tractive drive, friction self-oscillations, energy balance,
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THE INFLUENCE OF ELECTROMAGNETIC PROCESSES ON STABILITY OF LOCOMOTIVES TRACTION DRIVE IN THE SLIPPING MODE

Summary. On the model of a locomotive traction drive in a slipping mode, it is demonstrated how the electromagnetic transient processes in the traction motor may be accounted for using the energy balance method for assessing the stability of the system with respect to frictional self-oscillations. The regions of existence of frictional self-oscillations in the parameter space of the drive are built. The proposed method differs from the prior art in that it can be used to analyze the stability of a wide class of dynamical systems with small damping, containing both oscillatory and inertial units.

ВЛИЯНИЕ ЭЛЕКТРОМАГНИТНЫХ ПРОЦЕССОВ НА УСТОЙЧИВОСТЬ ТЯГОВОГО ПРИВОДА ЛОКОМОТИВА В РЕЖИМЕ БОКСОВАНИЯ

Аннотация. На примере модели тягового привода локомотива в режиме боксования показано, каким образом могут быть учтены электромагнитные переходные процессы в тяговом двигателе при использовании метода энергетического баланса для оценки устойчивости системы по отношению к фрикционным автоколебаниям. Построены области существования фрикционных автоколебаний в пространстве параметров привода. Предлагаемый метод может быть использован для анализа устойчивости широкого класса динамических систем с малой диссипацией, содержащих как колебательные, так и инерционные звенья.

1. INTRODUCTION

Slipping mode is one of the heaviest modes of operation of the locomotive traction drive. The reasons of the high loads are frictional auto-oscillations that develop in a rotating system of traction drive.

At the design stage or upgrading the traction drive is necessary to choose such of its elastic and dissipative parameters in which auto-oscillations are not possible.

Thus, the task of choosing rational parameters of the traction drive is to ensure the stability of a dynamical system with respect to the frictional self-oscillations in the mode of slippage.

In this paper we propose a method that allows for a given stiffness and inertial parameters of the drive to obtain such relations between the dissipative parameters and characteristics of the traction motor (TEM) that preclude the development of frictional auto-oscillations.

2. ANALYSIS OF RESEARCHES AND PUBLICATIONS

In [1] proposed a method for estimating the stability of dynamic systems based on energy balance of the oscillations in each of their own forms. The advantage of this method is that the stability conditions are recorded in the dissipative parameters of the dynamic system. However, with simplicity and clarity, it cannot be used when in the form in which it is interpreted in [1], to study the stability of systems containing both oscillators and inertial units. Therefore, the computational models of the locomotive traction drives, the stability of which was estimated by this method, the electromagnetic processes in the TEM excluded from consideration or been investigated separately.

In [2] attempts to take into account the dynamic model of the traction drive electromagnetic transients in TEM due to the introduction of an additional oscillator, the choice of parameters which may be a certain arbitrariness and loss of physical significance that cannot be considered correct. A simplified simulation approach is presented in [5].

3. THE PURPOSE OF RESEARCH

The purpose is – to assess the influence of electro-mechanical characteristics of the TEM on the stability of the traction drive with respect to self-oscillations in the mode of slipping by the method of energy balance. Thus - to confirm the possibility of applying this method to analyze the stability of oscillatory systems with small dissipation, containing inertional elements.

4. MATERIALS AND RESULTS OF RESEARCH

Slipping occurs when the traction torque M_T applied to a pair of wheel traction mode M_{*0}, is greater than the maximum total moment coupling of wheels to the rails M_C, resulting in an equilibrium mode $[M_*, \dot{\varphi}_*]$ which corresponds to the descending part of the characteristic traction wheels with rails – Fig. 1a.

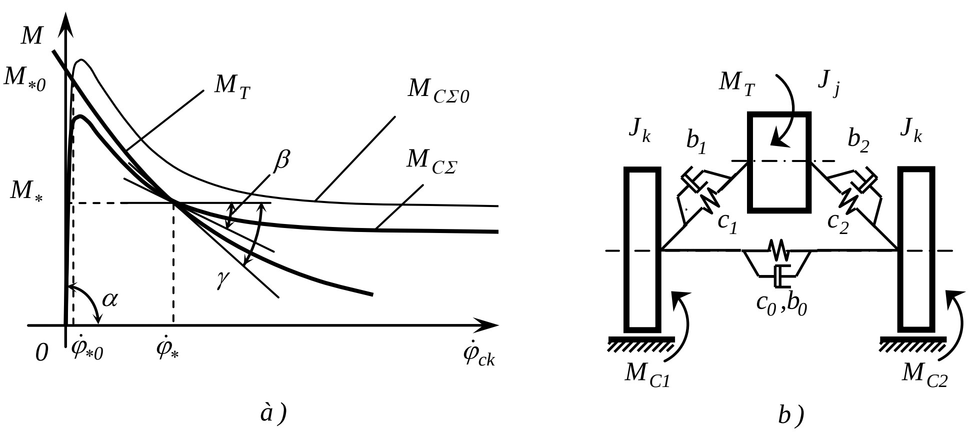

Fig. 1. The relative position of adhesion characteristics and traction characteristics in slipping mode - *a*. The scheme of model of traction drive - *b*

Рис. 1. Взаимное расположение характеристики сцепления и тяговой характеристики в режиме боксования - *a*. Схема модели тягового привода - *b*

Usually, slipping occurs when the motion locomotive occurs with low velocities and it is possible to neglect the influence of forced vibrations on the torsional system of drive.

Traction drive model can be represented in the form of a three-mass rotating system (Fig. 1b), whose motion is described by a system of differential equations:

$$\left.\begin{aligned}
&T_e\dot{M} + M = M_T(\dot{\varphi}_j);\\
&J_j\ddot{\varphi}_j + b_1\dot{\Delta}_1 + c_1\Delta_1 + b_2\dot{\Delta}_2 + c_2\Delta_2 = M;\\
&J_k\ddot{\varphi}_{k1} - b_1\dot{\Delta}_1 - c_1\Delta_1 + b_0\dot{\Delta}_0 + c_0\Delta_0 = -M_{C1}(\dot{\varphi}_{k1});\\
&J_k\ddot{\varphi}_{k2} - b_2\dot{\Delta}_2 - c_2\Delta_2 - b_0\dot{\Delta}_0 - c_0\Delta_0 = -M_{C2}(\dot{\varphi}_{k2}).
\end{aligned}\right\} \quad (1)$$

where: $\Delta_s = \varphi_j - \varphi_{ks}$, φ_j and φ_{ks} – the angular coordinates of the TEM rotor and s-th wheel of wheel pair (s=1,2) respectively; J_j and J_k – the moment of inertia of the TEM rotor referred to a wheel pair axle and the moment of inertia of the wheel of wheel pair relative to the axis of rotation; c_s and b_s – the elastic and damping characteristics of the connection between the TEM rotor and the s-th wheel; c_0 and b_0 – the elastic and damping characteristics of the axle of the wheel pair; $M_T(\dot{\varphi}_j)$ – the quasi-static traction characteristics of the TEM; $M_{Cs}(\dot{\varphi}_{ks})$ – the adhesion characteristics of s-th wheel of wheel pair; T_e – the time constant, which takes into account the electromagnetic transients in the TEM.

The first equation of system (1) reflects the electromagnetic processes in the TEM, as recommended in [1]. This approach is widely accepted for those cases when the research focuses on the mechanical part of the traction drive.

Building of the regions of existence of frictional self-oscillations in the parameter space of the drive is reduced to determining the value of the damping coefficients, in which the dynamical system is on the boundary of stability.

Consider the stability of the system "in the small" in the neighborhood of the equilibrium mode $[M_*, \dot{\varphi}_*]$ – Fig. 1a. For this, move to the new coordinates, which excludes the permanent components of the moments and angular velocities, and we linearize the system (1) in the vicinity of this mode.

The linearized system (1) becomes:

$$\left.\begin{aligned}
&T_e\dot{M} + M = -\gamma\dot{\varphi}_j;\\
&J_j\ddot{\varphi}_j + b_1\dot{\Delta}_1 + c_1\Delta_1 + b_2\dot{\Delta}_2 + c_2\Delta_2 = M;\\
&J_k\ddot{\varphi}_{k1} - b_1\dot{\Delta}_1 - c_1\Delta_1 + b_0\dot{\Delta}_0 + c_0\Delta_0 = \beta_1\dot{\varphi}_{k1};\\
&J_k\ddot{\varphi}_{k2} - b_2\dot{\Delta}_2 - c_2\Delta_2 - b_0\dot{\Delta}_0 - c_0\Delta_0 = \beta_2\dot{\varphi}_{k2}.
\end{aligned}\right\} \quad (2)$$

where: γ and $\beta_{1,2}$ – respectively, the slope coefficients of the traction characteristics and adhesion characteristics in the vicinity of the mode $[M_*, \dot{\varphi}_*]$.

We believe that the conditions of adhesion wheels with the rails are the same $M_{C1}(\dot{\varphi}_{k1}) = M_{C2}(\dot{\varphi}_{k2}) = 0{,}5M_{C\Sigma}(\dot{\varphi}_k)$. Therefore, in future take: $\beta_1 = \beta_2 = 0{,}5\beta$ – Fig. 1a.

If the system is on the boundary of stability, the condition of energy balance implies that the average power supplied to the system E_i^+ in the process of self-excited oscillations with i-th natural frequency equal to the average power dissipated in the process of self-oscillations E_i^- – [1].

Self-oscillations in systems with small dissipation similar to the harmonic and occur with frequencies approximately equal to the natural frequencies of vibration of the system. The influence of dissipative parameters on the natural frequencies and eigenvectors are negligible. And since the natural forms are orthogonal, it is possible to make the ratio of the energy balance separately for each of natural forms [1, 4, 6].

In view of the above considerations the natural frequencies of the dynamical system with small dissipation can be determined without taking into account the damping coefficients.

Assuming that the generalized coordinates of model vary harmonically, we define the average for the period of power supplied into the system in the process of self-excited oscillations with natural frequency ω_i:

$$E_i^+ = \frac{1}{T_i}\int_0^{T_i} \beta_1(\dot{\varphi}_{k1i}^2 + \dot{\varphi}_{k2i}^2)\sin^2(\omega_i t)dt = \frac{1}{2}\beta_1(\dot{\varphi}_{k1i}^2 + \dot{\varphi}_{k2i}^2), \tag{3}$$

where: $T_i = \dfrac{2\pi}{\omega_i}$ – the period of oscillation with a natural frequency ω_i.

The power E_{mi}^- dissipated due to mechanical damping in the elements of the drive is:

$$E_{mi}^- = \frac{1}{T_i}\int_0^{T_i} [b(\dot{\Delta}_{1i}^2 + \dot{\Delta}_{2i}^2) + b_0\dot{\Delta}_{0i}^2]\sin^2(\omega_i t)dt = \frac{1}{2}[b(\dot{\Delta}_{1i}^2 + \dot{\Delta}_{2i}^2) + b_0\dot{\Delta}_{0i}^2]. \tag{4}$$

To determine the average power E_{ei}^- dissipated in the TEM, consider the first equation of system (2). In it the variable M is dissipative torque, reflecting the electromechanical parameters of TEM. Then the average for the period of the power dissipated by this torque in the process of oscillations is:

$$E_{ei}^- = \frac{1}{T_i}\int_0^{T_i} M\dot{\varphi}_{ji}dt, \tag{5}$$

Assuming that $M(t) = M\sin(\omega_i t + \psi_i)$ we find the modulus of torque M, solve the first equation of system (2) by the method of complex amplitudes:

$$|M| = \gamma\dot{\varphi}_{ji}\frac{1}{\sqrt{(T_e\omega_i)^2 + 1}} = \gamma\dot{\varphi}_{ji}|W(\omega_i)|, \tag{6}$$

where: $|W(\omega_i)|$ – the amplitude-frequency characteristic (AFC) inertial link; $\psi_i = -arctg(T_e\omega_i)$ – phase angle between the speed $\dot{\varphi}_{ji}$ and torque M.

Then the expression (5) with (6) becomes:

$$E_{ei}^- = \frac{1}{T_i}\int_0^{T_i} |M|\dot{\varphi}_{ji}\sin(\omega_i t)\sin(\omega_i t + \psi_i)dt = \frac{1}{2}\gamma|W(\omega_i)|^2\dot{\varphi}_{ji}^2 = \frac{1}{2}\gamma_{*i}\dot{\varphi}_{ji}^2, \tag{7}$$

$$\gamma_{*i} = \gamma|W(\omega_i)|^2 = \gamma\frac{1}{(T_e\omega_i)^2 + 1}, \tag{8}$$

where: γ_{*i} – the dynamic slope coefficient of the traction characteristics of TEM, which depends from the time constant of the electrical machine and frequency of vibration of the rotor.

Arguing in terms of control theory, we note that the first equation of system (2) describes the inertial link of the first order.

Such a link may change the phase angle between input and output in the range $\psi = 0...\pi/2$.

With the assumption of small dissipation phase angle does not affect the frequency and the coefficients of forms of oscillations, but in calculating the power dissipation the phase angle must be considered.

In general, the power is: $E = |M|\cdot|\dot{\varphi}|\cos\psi$.

Because

$$\cos\psi = \cos[arctg(T\omega)] = \frac{1}{\sqrt{(T\omega)^2 + 1}},$$

then in the expression (8) the phase angle is implicitly taken into account.

AFC of first-order inertial link has the characteristic parameter – the frequency of conjugation $\omega_c = 1/T_e$, on which is the intersection of asymptotes of its logarithmic AFC.

If $\omega_i << \omega_c$, then you can take $\gamma_{*i} = \gamma$ and $\psi_i = 0$. If $\omega_i >> \omega_c$, then you can take $\gamma_{*i} = 0$ and $\psi_i = -\pi/2$. If the frequency ω_i is comparable with the frequency ω_c, it is advisable to calculate the γ_* use the expression (8). So if $\omega_i = \omega_c$, we have $\gamma_{*i} = 0{,}5\gamma$ and $\psi_i = -\pi/4$.

All of the average power dissipated in the drive during the oscillations is equal to:

$$E_i^- = E_{mi}^- + E_{ei}^- = \frac{1}{2}[\,b(\dot{\Delta}_{1i}^2 + \dot{\Delta}_{2i}^2) + b_0\dot{\Delta}_{0i}^2 + \gamma_{*i}\dot{\varphi}_{ji}^2\,]\,. \tag{9}$$

The condition at which the input power and the dissipated power are equal $E^+ = E^-$ and the system is on the boundary of stability is given by:

$$\beta_1(\dot{\varphi}_{k1i}^2 + \dot{\varphi}_{k2i}^2) = b(\dot{\Delta}_{1i}^2 + \dot{\Delta}_{2i}^2) + b_0\dot{\Delta}_{0i}^2 + \gamma_{*i}\dot{\varphi}_{ji}^2\,. \tag{10}$$

Equation (10) can be written using the coefficients of forms of vibration:

$$\beta_1(\mu_{k1i}^2 + \mu_{k2i}^2) = b[(\mu_{ji} - \mu_{k1i})^2 + (\mu_{ji} - \mu_{k2i})^2\,] + b_0(\mu_{k1i} - \mu_{k21})^2 + \gamma_{*i}\mu_{ji}^2\,, \tag{11}$$

from which we find boundary damping $\overline{b}_i$ for the oscillations with the natural frequency ω_i:

$$\overline{b}_i = \frac{\beta_1(\mu_{k1i}^2 + \mu_{k2i}^2) - b_0(\mu_{k1i} - \mu_{k21})^2 - \gamma_{*i}\mu_{ji}^2}{(\mu_{ji} - \mu_{k1i})^2 + (\mu_{ji} - \mu_{k2i})^2}\,. \tag{12}$$

For the bilateral traction drive there are relations: $c_1 = c_2 = c$ and $b_1 = b_2 = b$. It has two possible modes of vibration:

- the inphase mode (low frequency) at which the wheels of the wheel pair moving with the same phase, the oscillation frequency equal: $\omega_1 = \sqrt{2c(\frac{1}{J_j} + \frac{1}{2J_k})}$ and the coefficients of forms equal: $\mu_{k11} = \mu_{k22} = 1$ and $\mu_{j1} = -\frac{2J_k}{J_j}$;

- the antiphase mode (high frequency) with a node in a wheel pair, at which the wheels are moving in the opposite direction, the oscillation frequency equal: $\omega_2 = \sqrt{\frac{(c + 2c_0)}{J_k}}$ and the coefficients of the form equal: $\mu_{k22} = -\mu_{k12} = -1$ and $\mu_{j2} = 0$.

Note that the coefficients of the natural forms of the double-sided traction drive depend only on the ratio of inertial parameters and do not depend on the elastic parameters of its elements.

Damping in the wheel pair is negligible. Therefore, we can take: $b_0 = 0$. Then, for the low-frequency mode of vibration, expression (12) becomes:

$$\overline{b}_1 = \frac{2\beta_1 - \gamma_{*1}\mu_{j1}^2}{2(\mu_{ji} - 1)^2}\,. \tag{13}$$

For high-frequency mode of vibration from expression (12) we obtain:

$$\overline{b}_2 = \beta_1\,. \tag{14}$$

Comparing expressions (13) and (14), we can say that $\overline{b}_2 > \overline{b}_1$. This means that if in the bilateral drive is provided damping $b \ge \overline{b}_2$, precluding the development of high-frequency oscillation, then low-frequency oscillations will not occur, regardless of the electromagnetic processes in the TEM.

In unilateral traction drive $c_2 = 0$ and $b_2 = 0$. In this case the coefficients of natural forms depend on both the inertial parameters of drive, and on the coefficients of stiffness c_1 and c_0.

For unilateral drive with $b_0 = 0$ expression (12) takes the form:

$$\overline{b}_i = \frac{\beta_1(1+\mu_{k2i}^2) - \gamma_{*i}\mu_{ji}^2}{(\mu_{ji} - 1)^2}. \tag{15}$$

The natural frequencies of oscillations $\omega_{1,2}$ in unilateral drive can be found from the equation:

$$\omega^4 - [c(\frac{1}{J_k} + \frac{1}{J_j}) + 2\frac{c_0}{J_k}]\omega^2 + \frac{c_0 c}{J_j J_k}(2 + \frac{J_j}{J_k}) = 0. \tag{16}$$

The coefficients of natural forms: $\mu_{k1i} = 1$, $\mu_{k2i} = \dfrac{c_0}{-\omega_i^2 J_k + c_0}$, $\mu_{ji} = \dfrac{c_k}{-\omega_i^2 J_j + c_k}$.

Using of (15) and (16) for the unilateral traction drive plotted the natural frequencies of the system $\omega_{1,2}$ as a function of the relationship $\varepsilon = c/c_0$ (Fig. 2a) as well as plotted the regions of existence of frictional self-oscillations for $T_e = 0$, $T_e = 0{,}01\,s$ и $T_e = 0{,}3\,s$ – Fig. 2b.

The model parameters are taken close to the real parameters of the system "traction drive – the rails":

$J_j = 1{,}2tm^2$, $J_k = 0{,}2tm^2$, $c_0 = 10^4\ kNm$, $c = 10^3 ... 10^4\ kNm$, $\beta = 6\ kNms$, $\gamma = 10\ kNms$.

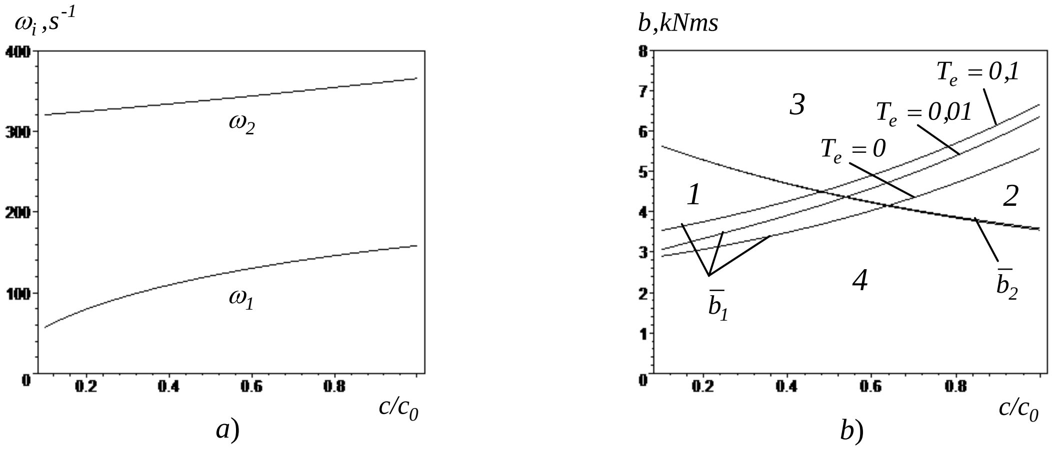

Fig. 2. The natural frequencies and the regions of existence of friction oscillations in the unilateral traction drive
Рис. 2. Собственные частоты и области существования фрикционных автоколебаний в одностороннем тяговом приводе

If the damping in the drive corresponds to the region located below the line $\overline{b}_1$ on Fig. 2b, then in system possible develop low-frequency oscillations. If the damping corresponds to the region located below the line $\overline{b}_2$, then in system possible develop high self-oscillation.

The lines $\overline{b}_1$ and $\overline{b}_2$ separate the area of the change parameter b and $\varepsilon = c/c_0$ on four zones.

In the zone 1 are possible only high-frequency self-oscillations. In zone 2 are possible only low-frequency self-oscillations. If the damping corresponds to the zone 3, then the self-oscillations in system do not develop. In zone 4 single-frequency (with low- or high-frequency) and two-frequency self-oscillatory regimes may develop.

In this case, it is important to note the influence of electromagnetic processes in TEM on the regions of existence of self-oscillations.

From Fig. 2b follows that the change of time constant of TEM practically does not influence upon boundary of the high-frequency oscillation – lines $\overline{b}_2$ for different values of T_e nearly coincide.

With increasing the time constant T_e decreases the value of γ_{*i}, which leads an increase to the coefficient of damping $\bar{b}_1$ for the low-frequency oscillations in accordance with expressions (8) and (15). Consequently, the increase in the time constant of TEM extends the zone of existence of low-frequency oscillations – Fig. 2b.

For the taken parameters of the traction drive when the time constant changes within $T_e = 0...0{,}3\,s$ the change of coefficient of damping $\bar{b}_1$ does not exceed 10-12%.

From expression (12) follows that influence γ_{*i} on value of the damping coefficient $\bar{b}_i$ depends on the coefficient of form μ_{ji}. So, than inertial moment of the rotor TEM J_j less , that more coefficient of form μ_{ji}, and that greater influence render the electromagnetic processes on stability of the tractive drive in respect to the frictional self-oscillations.

5. CONCLUSIONS

1. The grounded possibility of using the method of the energy balance for estimation of stability of the oscillatory systems with small dissipation, containing a periodic inertional links.
2. Based on the energy balance of the natural modes of oscillations for the model of a locomotive traction drive are obtained expressions for determining the influence of electromagnetic parameters (time constant of TEM) on its stability in relation to friction self-oscillations.
3. For real stiffness and inertial parameters of the traction drive the value of time constant of TEM has virtually no effect on the stability of the high-frequency oscillation with a knot in the axis of the wheel pair.
4. Increasing the time constant of TEM expands region of existence of low-frequency oscillations in the space of elastic-dissipative parameters of the drive.
5. Effect of time constant of TEM on the stability of the traction drive increases with decreasing the moment of inertia of the rotor TEM.
6. The proposed method of assessing the stability differs from the prior art in that it can be used to analyze the stability of a wide class of dynamical systems with small damping, containing both oscillatory and inertial units.
7. Obtained on the model of a locomotive traction drive the regularities can be used to assess the stability of a wide class of dynamical systems.

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