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HYBRID ANALYSIS OF VIBRATION OF THE OVERHEAD TRAVELLING CRANE

Summary. The article presents the results of numerical experiment using a hybrid approach of finite element method and system dynamics simulation of the overhead traveling crane in the case of lifting load. The aim of the analysis was to examine the impact of the load on the crane structure, while lifting the load and vibration measurements in selected areas of the structure. For this purpose were built and used FE model of the object and an example of the proposed phenomenological model of the hoisting mechanism of crane. The presented method is applicable when vibrations are becoming of interest in the desired location points on the structure, which allows to determine the dynamic properties of supporting structure of the crane at the stage of its construction, which in the end allows you to reduce the costs associated with research on the manufactured experimental objects. Depending on the degree of accuracy in representation of real object and the phenomenological and FE model, the calculation accuracy is obtained with a degree of more or less satisfactory, which is dependent on the desired results and application. This method is characterized by two defects, time-consuming calculations because of the need for a very large number of calculation steps in FEA and high accuracy of modeling depending on both the FEM and rigid mass model to the results of the calculations. The problem may occur in the case of complex structures, where is a need to determine the phenomenological model parameters, such as the replacement mass, stiffness and damping.

ANALIZA HYBRYDOWA DRGAŃ SUWNICY POMOSTOWEJ

Streszczenie. W artykule przedstawiono wyniki eksperymentu numerycznego z wykorzystaniem hybrydowego ujęcia metody elementów skończonych oraz symulacji dynamiki ustroju nośnego podczas unoszenia ładunku. Celami analizy były: zbadanie wpływu obciążenia ładunkiem na ustrój nośny suwnicy w trakcie unoszenia ładunku oraz pomiar drgań w wybranych miejscach konstrukcji. W tym celu zbudowano i wykorzystano model MES badanego obiektu oraz przedstawiono przykładowy proponowany model fenomenologiczny mechanizmu podnoszenia suwnicy. Przedstawiona metoda znajduje zastosowanie, w przypadku gdy interesujące stają się drgania w wybranym miejscu konstrukcji, co pozwala na określenie własności dynamicznych ustroju nośnego dźwigni już na etapie jej konstrukcji i w efekcie umożliwia zredukowanie kosztów związanych z badaniami na wytworzonych obiektach eksperymentalnych. W zależności od stopnia dokładności odwzorowania obiektu rzeczywistego w modelu MES oraz fenomenologicznym otrzymuje się dokładność obliczeń o stopniu mniej lub bardziej zadowalającym, co jest zależne od oczekiwanych

efektów i zastosowania. Metoda ta cechuje się dwiema wadami: czasochłonnością obliczeń ze względu na konieczność zastosowania bardzo dużej liczby kroków obliczeniowych MES oraz dużą zależnością dokładności modelowania zarówno od modelu MES, jak i sztywnych mas na wyniki obliczeń. W przypadku złożonych konstrukcji, problemem może okazać się ustalenie parametrów modelu fenomenologicznego, takich jak: zastępcza masa, sztywność czy tłumienie.

1. INTRODUCTION

By analyzing the structure using FEM, it is possible to get the basic information on stress, the form and frequency of natural vibration or deflection caused by the load mass of its own accessories or cargo. In the case of FEM analysis, difficulties arise when it is necessary to model for example extortion caused by a moving trolley or lifting load from the ground. Therefore, in the case of the analysis of complex movement and vibration measurements, FEM modeling becomes insufficient [17]. It is proposed to use a hybrid model involving the coupling of two or more models for the description of physical processes in various ways. Therefore simulations of the dynamics of lifting loads using Matlab Simulink and FEM software (Abaqus) was carried out. To perform these simulations it was necessary to apply the hybrid calculations, where the system response generated from a solved system of differential equations in Matlab Simulink has been used to excite the crane supporting structure during the FEM simulation, which allows to determine the dynamic properties of supporting structure of the crane at the stage of its construction. Application that describes the dynamics of cranes, models with rigid masses and elastic-dissipative elements, with limited to a few (several) number of degrees of freedom does not give complete information about the vibration of the whole structure [11, 12, 15], but by using the hybrid method, after application vibration extortion to supporting structure in the FEM model, it can be observed about the behaviour of the whole structure with accuracy to detail of models.

In presented article, there are proposed the simplest phenomenological models of lifting mechanism. Using such kind of models is justified due to preliminary character of researches on hybrid approach method for connecting phenomenological models and FE models. In future publications, author wants to include a more sophisticated dynamic models of lifting mechanism. They will include more degrees of freedom, and of course more developed model of wire rope which has a very big influence on the dynamic surplus ratio in crane lifting mechanism. Some of proposed models are included in this paper [5, 7, 10]

2. OBJECT OF RESEARCH

In this paper are shown a hybrid vibration analysis of overhead traveling crane with a lifting capacity of 5000 [kg] and the span of the bridge 20 [m]. The described construction (Fig. 1) has one girder and a trolley winch construction which is supported on girder with special supporting arm, that is sliding along the side edge of the girder [6].

In table 1 is shown the general characteristics of the tested overhead traveling crane, prepared on the basis of the technical documentation.



Fig. 1. Experimental research crane – OBRDiUT „Detrans” in Bytom

Rys. 1. Eksperymentalna suwnica badawcza – OBRDiUT „Detrans” w Bytomiu

Table 1

Characteristics of experimental crane

description		symbol	dimension	value
lifting capacity		Q	[kg]	5000
span		L_{mostu}	[m]	20
lifting height		$H_{p\ max}$	[m]	16
operating speed	lifting	v_p	[m/s]	0,208
	winch driving	v_{jw}	[m/s]	0,625
	bridge driving	v_{jm}	[m/s]	0,472
supply voltage		U	[V]	380
maximum load per wheel		P_{max}	[N]	56e3
maximum impact force in the buffer at $0,5 V_{jm}$		Z_u	[N]	30e3

3. HYBRID METHOD FOR MODELLING AND MEASUREMENT OF VIBRATIONS

Now for analysis of dynamical systems used various modifications of the hybrid simulation [18]. In fig. 2 is shown proposed process of modeling and vibration measurements, based on a combination of finite element method and the modeling method of rigid masses.

Determining some important properties of the object in the dynamic model (depending on the expected information) it is possible to determine (with some approximation) the dynamic properties of the supporting structure of crane, or the influence for example the design of the drive system on crane structure, or the driver's cab and itself. In this case the author has obtained on the basis of experimental (involving the lifting process with the tensioned and loosened wire ropes in start-up phase), the acceleration signals of girder center (Fig. 2). These signals will be used for later comparison with those obtained by the simulation and therefore to verify the accuracy and usefulness of the method and of its application.

The basis of the proposed method are two models, the FEM model of the object and phenomenological model. FEM model is used to determine the reduced model parameters. This model allows to obtain information about the static deflections and stresses. The role of the FEM model is giving the opportunity to run an experiment, which allows to simulate the behavior of structures under the influence of for example start-up of lifting process. This will allow the determination of stresses, deflection, or acceleration in the process of lifting the load. The role of the phenomenological model is to generate a response in the form of kinematic extortion used in the FEM model.

The next step is to build a phenomenological model of the object, which allows to simulate the processes in a given object. In this case, it is the process of lifting the load, i.e. its effect on the dynamic deflection of the girder. So it is possible to determine the coefficient of dynamic surplus so

important for the safety of structures of overhead cranes. Depending on the number of considered degrees of freedom of model are obtained more or less accurate results.

Simulation of lifting load allows to use the generated data in the form of kinematic extortion on the structure of crane built by using the FEM. Thus we get the opportunity to obtain a more complete information about the vibration of the whole structure, and the opportunity to make measurements of selected parameters at any points such as acceleration. Then, by performing analysis for example FFT, is possible to get the spectrum of amplitude-frequency representing the oscillation frequency, which are components of this spectrum, essential for verification of resonant zones and for determining the effects on for example operator or structure [4, 9].

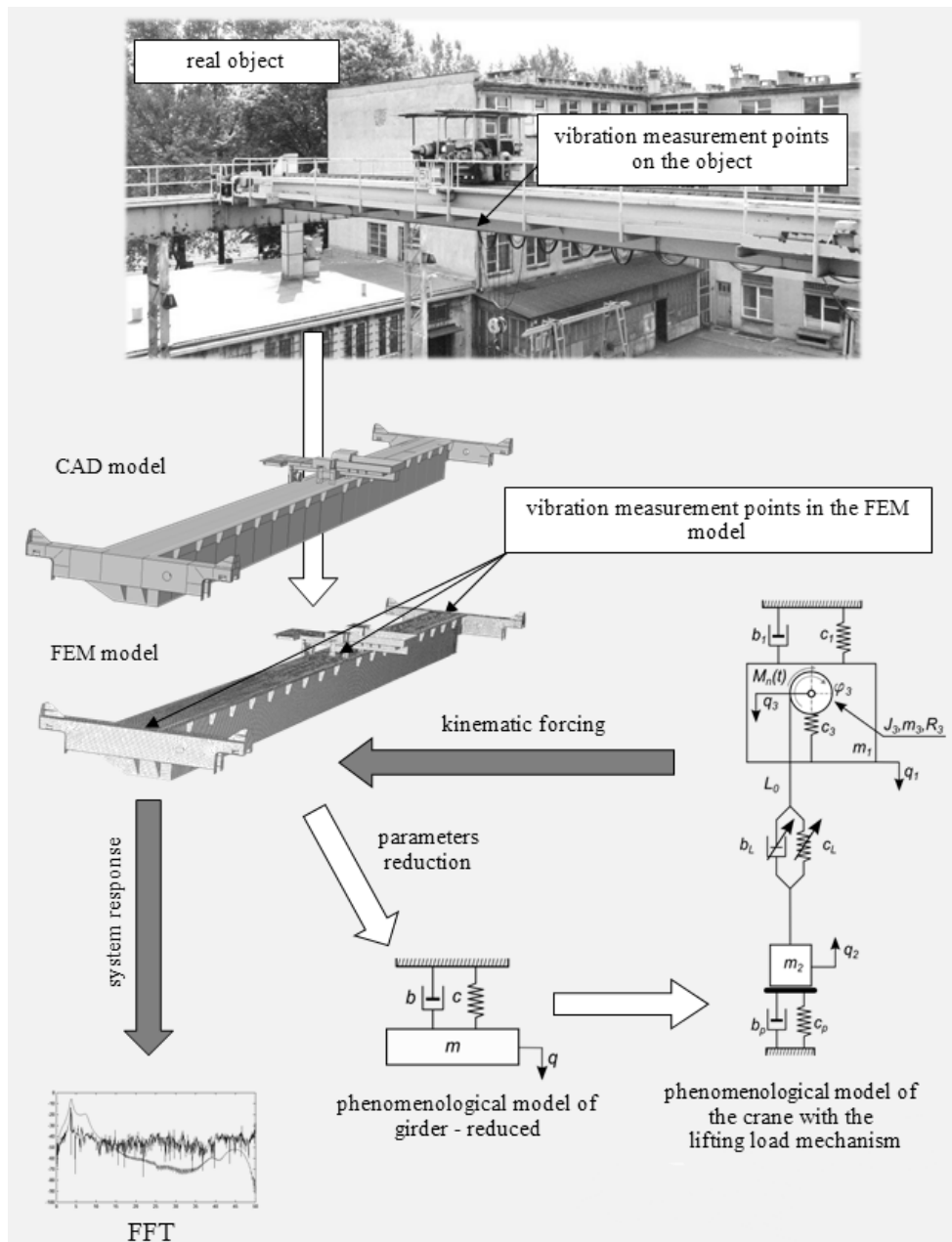


Fig. 2. A hybrid modeling method for vibration measuring

Rys. 2. Schemat hybrydowej metody modelowania i pomiaru drgań

4. PHENOMENOLOGICAL AND FEM MODEL OF CRANE

In the case of using the hybrid method, as mentioned on the introduction, there was a need to create two models, FEM and phenomenological model of overhead traveling crane. For the research, the same FEA model was used that was shown in the work [6]. Model was mainly built with using shell elements S3, S4, and beam type elements B31. Mesh was made up of 138155 elements. Construction of the tested object was built with parts made mainly from steel S235. Figure 3 shows the FEM model of the tested crane, on which the support points are shown (1-4). In support locations (no. 4) six degrees of freedom have been received, in place of support at no.1, 2 has been received only two degrees of freedom: translations in the y-axis (in the direction of movement of the girder) and z-axis (in a direction perpendicular to the plane xy), while at no.3 three degrees of freedom have been received (translations of the x axis, y and z).

A dynamic model of the crane with the lifting mechanism is shown in Figure 4. This model includes in its structure elements such as girder, rope drum, wire rope and ground.

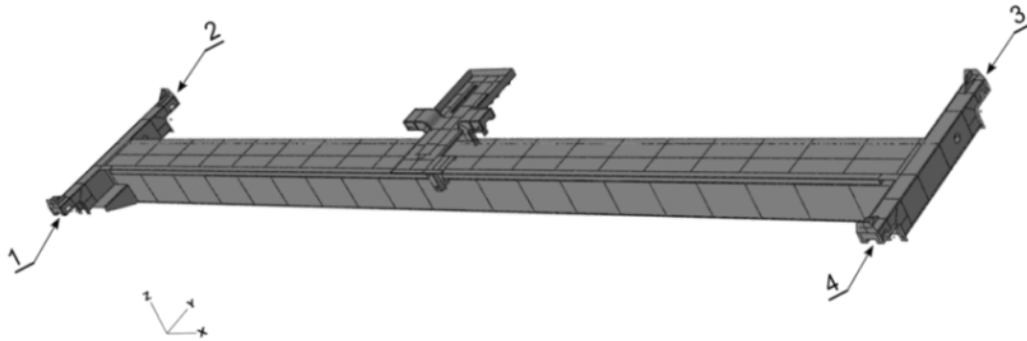


Fig. 3. Boundary conditions
Rys. 3. Warunki brzegowe

Because this system has the ability to directly control the movement of load through the cable drum drive, as an alternative to the solutions of the equation taking into account the additional controls based on the time constants [5], it becomes necessary to take into account, support of load by a ground [14, 15] in the initial phase of the movement of the masses. The value of the stiffness and damping of the ground hasn't significantly impact on the test system (because the load impact on the ground or its blocking, as in the case of freezing is not being taken into account in this paper). The ground acts the role of a platform mainly holding the cargo.

On the basis of the concept of generalized coordinates, and phenomenological model shown in Figure 4, the equations of motion can be written as second type Lagrange equations [1,3]:

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{q}_j} \right) - \frac{\partial E_k}{\partial q_j} + \frac{\partial E_p}{\partial q_j} + \frac{\partial E_R}{\partial \dot{q}_j} = F_j, \quad j = 1, 2, \dots, n \quad (1)$$

where: t – time, q_j – generalized displacement, \dot{q}_j – generalized velocity, n – number of degrees of freedom, F_j – generalized force, E_k – kinetic energy, E_p – potential energy, E_R – energy dissipation function.

This approach allows to obtain the differential equations of motion in the form of kinetic energy of the system:

$$E_k = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2 + \frac{1}{2} J_3 \dot{\phi}_3^2 \quad (2)$$

where: m_1 – reduced mass of girder, m_2 – mass of load, m_3 – mass of the rope drum, J_3 – mass moment of inertia of the rope drum, $\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{\phi}_3$ – generalized velocity.

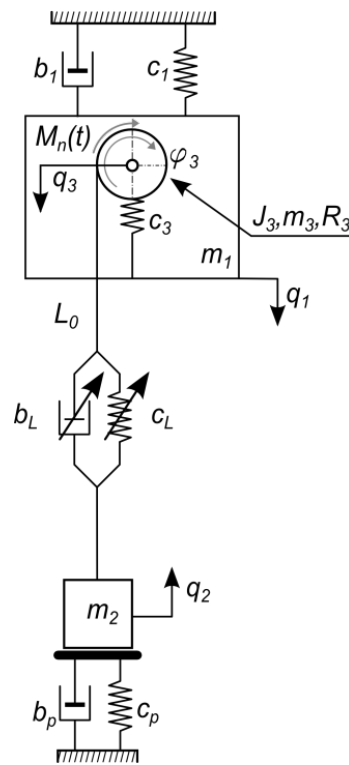


Fig. 4. Simplified phenomenological model of examined overhead traveling crane, which include Kelvin-Voigt model of wire rope

Rys. 4. Uproszczony model fenomenologiczny badanej suwnicy pomostowej, zawierający w gałęzi linowej model Kelvina-Voigta

The potential energy of the system:

$$E_p = \frac{1}{2}c_3(q_1 - q_3)^2 + \frac{1}{2}c_p q_2^2 + \frac{1}{2}c_1 q_1^2 + \frac{1}{2}c_L \left(-q_2 - q_3 + \frac{R_3 \phi_3}{i_w} \right)^2 \quad (3)$$

where: c_3 – stiffness coefficient of the cable drum axle, c_p – stiffness coefficient of ground, c_1 – stiffness coefficient of girder, c_L – stiffness coefficient of wire rope, R_3 – radius of the cable drum, i_w – gear ratio of pulley blocks, q_1 , q_2 , q_3 , ϕ_3 – generalized displacements.

and energy dissipation function:

$$E_R = \frac{1}{2}b_1 \dot{q}_1^2 + \frac{1}{2}b_p \dot{q}_2^2 + \frac{1}{2}b_L \left(-\dot{q}_2 - \dot{q}_3 + \frac{R_3 \dot{\phi}_3}{i_w} \right)^2 \quad (4)$$

where: b_1 – girder damping ratio, b_p – ground damping ratio, b_L – wire rope damping ratio, m_{iny} – mass of rope.

Rope stiffness coefficient defined by the relation (5), and its value depends on the length of the rope:

$$c_L = (n_{lin} \cdot A_l E_l) (L_0 - R_3 \phi_3)^{-1} \text{ gdzie: } E_l = (0,4 \div 0,65) E_s \quad (5)$$

where: n_{lin} – number of bands of wire rope, L_0 – initial length of the rope, E_l – modulus of elasticity, A_l – metallic cross sectional area of wire rope, E_s – Young modulus for steel.

Variable damping coefficient of wire rope strand, are taken from the publications [5, 8]:

$$b_L = 2\zeta \sqrt{c_L (m_2 + m_{iny})} \text{ gdzie: } m_{iny} = n_{lin} \rho_l A_l (L_0 - R_3 \phi_3) \quad (6)$$

where: ζ – dimensionless coefficient, ρ_l – density of steel.

The reaction of a ground N , was made dependent on displacement. At rest, the elastic force and the damping force of the ground affects the cargo. At the time of lifting load, force will be switched off from the system. The value of this reaction is described below:

$$N = \begin{cases} 0 & q_2 \geq 0 \\ -c_p q_2 & q_2 < 0 \end{cases} \quad (7)$$

where: N – elastic response of ground.

In Matlab-Simulink environment the dynamic model was formulated in the form of a flowchart. Table 2 shows the physical parameters describing the considered vibrating model, which are estimated on the basis of the technical documentation of overhead traveling crane, and own research. Then the numerical experiments for the data presented and the assumed initial conditions with the classical model of elastic-damping model (Kelvin-Voigt) for the wire rope was performed.

Due to the character of article, ie, including preliminary studies and somehow a proposal for hybrid modeling, the author considers as the extortion signal a constant driving torque corresponding to very fast start of the engine, without the control system, ie the worst case. However, there are no obstacles to apply the model of an asynchronous machine with the control the motor angular velocity, which was applied by the author in [10, 7]. In accordance with the applicable standard (PN-EN 13001-2:2011 – "Crane safety – General design — Part 2: Load effects" [13]) the hoist drive class HD1, for the lifting mechanisms without creep speed was examined.

Table 2

Physical parameters describing the dynamic system

No.	Symbol	Value	Unit	No.	Symbol	Value	Unit
1	m_1	5000	[kg]	14	L_0	10	[m]
2	m_2	1800	[kg]	15	A_1	5,53e-5	[m ²]
3	m_3	280	[kg]	16	ρ_1	7850	[kg/m ³]
4	m_{liny}	20,5	[kg]	17	E_s	2,1e011	[Pa]
5	J_3	16,15	[kgm ²]	18	E_1	1,155e011	[Pa]
6	c_1	4,6e6	[N/m]	19	g	9,81	[m/s ²]
7	c_p	2,0e8	[N/m]	20	V_p	0,208	[m/s]
8	c_3	1,8e8	[N/m]	22	ω_b	1,67	[rad/s]
9	b_p	1,0e6	[Ns/m]	23	n_{lin}	4	[-]
10	b_1	2,3e4	[Ns/m]	24	ζ	0,07	[-]
11	R_3	0,25	[m]	25	i_p	60	[-]
12	R_{3w}	0,23	[m]	26	i_w	2	[-]
13	d_1	0,012	[m]				

Simulations were carried out using algorithm ode4, with constant step of integration 1E-04 [s]. Simulations were performed for load value of 1800 [kg]. After a series of numerical experiments, many model parameters were obtained, such as girder acceleration and cargo load, and the waveforms changes of forces in wire rope strand. Obtained acceleration waveforms are shown in the next section.

5. SIMULATION RESULTS

Below are shown the results of a hybrid simulation, of examined construction in the form of the dynamic response at selected points, to an applied kinematic extortion (dynamic deflection were determined using Matlab-Simulink software for a phenomenological model, described in point 4). Responses to selected measurement points of the FEM model will be used to determine the main oscillation frequency using Fast Fourier Transform. Plan which shows an arrangement of measurement points in FE model, is shown in Fig. 5.

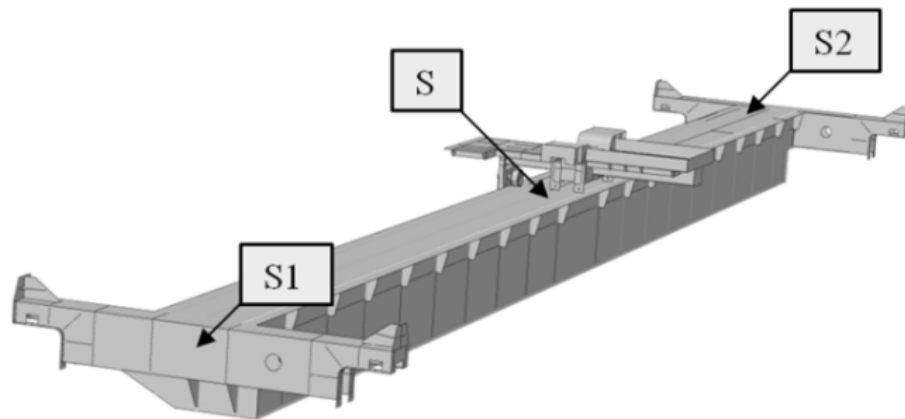


Fig. 5. Scheme of vibration measurements signals in FEM
 Rys. 5. Plan pomiarów sygnałów drganiowych w MES

Calculations were performed using a computing cluster IBM BladeCenter HS21. Calculations were carried out in the Abaqus FEA software, made available through grant: MNiSW/IBM_BC_HS21/PŚląska/021/2010 [19]. The calculations were made for the two load cases: the rope are loose and pre-tensioned in the startup phase. Figure 6 shows the form of the deformation of the supporting structure under the influence of excitation amplitude, for the two considered cases. As can be seen when the wire rope strands are loose in the startup phase, the dynamic deflection is much bigger than in the case of dynamic deflections for the pre-strained wire rope strands. Deflection values for the case when the wire rope strands are pre-tensioned and are 28.25% smaller than the loose wire rope strands. Dynamic deflection is bigger by 52.2% from static deflection for the loose wire rope strands and 33.4% bigger for the strained wire rope strands in the startup phase.

In Fig. 7 - 9, acceleration waveforms, obtained on the research the actual object, and received on the basis of vibration measurements in the FEM (Fig. 5 pts. S) are shown. As can be seen, the waveforms are similar to each other. In addition, the waveforms of vibrations, measured in FEM model for extreme girder points, ie the point. S1 and S2 (Fig. 5) are shown.

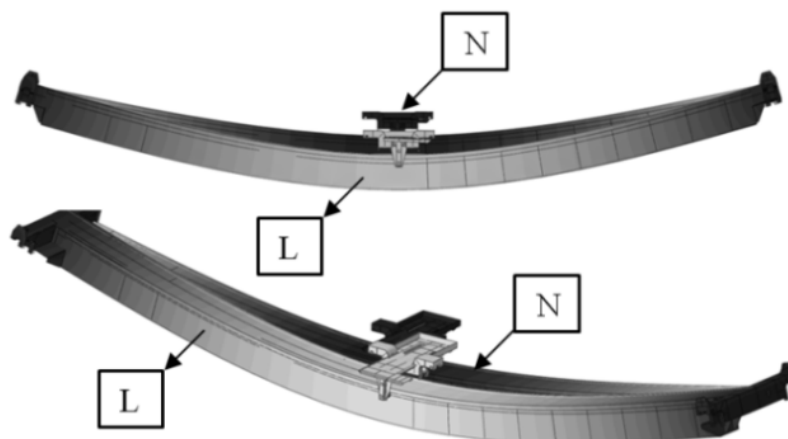


Fig. 6. Form of bent structure under the influence of the maximum amplitude of the load, L – case for loose ropes during start-up, N – the case of the strained ropes in start-up

Rys. 6. Postać ugięta konstrukcji pod wpływem maksymalnej amplitudy wymuszenia, L – przypadek dla cięgien luźnych w czasie rozruchu, N – przypadek dla cięgien napiętych w fazie rozruchu

On the crane operator not only the values of amplitude of vibration or acceleration but also the frequencies of those signals may influence. To determine the natural frequency at assumed measuring points in structure (Fig. 5), a fast Fourier transform has been used. By plotting output values (value of the normalized module of the complex number [16]) FFT as a function of the frequency, amplitude spectrum of vibration signal was obtained, on the basis of which fundamental oscillation frequency in the specified locations were determined.

For the shown vibration signals the FFT amplitude spectra are presented (Fig. 10, 11) for vibration signals extracted from the FEM (point S1, S2 Fig. 5) and the real object at the point, which is on half the span of girder (point S Fig. 5). As can be seen, the main oscillation frequency overlap each other.

It can be seen that primary oscillation frequency appearing at the ends of the girder is 3.8 Hz and average value is 7.5 Hz in the direction of lifting load. As can be seen from the data presented in the paper [2], frequencies from 4 to 10 Hz are the cause of increasing the burdensome work and the emergence of feelings of pain in the chest and abdominal cavities, resonant vibration of the head is in a range of 8 to 27 Hz and can be the cause of visual acuity decrease. It should be noted that the frequency of 3.8 Hz at the place of fixing cargo coincides with the frequency of vibration (corresponding to the form of lifting) defined analytically, which would indicate the accuracy of the analyzes.

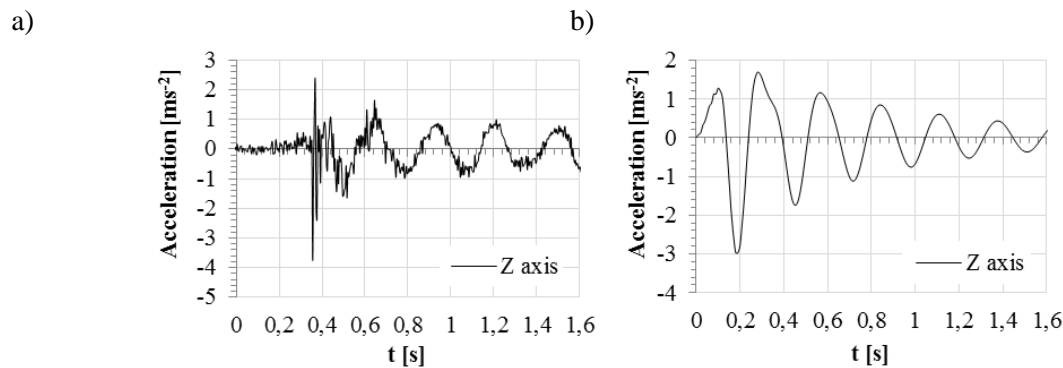


Fig. 7. Acceleration waveforms measured in the middle of the span of girder for the case when the ropes are loose at start-up: a) real signal, b) the signal from the measurements in FEM

Rys. 7. Przebiegi czasowe przyspieszeń mierzone w środku rozpiętości dźwigara dla przypadku, gdy ciągną są luzne w fazie rozruchu: a) sygnał rzeczywisty, b) sygnał otrzymany na drodze pomiaru hybrydowego

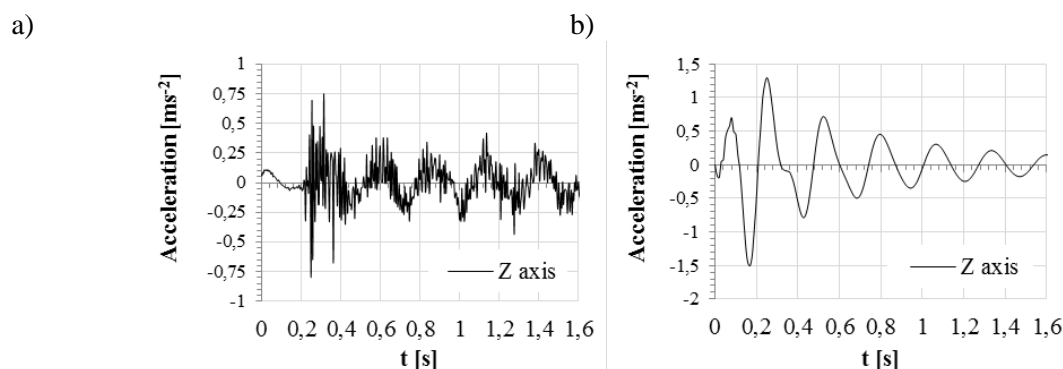


Fig. 8. Acceleration waveforms measured in the middle of the span of girder (point S fig. 5) for the case when the ropes are strained at start-up: a) real signal, b) the signal from the measurements in FEM

Rys. 8. Przebiegi czasowe przyspieszeń mierzone w środku rozpiętości dźwigara (pkt S, rys. 5) dla przypadku, gdy ciągną są napięte w fazie rozruchu: a) sygnał rzeczywisty, b) sygnał otrzymany na drodze pomiaru hybrydowego

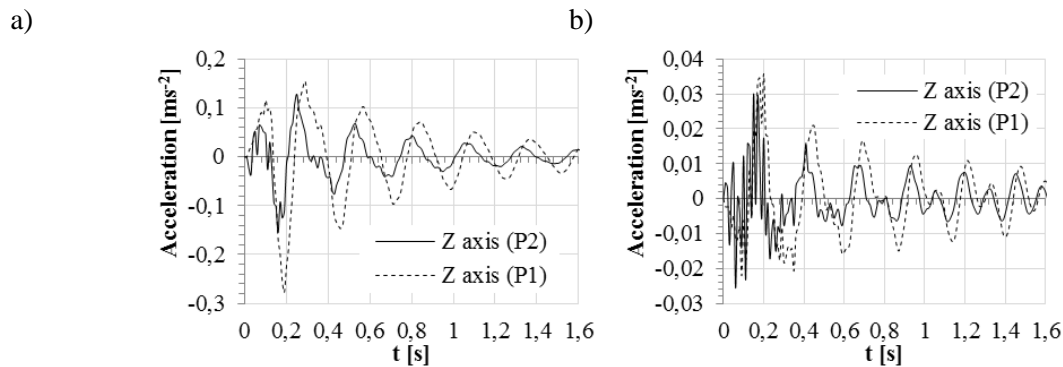


Fig. 9. Acceleration waveforms measured in the extreme positions of the girder for: a) signal obtained by using the hybrid model - P1 when the ropes are loose in the start-up phase, P2 - when the ropes are strained in the start-up phase (point S1 fig. 5) b) signal obtained by using the hybrid model (point S2 fig. 5)

Rys. 9. Przebiegi czasowe przyspieszeń mierzone w skrajnych położeniach dźwigara dla przypadku: a) sygnał otrzymany na drodze pomiaru hybrydowego – P1, gdy ciężna są luźne w fazie rozruchu, P2 – gdy ciężna są napięte w fazie rozruchu (pkt S1, rys. 5) b) sygnał otrzymany na drodze pomiaru hybrydowego (pkt S2, rys. 5)

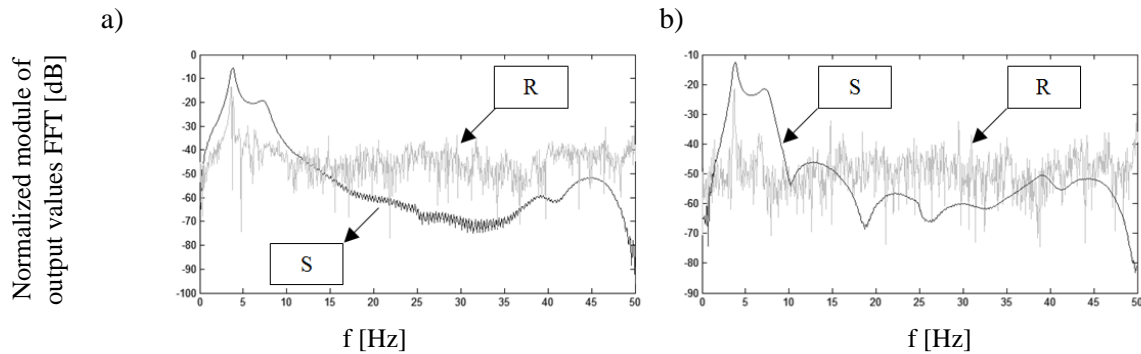


Fig. 10. Amplitude spectrum of the vibration signal measured in the middle of the girder: a) in the case of loose ropes in start-up phase, b) in the case of strained ropes in start-up phase, where S - the spectrum obtained by simulation, R - spectrum obtained by measuring's at the object

Rys. 10. Widmo amplitudowe dla sygnału drganiowego mierzonego w środku dźwigara: a) przypadek luźnych ciężen w fazie rozruchu, b) przypadek napiętych ciężen w fazie rozruchu, gdzie S – widmo otrzymane na drodze symulacji, R – widmo otrzymane na drodze pomiaru na obiekcie

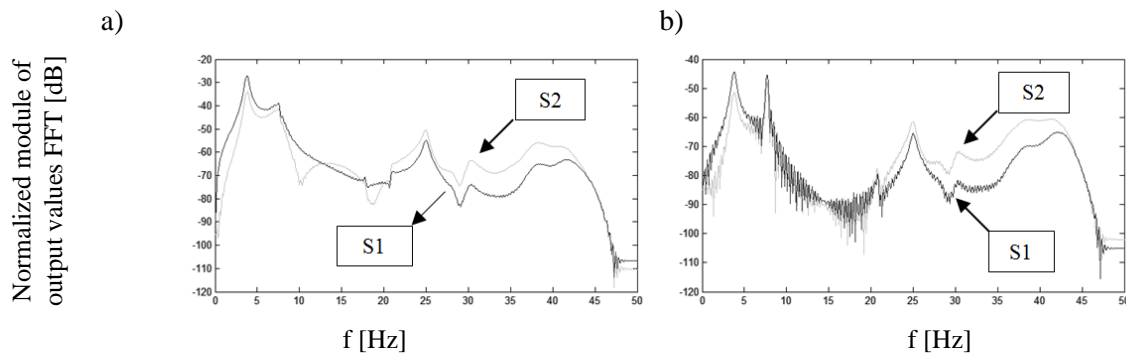


Fig. 11. Amplitude spectrum of the vibration signal measured in points S1, S2 (fig. 5): a) in the case of loose ropes in start-up phase, b) in the case of strained ropes in start-up phase

Rys. 11. Widmo amplitudowe dla sygnału drganiowego mierzonego w punktach S1, S2 (rys. 5): a) przypadek luźnych ciężen w fazie rozruchu, b) przypadek napiętych ciężen w fazie rozruchu

6. CONCLUSIONS

Presented FE and rigid masses models can be the basis for research, based on a so called hybrid approach, which combines the dynamic model with the FE model. As presented, vibration signals measured on FE model are similar to those obtained by experiment on real object, despite using a very simple phenomenological model of overhead traveling crane with a lifting mechanism. Usage of correspondingly more complex models, taking into account the multi-mass girder and drive system, and accurately replicated model of a wire ropes, allows to obtain more accurate results.

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