trip coordination, municipal passenger transport

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TRIP COORDINATION IN MUNICIPAL PASSENGER TRANSPORT

Summary. Routes of bus or tram lines have often common passages with non negligible length. It is desirable that the time intervals between subsequent departure times of trips of all lines on such a common passage are distributed as equally as possible. This paper studies coordination problem on an isolated common passage and also more complicated coordination problem in transportation network.

KOORDYNACJA PODRÓŻY W ZBIOROWYM TRANSPORCIE PASAŻERSKIM

Streszczenie. Trasy autobusów lub tramwajów mają często wspólne fragmenty przejazdu o nieznikomej długości. Pożądane jest, by odstępy czasowe pomiędzy kolejnymi czasami odjazdów wszystkich linii takiego wspólnego przejazdu były rozmieszczone tak równo, jak jest to tylko możliwe. Artykuł przedstawia problem koordynacji na pojedynczym wspólnym przejeździe, a także bardziej złożone zagadnienie koordynacji w sieci transportowej.

1. INTRODUCTION

A regional or municipal regular passenger transport takes place on a transportation network. Individual trips are organized into lines – sets of trips with the same (or very similar) routes. These routes have often common passages – the same sequences of bus (or tram) stops. If such a passage is long enough there is large number of passengers travelling only between bus stops falling into this passage. Such passengers can use vehicle of any line travelling along common passage.

Every passenger would prefer to have a regular sequence of departure times of "his" vehicles – i.e. to have equal intervals between subsequent departure times. Therefore time tables in urban areas are constructed as regular-interval time tables where trip depart from their starting stop several times in an hour and time interval between subsequent departures are the same.

Departure times of a regular-interval time table of a line with n trips an hour can be represented as n vertices of a regular n-gon on a circle. Circle can be thought as a clock and vertices as minute hand positions corresponding to trip departure times. This representation introduced Černý in [2] and [3].

2. REGULARITY MEASURES

It is relatively easy to construct a regular-interval line timetable. Unfortunately, the departure time sequence of all trips of all lines on a common passage are not generally regular.
Let us have three lines:

Line 1 with 12 minute interval,
Line 2 with 10 minutes interval and
Line 3 with 15 minute interval

With departures from the first bus stop of a common passage:

Line 1: 6:00, 6:12, 6:24, 6:36, 6:48, 7:00, …
Line 2: 6:00, 6:10, 6:20, 6:30, 6:40, 6:50, 7:00, …
Line 3: 6:00, 6:15, 6:30, 6:45, 7:00, …

The departure time sequence is

6:00, 6:00, 6:00, 6:10, 6:12, 6:15, 6:20, 6:24, 6:30, 6:30, 6:36, 6:40, 6:45, 6:48, 6:50, 7:00, 7:00, 7:00,
6:00, 6:04, 6:05, 6:12, 6:15, 6:19, 6:24, 6:25, 6:34, 6:35, 6:36, 6:45, 6:48, 6:49, 6:55, 7:00, 7:04, 7:05

Corresponding time intervals are

0, 0, 0, 10, 2, 3, 5, 4, 6, 0, 0, 6, 4, 5, 3, 2, 0, 0, 0,

It is evident, that the regularity on common segment is not good – the length of interval between subsequent departure times varies from 0 to 10.

Let us shift in time the timetable of line 2 by 5 minutes and the timetable of line 3 by 4 minutes:

Line 1: 6:00, 6:12, 6:24, 6:36, 6:48, 7:00, …
Line 2: 6:05, 6:15, 6:25, 6:35, 6:45, 6:55, 7:05, …
Line 3: 6:04, 6:19, 6:34, 6:49, 7:04, …

The departure time sequence is

6:00, 6:04, 6:05, 6:12, 6:15, 6:19, 6:24, 6:25, 6:34, 6:35, 6:36, 6:45, 6:48, 6:49, 6:55, 7:00, 7:04, 7:05

And corresponding time intervals are

4, 1, 7, 3, 4, 1, 9, 1, 1, 9, 3, 1, 6, 5, 4, 1, …

what seems to be much better than before.

There are several reasons why we are looking for regularity of departure times on a common passage. Some of them are

- Passengers like regular time intervals.
- Transport providers endeavour to minimize zero or short intervals because such short intervals cause vehicle jams on bus or tram stops.
- A lot of passengers comes to the bus stop during long time interval. A consequence is that the trip arriving to the bus stop at the end of a long interval is overcrowded. Moreover, a lot of ingoing and outgoing passenger causes delays.
- Minimization of passenger waiting time on bus or tram stops.

We can have several optimization criterions:

1. The shorter is the minimal interval, the worse solution
2. The longer is the maximal interval, the worse solution
3. The smaller is total passengers waiting time, the better solution

2.1. Maximization of shortest time interval

The first criterion can be refined as follows:

- Let $c_1, c_2, \dots, c_n$ be a sequence of time intervals between subsequent trips arranged in ascending order corresponding to a solution 1.
- Let $d_1, d_2, \dots, d_n$ be a sequence of time intervals between subsequent trips arranged in ascending order corresponding to a solution 2.

Then the solution 1 is worse than the solution 2 if $c_1 < d_1$ or if there exists an integer $k \in \langle 1, n-1 \rangle$ such that $c_1 = d_1$, $c_2 = d_2$, … , $c_k = d_k$ and $c_{k+1} < d_{k+1}$.

An optimum solution with respect to the first criterion can be obtained by maximization of objective function

$$F(c_1, c_2, \dots, c_n) = \sum_{i=1}^{n} L^{(n-i)} c_i \quad (1)$$

where $c_1 \le c_2 \le \dots \le c_n$ is a sequence of time intervals between subsequent trips arranged in ascending order and L is a large number.

2.2. Minimization of longest time interval

The second criterion can be refined as follows:

- Let $c_1, c_2, \dots, c_n$ be a sequence of time intervals between subsequent trips arranged in descending order corresponding to a solution 1.
- Let $d_1, d_2, \dots, d_n$ be a sequence of time intervals between subsequent trips arranged in descending order corresponding to a solution 2.

Then the solution 1 is worse than the solution 2 if $c_1 > d_1$ or if there exists an integer $k \in \langle 1, n-1 \rangle$ such that $c_1 = d_1$, $c_2 = d_2$, … , $c_k = d_k$ and $c_{k+1} > d_{k+1}$.

An optimum solution with respect to the second criterion can be obtained by minimization of objective function

$$F(c_1, c_2, \dots, c_n) = \sum_{i=1}^{n} L^{(n-i)} c_i \quad (2)$$

where $c_1 \ge c_2 \ge \dots \ge c_n$ is a sequence of time intervals between subsequent trips arranged in descending order and L is a large number.

2.3. Minimization of total passenger waiting time

Passengers are arriving to a bus stop of a line within the time interval $\langle t_1 t_2 \rangle$ with density $f(t)$. Total number of passengers arriving to the bus stop during whole interval $\langle t_k t_{k+1} \rangle$ can be expressed as $\int_{t_k}^{t_{k+1}} f(t)dt$ and the waiting time of all passengers arriving to a bus stop during interval $\langle t_k t_{k+1} \rangle$ can be calculated as $\int_{t_k}^{t_{k+1}} f(t)(t - t_1)dt$.

Suppose that $t_0, t_1, \dots, t_n$ are departures of all trips from the considered bus stop sorted in ascending order. The total waiting time of all passengers during the whole day is

$$W(t_0, t_1, \dots, t_n) = \sum_{k=0}^{n-1} \int_{t_k}^{t_{k+1}} f(t)(t - t_k)dt \quad (3)$$

It holds for constant passenger density function, i.e. if $f(t) = f = const$:

$$W(t_0, t_1, \dots, t_n) = \sum_{k=0}^{n-1} \int_{t_k}^{t_{k+1}} f.(t - t_k)dt = f.\sum_{k=0}^{n-1} \frac{1}{2}(t_{k+1} - t_k)^2 \quad (4)$$

The following figure (taken over from [5]) illustrates the total passenger waiting time by shaded area.

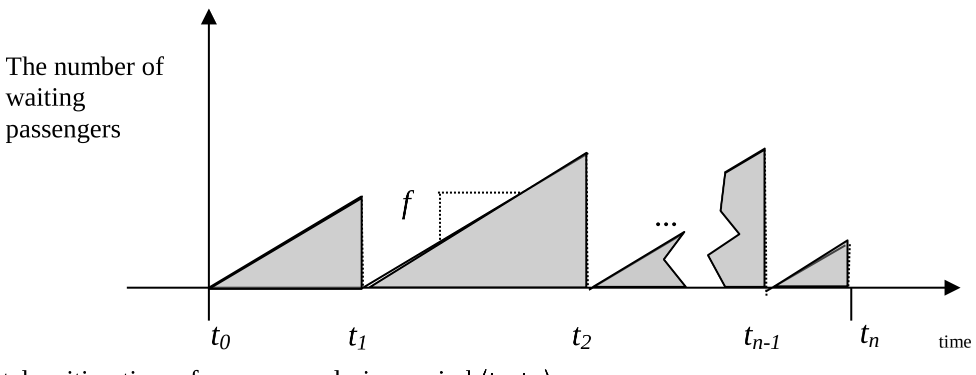


Fig. 1. The total waiting time of passengers during period $\langle t_0, t_n \rangle$

Rys. 1. Całkowity czas oczekiwania pasażerów podczas okresu $\langle t_0, t_n \rangle$

Let $b_1, b_2, \dots, b_n$ be a sequence of time intervals between subsequent trips, $b_i = t_i - t_{i-1}$. Denote

$$V(b_1, b_2, \dots, b_n) = f.\sum_{i=1}^{n} b_i^2 \tag{5}$$

Then the total waiting time of all passengers during whole day can be rewritten as follows:

$$W(t_0, t_1, \dots, t_n) = V\big((t_1 - t_0), (t_2 - t_1), \dots (t_n - t_{n-1})\big) = V(b_1, b_2, \dots, b_n) = f.\sum_{i=1}^{n} b_i^2$$

An optimum solution minimizing total waiting time of passengers can be found by minimizing the objective function (5), where $b_1, b_2, \dots, b_n$ is a sequence of time intervals between subsequent trips. Since f is a constant, the optimum does not depend on f and therefore we can use as objective function

$$V(b_1, b_2, \dots, b_n) = \sum_{i=1}^{n} b_i^2 \tag{6}$$

3. TRIP COORDINATION ON AN ISOLATED PASSAGE

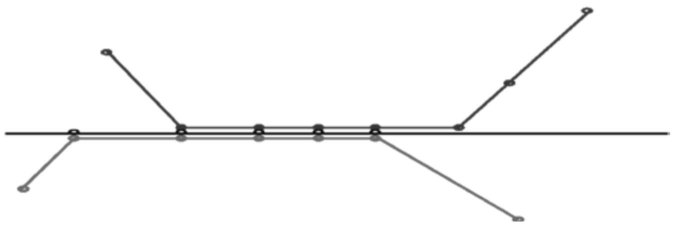


Fig. 2. Common passage of three bus lines
Rys. 2. Wspólny przejazd trzech linii autobusowych

Let us have k lines with regular-interval timetable with common passage. Suppose that line intervals have to be preserved. Then the only possibility how to improve the regularity on common passage is to change time offset between every two of them by shifting their whole timetables in time.

As we have mentioned at the end of the introduction, departure times of every considered bus line can be represented as vertices of a regular polygon on a circle. Thus we have several bus lines on a common passage represented as several polygons on a circle. It is convenient to consider a circle having the length equal to 60. Then the length of the arc between two adjacent vertices on a circle is equal to the length of time interval between corresponding trip departures. (In this case, the sector angle of an arc 1 unit long is equal to 60 degrees – since 60=360/60.)

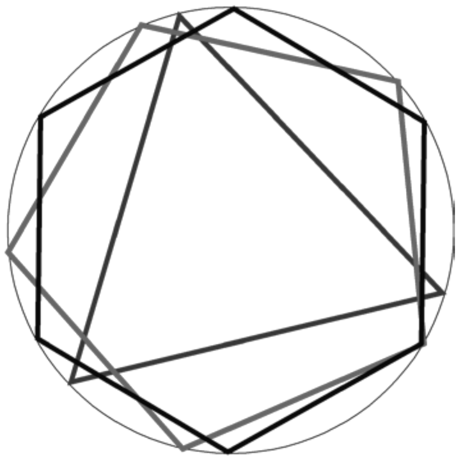


Fig. 3. Representation of three regular-interval lines on a circle
Rys. 3. Przedstawienie trzech regularnych przedziałów linii na cykl

To find an optimum solution means to find for every polygon an angle by which turn this polygon such that arc lengths between subsequent vertices on the circle minimize one of objectives (1), (2), (6). An optimum solution can be found by the following full search algorithm.

1. Define one point of the circle as the point 0.
2. Fix the position of polygon with the least number of vertices such that it’s one vertex is point 0. This polygon will be referred to as a fixed polygon all other polygons will be called free polygons.
3. Examine combinations of all possible positions of all free polygons and choose an optimum position.

A m-gon on the circle can have at most 60/m positions with different values of objectives (1), (2) or (6). Almost always is $m \geq 3$ and therefore the number of positions is less or equal to 20. The number of lines on a common passage is seldom greater than 5. Hence the number of combinations does not exceed $20^4 = 160000$. Today’s computers can realize such calculation within seconds or minutes.

The problem of line coordination was introduced by Černý and Guldan in [2] and is known as the problem of Žilina. Line coordination problem is mentioned also in [1], [3] and [4].

4. TRIP COORDINATION ON WHOLE TRANSPORTATION NETWORK

Considerably complicated situation occurs in greater towns with dense transportation network. One bus or tram line can occur in several distinct passages. The structure of common passages can be very complex as shown in fig. 4.

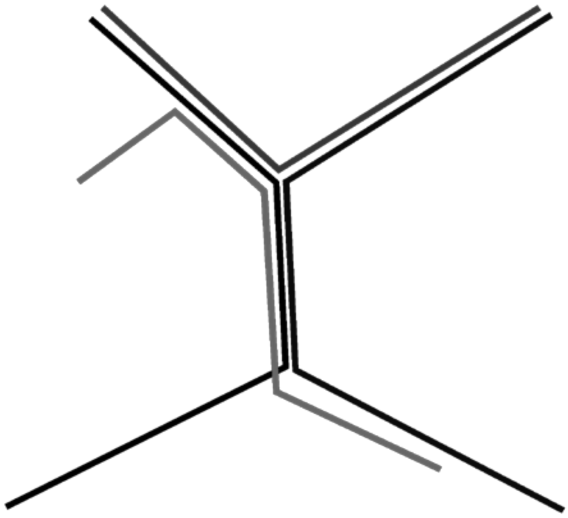


Fig. 4. The structure of common passages of four lines
Rys. 4. Struktura wspólnego przejazdu czterech linii

Identification of common passages in such complicated transportation networks can be done by a computer program as the common segments of the network with at least two lines containing at least four bus stops. It is necessary to assess the density of passengers on every passage. If no data is available it is useful to presume that the passenger density is directly proportional to the length of the common passage and to the number of trips per hour travelling along this passage.

It is desirable to make timetables of all lines such that the trip departures on all selected common passages are as regular as possible. Now the only applicable measure of regularity is the total waiting time of all passengers.

Let us have p common passages $P_1, P_2, \dots, P_n$. Denote by $t_1^{(i)}, t_2^{(i)}, \dots, t_{k_i}^{(i)}$ departures of all trips from the first bus stop of the segment P_i sorted in ascending order. Let f_i be the passenger density on the segment P_i. Then the waiting time of passengers on the segment P_i is

$$W_i\left(t_1^{(i)}, t_2^{(i)}, \dots, t_{k_i}^{(i)}\right) = f_i. \sum_{j=1}^{k_i-1} \left(t_{j+1}^{(i)} - t_j^{(i)}\right)^2$$

and the total waiting time of all passengers on all passages is

$$T = \sum_{i=1}^{n} W_i\left(t_1^{(i)}, t_2^{(i)}, \dots, t_{k_i}^{(i)}\right) = \sum_{i=1}^{n} f_i. \sum_{j=1}^{k_i-1} \left(t_{j+1}^{(i)} - t_j^{(i)}\right)^2$$

We used the following suboptimal neighbourhood search algorithm for finding suboptimal time offsets of line timetables:

1. Start with all line timetables with zero offset
2. Find a line timetable which 1 minute shift forth or back can reduce total waiting time T.
3. If such a line timetable does exist, realize time shift which improves T and go to step 2.
4. If such a line timetable does not exist, STOP, you have a suboptimal solution.

Practical experiences showed that in several cases regularity on common passages achieved by shifting all line timetables is not sufficient. In such cases it is possible to increase degrees of freedom of neighbourhood search algorithm as follows

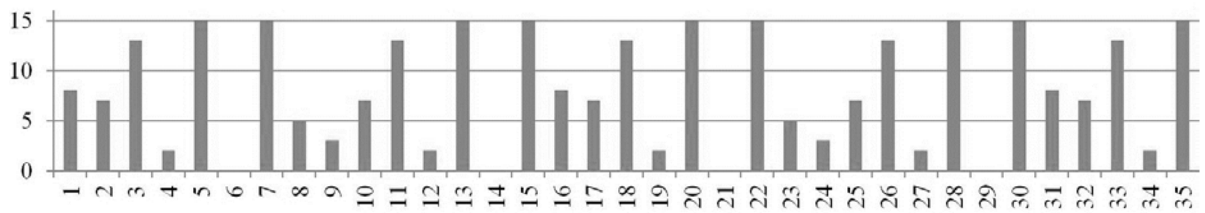
1. Start with a suboptimal timetable obtained by shifting whole line timetables.
2. Find an individual trip whose 1 minute shift forth or back can reduce total waiting time T.
3. If such a trip does exist, realize time shift which improves T and go to step 2.
4. If such a trip does not exist, STOP, you have a suboptimal solution.

However, this attitude can spoil the regularity of line timetables. One way how to ensure only small deterioration of regularity of line timetables is to allow trip shifting in only small intervals and/or to include into computation all line routes as special “one line” common passages.

Computation time of algorithm optimizing regularity on whole network by shifting individual trips was longer – it took from ten minutes to several hours depending on number of lines, trips and passages.

The following figure presents the lengths of intervals between subsequent trips before and after coordination. Numbers on the x-axis denote the sequence number of interval, the height of corresponding column means the length of this interval in minutes. 6-th, 14-th, 21-th and 29-th columns in the graph before coordination have zero height which means that the corresponding intervals have zero length – the buses of two trips came to the bus stop in the same time. Presented results were computed and presented in [6] by my student Ovsák.

Length if i-th interval in minutes between trips before coordination



Length if i-th interval in minutes between trips after coordination

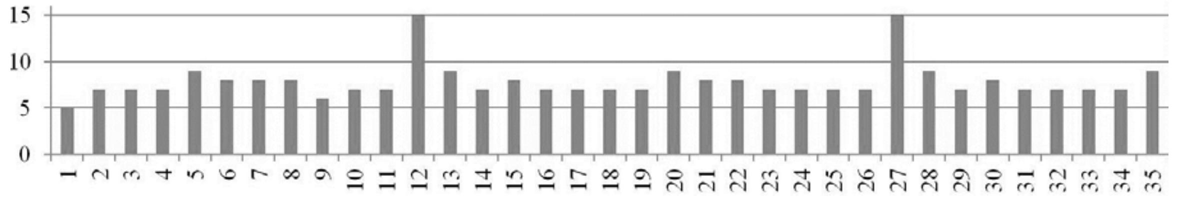


Fig. 5. Illustration of the coordination effect on a common passage of several lines
Rys. 5. Zobrazowanie efektu koordynacji na wspólnym przejeździe kilku linii

Presented results were computed and presented in diploma thesis [6] by my student Ovsák.

5. CONCLUSION

Just described procedures were successfully applied in several municipal bus transport in Slovakia. The effect is apparent improvement of regularity on common passages. One serious problem appeared after application in towns Martin – Vrútky.

Suppose that two lines run along a common passage, line 1 (referred to as the long line) is an extension of line 2 (referred to as the short line). If a bus of a short line arrives to a common passage shortly before a bus of a long line, passengers travelling to short distances can occupy all places in this bus, therefore no place remains for long distance passengers. The bus of the long line comes shortly after the bus of the short line, but it stays half-empty since it is not suitable for long distance passengers. Therefore the regularity improvement algorithms have to be modified in order to avoid such cases.

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