minimal computational cost and has the satisfactory quality of the correlation function reproducing. The question of choosing the optimal size of sampling cell arises and is considered applying the geophysical data in [9].

4. ALGORITHM OF OPTIMAL SAMPLING INTERVAL DETERMINATION

If there are several different data types of physical nature and of different resolution, generally the nodal points may not coincide. It is necessary to determine the optimal strategy: downscaling of data to the lowest sampling interval or upscaling to the maximum one, or all data must be resampling by some average. Let's use the following assumptions: all data is obtained for the same area of terrain by size $L_x \times L_y$. Let the cell size of each field is determined as c_{ix}, c_{iy}, where the index indicates the field type.

To simplify the problem, the same sampling interval of the horizontal and vertical directions is used, i.e. $c_{ix} = c_{iy} = c_i$. Since the key problems in CENS is the use of correlation functions of geophysical field as the basic navigation parameter, the criterion of optimal interval for unified template is selected as the root mean square difference between the correlation functions of the original (mapped) field B_i and the approximated field B_i':

$$\min \delta_0 = \sum_{i=1}^{N} \left(B_i - B_i' \right)^2 , \tag{4}$$

where the subscript i indicates the type of fields.

Obviously, the correlation function of approximated field will be some function of the field sampling interval $B_i' = f(c_i)$. The correlation function of two-dimensional (surface) geophysical field generally can be found only by the numerical methods, e.g. as described in [10]. To find the optimal sampling interval by criterion (4) let's represent the correlation function (one-dimensional) as Fourier series:

$$B_i(\Delta x) = \sum_{j=1}^{M} (a_j \cos \omega x + b_j \sin \omega x) . \tag{5}$$

Here coefficients a_j, b_j will determine the measure of distortion of correlation function.

The proposed algorithm of optimal sampling interval determination for complex CENS is the following:

1) The mathematical expectations $\mu_i = f(x, y)$ are determined for each geophysical field according to mapping data for the given area $L_x \times L_y$, and in case of non-isotropic fields they are calculated separately for each direction as:

$$\mu_{ix}(n) == \frac{1}{m} \sum_{j=1}^{m} s(nx, y + jc_i), \mu_{iy}(m) == \frac{1}{n} \sum_{j=1}^{n} s(x + jc_i, my) \tag{6}$$

Here the parameters n, m are numbers of possible trajectories in the horizontal and vertical directions and depend on the sampling interval as $n = \frac{L_x}{c_i}, m = \frac{L_y}{c_i}$.

2) Calculation of correlation functions of geophysical fields by the original mapping data. In general, the normalized correlation function has the form:

$$B(\Delta x, \Delta y) = \frac{1}{K-1} \sum_{i=1}^{K} \frac{\tilde{s}(x, y) \cdot \tilde{s}(x + i\Delta x, y + i\Delta y)}{\sigma_{\Delta x} \sigma_{\Delta y}} , \tag{7}$$

where $\Delta x, \Delta y$ - correlation intervals, K - number of points along the chosen direction, the sign "~" designates centered field, $\sigma_{\Delta x} \sigma_{\Delta y}$ - variance along the chosen direction.

3) Approximation of correlation functions along each orthogonal directions (x, y) by series (5) for the initial sampling interval (corresponds to the mapping conditions).

4) Resampling of mapping data of all fields for the largest sampling interval $c_{\max} = \max(c_1, c_2, .., c_N)$ using the bilinear interpolation method.

5) Repeat steps 1-3 for new data of geophysical fields.

6) Calculation of the criteria function (4), expressed in terms of coefficients of the approximation a_j, b_j as

$$\delta_0 = \sum_{i=1}^{N} \sum_{j=1}^{M} \left(a_{ij} - a_{ij}' \right)^2 + \left(b_{ij} - b_{ij}' \right)^2 . \tag{8}$$

If the obtained value is less than given threshold, then the sampling interval is reduced to the next maximum value among $c_{\max} = \max(c_1, c_2, .., c_N)$, and respectively, steps 1-6 are repeated until achieving the prescribed accuracy of correlation function reproducing.

5. ANALYSIS OF SIMULATION RESULTS

For simulation the data of anomalous magnetic field of Alaska regions is used that is available in (Data of anomalous magnetic field of Alaska regions, Internet resource, [11]). Along each flight path the trajectory is formed with data of anomalous component of the geomagnetic field. In calculating the statistical parameters of the trajectory Matlab 2007 software is used. Calculations by proposed algorithm is done for single trajectory (Line 1000 – L1000) on section AK -3019 with the determination of the maximal possible sampling interval at which the criterion function (8) does not exceed the value 3σ.

Correlation function of the trajectory (299 nodal points) and its approximation by series (5) with three harmonics are shown in Fig. 1. Approximation coefficients obtained in Matlab 2007 are shown in Table 1. Further the reproducing quality of the correlation function is studied with decreasing sampling interval up to 10 times (Fig. 2 - 5).

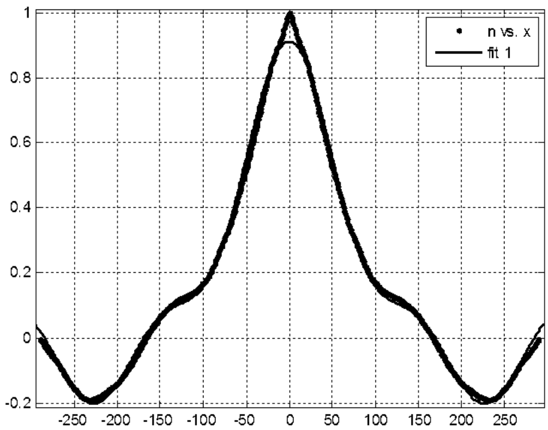

Fig. 1. Correlation function of trajectory for the initial mapping interval (latitude is varied with intervals $\varphi = 0.0008'$, longitude is maintained constant with changes not more than $\lambda < 0.0001'$)

Рис. 1. Корреляционная функция траектории для начального интервала картографирования (широта изменяется в пределах $\varphi = 0.0008'$, долгота выдержана постоянной с изменениями не более чем $\lambda < 0.0001'$)

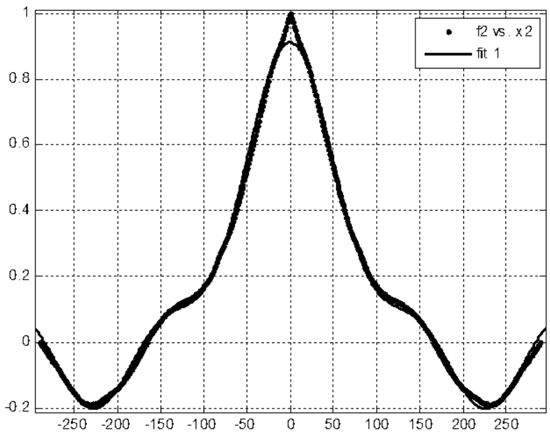

Fig. 2. Correlation function of trajectory for reduced mapping interval by 75%

Рис. 2. Корреляционная функция траектории для уменьшенного на 75% интервала картографирования

As can be seen from Fig. 6, the reproducing accuracy of correlation function is practically stable with reducing the sampling interval twice as much, and the value of criterion function (8) degrades by logarithmic dependence (Table 1).

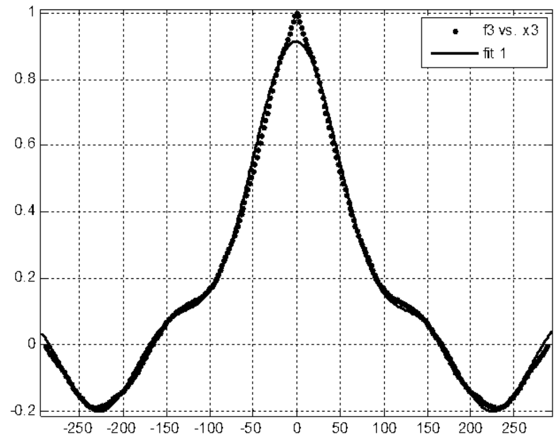

Fig. 3. Correlation function of trajectory for reduced mapping interval by 50%

Рис. 3. Корреляционная функция траектории для Уменьшенного на 50% интервала картографирования

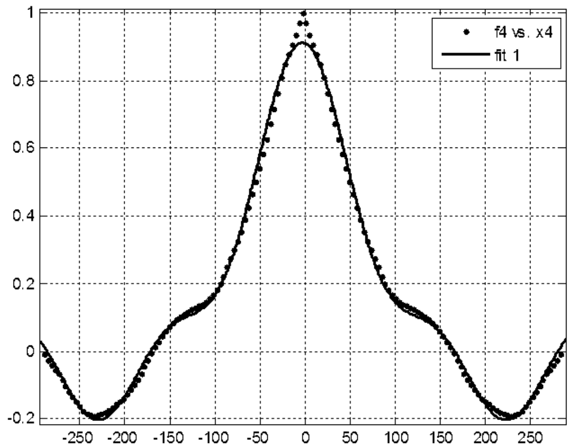

Fig. 4. Correlation function of trajectory for reduced mapping interval by 25%

Рис. 4. Корреляционная функция траектории для уменьшенного на 25% интервала картографирования

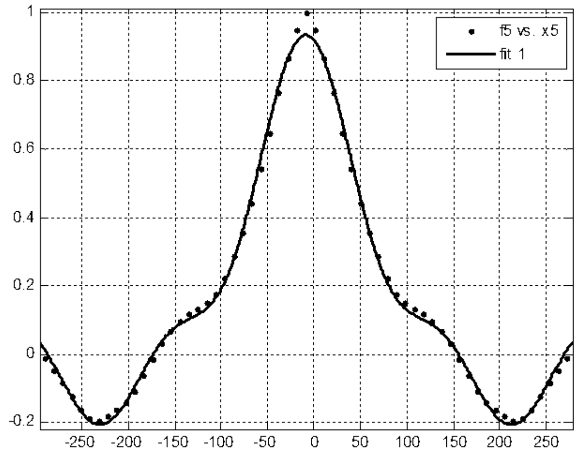

Fig. 5. Correlation function of trajectory for reduced mapping interval in 10 times

Рис. 5. Корреляционная функция траектории для уменьшенного в 10 раз интервала картографирования

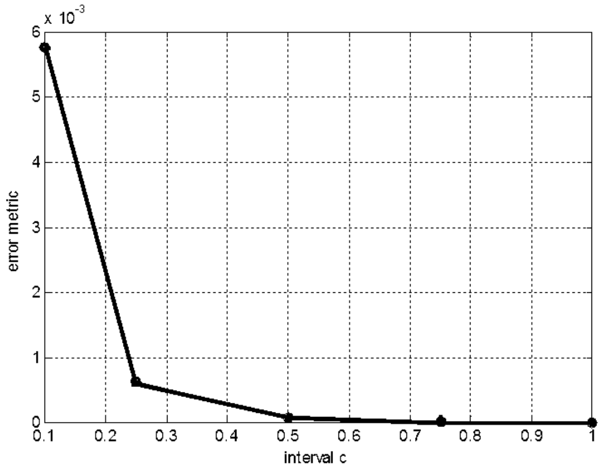

Fig. 6. Dependence of the reproducing accuracy of the correlation function on the sampling interval

Рис. 6. Зависимость точности воспроизведения корреляционной функции на интервале дискретизации

Table 1

Reproducing quality of the correlation function for change the sampling interval

Sampling interval $\omega = 0.01382$	Fourier series coefficients							Value of criterion function
	a_0	a_1	b_1	a_2	b_2	a_3	b_3	
c	0.2388	0.4497	≈ 0	0.1163	≈ 0	0.1067	≈ 0	0
$0.75c$	0.2392	0.4505	-0.0021	0.1165	-0.0011	0.1067	-0.0015	$8.71 \cdot 10^{-6}$
$0.5c$	0.2393	0.4509	-0.0062	0.1165	-0.0032	0.1068	-0.0044	$6.98 \cdot 10^{-5}$
$0.25c$	0.2381	0.4494	-0.0186	0.1159	-0.0096	0.1069	-0.0134	$6.19 \cdot 10^{-4}$
$0.1c$	0.2444	0.4591	-0.0569	0.1155	-0.0291	0.1002	-0.0389	0.0058

6. CONCLUSIONS

The algorithm of determination of maximal available sampling interval for several types of geophysical fields is developed, that makes possible to use the single unified template for mapping data in complex CENS. The example of calculation of optimal sampling interval is represented using the mean square error metrics of correlation function approximation.

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Received 22.01.2014; accepted in revised form 04.09.2015