long and heavy trains; fuel and energy resources; performance charts; traction mode; longitudinal dynamic efforts; mathematical modeling

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METHODS OF CALCULATION LINE OPTIMUM TRAVEL OF TRAINS WITH CONSIDERATION OF LONGITUDINAL DYNAMIC EFFORTS

Summary. The paper examines the main provisions of the methods used for optimization of station-to-station travel time taken (according to the criteria of minimum fuel consumption for haulage) for freight long train timing with consideration of longitudinal dynamic efforts. A hybrid mathematical model of long train travel is developed, where in one case of travel the train is considered as a flexible non-extensible line with a variable running weight, in other cases the train is considered as a system of weights connected by nonlinear deformable elements with a consideration of gap clearance in a draft. The model allows essentially reducing the time of solving the problem. The problem of selecting an optimum mode of train operation with consideration of longitudinal dynamic efforts and use of method of dynamic programming in a discrete form was set.

МЕТОДИКА РАСЧЕТА ОПТИМАЛЬНЫХ ПЕРЕГОННЫХ ВРЕМЕН ХОДА ПОЕЗДОВ С УЧЁТОМ ПРОДОЛЬНЫХ ДИНАМИЧЕСКИХ УСИЛИЙ

Резюме. Рассмотрены основные положения методики оптимизации (по критерию минимума расхода топлива на тягу) перегонных времен хода к графику движения грузовых длинносоставных поездов с учётом продольных динамических усилий. Разработана гибридная математическая модель движения длинносоставного поезда, в которой в одних случаях движения поезд рассматривается, как гибкая нерастяжимая нить с переменной погонной массой, в других, как система масс, соединенных нелинейными деформируемыми элементами, учитывающими наличие зазоров в упряжи. Такая модель позволяет существенно сократить время решения задачи. Поставлена задача выбора оптимального режима ведения поезда с учетом продольных динамических усилий с использованием метода динамического программирования в дискретной форме.

1. INTRODUCTION

Increase of traffic-carrying and train-handling capacities shall be considered as the basic direction of development of the railway transportation in Lithuania. Therefore, the increase of weight and length of freight trains is one of the important and effective directions of engineering policy in transport. The increase of traffic-carrying capacities of the railways can be achieved by organizing regular traffic of long trains. When long trains are travelling on brakes in track grading elevations, longitudinal dynamic efforts can reach dangerous values (in respect of durability and stability of vehicles in a track).

Therefore, the mode of train operation shall be selected with consideration of dynamic loading of the rolling-stock at each specific section.

The analysis of passenger and freight trains [2, 5–10] timing shows that it is economically reasonable to organize train traffic for trains weighting up to 8,000 tons on the main corridors of Lithuanian railways in case of the estimated increase of freight traffic by 15–20%.

The relevance of the organization of regular traffic of heavy trains weighting up to 8,000 tons on the main corridors of Lithuanian railways is also conditioned by the fact that in 2012 Joint Stock Company "Russian Railways" would complete preparation of engineering and production regulation for the regular service of freight trains weighting up to 9,000 tons.

Along with the traffic-carrying and train-handling capacities of the railways, the task of conservation of fuel and energy resources is set.

The existing methods of producing hauling calculations [13] for train schedules allow determining only minimal station-to-station travel times taken and do not provide optimization of the same in respect of fuel consumption for haulage.

Therefore, it is reasonable to develop such methods that would allow calculating station-to-station travel times taken (optimum times taken in regards of fuel consumption for haulage) and their realization (optimum performance charts) with consideration of longitudinal dynamic efforts using unified positions.

The offered methods [1, 4] allow calculating station-to-station travel time taken (optimum times taken in regards of fuel consumption for haulage) and their realization (optimum performance charts), yet without taking into consideration longitudinal dynamic efforts.

When solving the problem with consideration of longitudinal dynamics [11, 12, 14], the train shall be considered as a system of discrete weights. We need to integrate the system of differential equations, the 2n degree of which can reach several hundreds. The expenditures of computing time for calculation essentially increase with the increase in the degree of the system of equations and the length of integration interval. The problem examined herein excludes an opportunity to reduce the length of integration interval; therefore, seeking to reduce the expenditures of computing time we need to use hybrid mathematical model of train (flexible non-extensible line with a variable running weight, in other cases the train is considered as a system of the weights connected by nonlinear deformable elements with a consideration of gap clearance in a draft).

The mathematical model of train in the form of flexible non-extensible line with a variable running weight is described in [3]; therefore, this paper is limited to a more detailed mathematical model of train in the form of system of discrete weights connected by nonlinear deformable elements with a consideration of gap clearance in a draft.

2. MATHEMATICAL MODEL

To determine the longitudinal efforts affecting the vehicles, we shall represent the train in the form of a chain of solid bodies (Fig. 1) connected by the nonlinear deformable elements with a consideration of gap clearance in a draft [14].

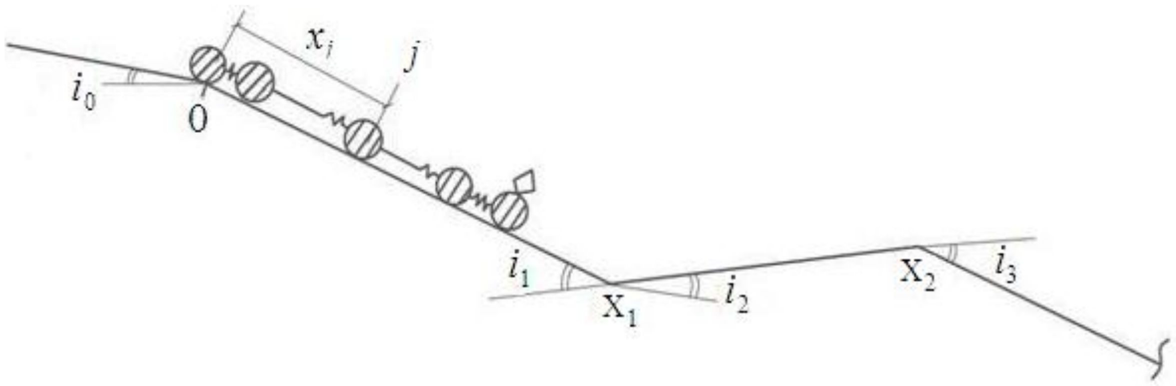

Fig. 1. Design diagram of a train
Рис. 1. Расчётная схема поезда

Travel of a train as a system of discrete weights can be described using the following differential equations:

$$\left.\begin{cases} \dot{v}_j(t) = \left[S_j(t) - S_{j-1}(t) + F_j(t, v_j, x, u(x), U_c)\right]/m_j, \ (j = 1 \div n, \ S_1 = S_{n+1} = 0); \\ \dot{q}_1(t) = \dot{x}_1(t) = v_1(t); \\ \dot{q}_j(t) = v_{j-1}(t) - v_j(t), \ (j = 2 \div n); \\ \text{entry conditions } v_j(0) = v_{j0}; \ q_j(0) = q_{j0}; \text{ aditional conditions } S_j(0) = S_{j0}; \end{cases}\right\}; \quad (1)$$

here n is the number of vehicles in the train; $q_j = x_{j-1} - x_j - 0{,}5(l_{j-1} - l_j)$; q_j is relative displacement of the centers of inertia of two next vehicles (deformation of connection between vehicles (j-1) and j); l_j is length of j vehicle; x_j is coordinate of its center of inertia calculated according to trajectory of travel; m_j is weight of element j; v_j is its speed; S_j and S_{j-1} are longitudinal efforts affecting the vehicle (these efforts are internal efforts in relation to the system); F_j is the resultant of the external efforts applied to the vehicle j.

The problem of determination of the longitudinal efforts affecting the vehicles of the train during travel is limited to the solution of Cauchy problem (initial value problem) for a system of ordinary differential 2n degree equations (1) under the given initial and several additional conditions due to specification of the problem of function $S_j(q_j, \dot{q}_j)$. The solution of the system of the differential equations (1) is determined by its numerical integration with application of predictor-corrector method based on two-step difference formulas, such as Adams-Bashforth and Adams-Moulton process.

When modeling train travel, we shall assume that the vehicles are equipped with the most widespread friction-type cushioning devices, while the coupling gears always have gap clearances. In this case, the power characteristic $S_j(q_j, \dot{q}_j)$ interwagon coupling j is possible to present as Fig. 2 and mathematical dependence [14].

$$S_j\left(q_j,\dot{q}_j\right)=\begin{cases} 0,\text{ if } 0<q_j\le\delta_{0j}; \\ \text{under } \delta_{0j}^{'}<\left|q_j\right|<\Delta+\delta_{0j}\text{ (where }\delta_{0j}^{'}=0\text{ under }q_j<0\text{ and }\delta_{0j}^{'}=\delta_{0j}\text{ under }q_j>0) \\ \text{and } q_j\,\dot{q}_j>0: \\ k_{nj}(q_j-\delta_{0j}),\text{ if }\left|k_{nj}(q_j-\delta_{0j})\right|\le\left|k_{kj}(q_j-q_{pj})+k_{pj}(q_{pj}-\delta_{0j}^{'})\right|; \\ k_{kj}(q_j-q_{pj})+k_{pj}(q_{pj}-\delta_{0j}^{'})+\beta\dot{q}_j, \\ \text{if }\left|k_{nj}(q_j-\delta_{0j}^{'})\right|>\left|k_{kj}(q_j-q_{pj})+k_{pj}(q_{pj}-\delta_{0j})\right|; \\ \text{under }\delta_{0j}<\left|q_j\right|<\Delta_j+\delta_{0j}^{'}\text{ and }q_j\,\dot{q}_j<0: \\ k_{pj}(q_j-\delta_{0j}^{'}),\text{if }\left|k_{pj}(q_j-\delta_{0j}^{'})\right|>\left|k_{kj}(q_j-q_{nj})+k_{nj}(q_{nj}-\delta_{oj}^{'})\right|; \\ k_{kj}(q_j-q_{nj})+k_{nj}(q_{nj}-\delta_{0j}^{'})+\beta\dot{q}_j, \\ \left|\text{if }\left|k_{pj}(q_j-\delta_{0j})\right|\le k_{kj}(q_j-q_{nj})+k_{nj}(q_{nj}-\delta_{oj}^{'})\right|; \\ \text{under }(k_{kj}-k_{nj})\Delta_j/(k_{kj}-k_{pj})\le\left|q_j-\delta_{0j}^{'}\right|\le\Delta_j\text{ and }\left|q_j-\delta_{0j}\right|>\Delta_j: \\ k_{kj}(q_j-\Delta_j-\delta_{0j}^{'})+k_{nj}\Delta_j-\beta\dot{q}_j. \end{cases};\qquad(2)$$

where: δ_{0j} is size of gap clearance in the coupling; q_{nj}, q_{pj} are q_j values; at the moment of change in product sign $q_j\cdot\dot{q}_j$; k_{nj}, k_{pj} coupling rigidity at loading and unloading; k_{nj} – rigidity of wagon structure; β – coefficient of viscosity; $\Delta_j=S_{\Delta_j}/k_{nj}$. As the paper [7] demonstrates, equations (2) reflect the actual power performance fully enough. As it can be seen in Fig. 2, functions $S_j\left(q_j,\dot{q}_j\right)$ ambiguously depend on $q_j\in(\delta_{0j},\delta_{0j+\Delta j})$ when $q_j>0$ and on $q_j\in\left(0-\Delta j\right)$ when $q_j<0$. Therefore, to make the assignment of the entry conditions of the problem unambiguous, it is necessary to specify additional requirements $q_j<0$ that determine the values $S_j\left(q_j,\dot{q}_j\right)$ at the moment of time t = 0.

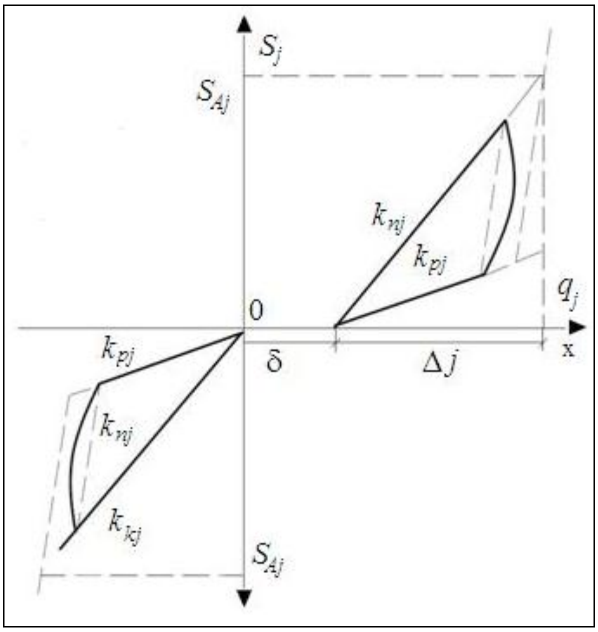

Fig. 2. Power characteristic of interwagon coupling
Рис. 2. Силовая характеристика межвагонного соединения

The sum of the projections of all external efforts to the tangent to the trajectory of travel shall be represented as follows:

$$F_j = P_j + T_j + B_j + W_j\,; \tag{3}$$

here P_j is the projection of gravitation of vehicle j on the tangent to the trajectory of travel; T_j is the hauling capacity of the locomotive calculated according to the algorithm described in [4]; B_j is the braking force affecting the vehicle; W_j is the force of the basic resistance against forward motion of the vehicle j determined according to [4]. As the lateral view of the examined track section is supposed to consist of rectilinear components k with grades i_v and coordinates of the ends $x_v(v=0,\ K-1;\ x_0=0)$, then the analytical expression P_j can be presented as follows:

$$P_j(x_j)=\begin{Bmatrix} m_j q_{j0}, \text{ if } x_j \le 0, \text{under } x_j > 0; \\ m_j q_{jv}, \text{ if } x_j \ge x_j, x_{v-1}\ (v=1, k-1) \end{Bmatrix}; \tag{4}$$

we shall consider the value of down grade (the component of the vehicle's gravitation is functioning as accelerating force) as positive, while the value of upgrade as negative.

We shall present the force affecting the vehicle during braking using two components:

$$B_j = B_{1j} + B_{2j}\,; \tag{5}$$

where: B_{1j} is the force affecting the vehicle during electrical (regenerative or dynamic) braking, which is set according to the algorithm [4].

We shall determine the forces B_{2j} according to [14] according to the following formula:

$$B_{2j} = -n_j \varphi_j k_j(t); \tag{6}$$

where: n_j is the number of brake blocks in the vehicle; φ_j is the friction coefficient of a brake block against the wheel; $k_j(t)$ is the force of pressure onto one brake block. It is convenient to change the formula (5) to the following [14]:

$$\varphi_j = A\left[(k_j + B')/(k_j + C')\right]\left[(v_j + D')/v_j + E'\right]; \tag{7}$$

where: v_j is measured in m/s^{-1}. The law of variation of pressure force $k_j(t)$ in proportion to air pressure in the brake cylinder depends on type of air distributor and type of braking (adjusting, full service, emergency). Values A, B, C, D are determined on empirical formulas, they affect the regenerative and conventional breaking. During modeling [14], we shall assume that variation of time of pressure force onto the brake block corresponds to the diagram presented in Fig. 3.

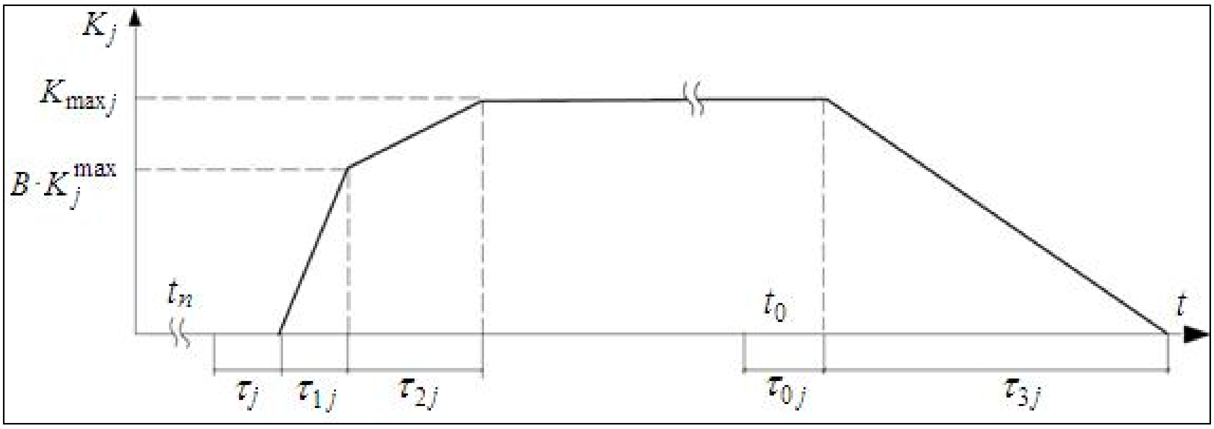

Fig. 3. Approximation of the diagram of filling and release of the brake cylinder
Рис. 3. Аппроксимация диаграммы наполнения и отпуска тормозного цилиндра

Here t is current travel time; t_n is the moment of the beginning of pneumatic brake actuation, whereas t_0 is the moment of the beginning of releasing the brakes of the head train vehicle; τ_j is the time interval, after which the brake wave with propagation velocity C_τ would reach section j from the head section; τ_{0j} is the time interval, after which the releasing wave with propagation velocity $C_{0\tau}$ would reach section j from the head section; τ_{1j} is the time interval, during which the pressure force applied to the brake block would increase from zero up to the value $k_{1j} = b_j + k_{\max j}$, ($b_j$ is the set values), whereas τ_{2j} is the time period of increase from value τ_{2j} up to value $k_{\max j}$; τ_{3j} is the time interval, during which the realizing of brakes of the vehicle j takes places. It is necessary to emphasize that when modeling the braking using pneumatic brakes, variation of the values τ_{1j}, τ_{2j}, τ_{3j}, along the train length shall be considered.

The analytical dependence $k_j(t)$ can be presented using the following expression:

$$k_j(t) = \left\{ \begin{array}{l} 0, \text{if } t - t_n \le t_j; \text{under } t - t_n > t_j \text{ and } t < t_0; \\ b_j \cdot K_{\max j}(t - t_n - t_j)/\tau_{1j}, \text{if } \tau_j < t - t_n \le \tau_j + \tau_{1j}; \\ K_{\max j} \cdot [b_j + (1 - b_j) \cdot (t - t_n - t_j - t_{1j})/\tau_{2j}], \\ \text{if } \tau_j + \tau_{1j} < t - t_n \le \tau_j + \tau_{1j} + \tau_{2j}; \\ K_{\max j}, \text{if } \tau_j + \tau_{1j} + \tau_{2j} < t - t_n \le \tau_0 + \tau_{0j}; \\ K_{\max j} \cdot [1 - (t - t_0 - t_{0j})/\tau_{3j}], \text{if } \tau_{0j} < t - t_n \le \tau_0 + \tau_{3j} \\ 0, \text{if } \tau_0 + \tau_{3j} < t - t_0. \end{array} \right\}. \quad (8)$$

While solving the problems regarding travel of a train as a system of discrete weights, it is necessary to integrate a system of the differential equations, the 2n degree of which can reach several hundreds. The expenditures of computing time for calculation essentially increase with the increase in the degree of the system of equations and the length of integration interval. The problem examined herein excludes an opportunity to reduce the length of integration interval; therefore, seeking to reduce the expenditures of computing time we need to use the method of reducing the degree of system of the differential equations.

The mathematical model of train in the form of flexible non-extensible line with a variable running weight is described in [4]; therefore, this paper is limited to a more detailed mathematical model of train in the form of system of discrete weights connected by nonlinear deformable elements with a consideration of gap clearance in a draft.

In regards to railway trains with gap clearances in a draft, this issue is examined in the papers [14]. The reduction of the degree of system of the differential equations (1) shall be done by means of appropriate grouping ($\tilde{n}$ vehicles in each group).

Thus, the length and weight of the group equals the sum of lengths and weights of the wagons in the group, whereas the new system of the differential equations describes sort of displacement of centers of inertia of groups consisting of *n* wagons. The forces originating during deformation of the elements interconnecting the groups of wagons shall equal the forces affecting the wagons at corresponding sections of the train. According to the changes described herein, it turns out that the gap clearance in the equivalent coupling of the groups equals the sum of gap clearances in interwagon coupling, whereas the rigidity of group coupling $k_{j\omega} = \left(\sum_{\omega - 1} k_{j\omega}^{-1} \right)$, where $k_{j\omega}$ is rigidity of interwagon coupling; *w* is the number of the wagon in the group; *j* is the group's number. The external efforts applied to centers of inertia of the wagon groups equal:

$$F_j(x_j, t) = \sum_{\omega=1}^{\tilde{n}} F_{j\omega}(x_{j\omega}, t)\,; \tag{9}$$

where: $F_{j\omega}(x_{j\omega}, t)$ is the force affecting the wagon number ω in the group *j*. As demonstrated in the papers [11], it is reasonable to reduce the degree of the system of equations (1) 6÷8 times, which leads to reducing the expenditures of computing time to solve the problem 10÷15 times. Meanwhile, the time characteristics of the changing process change; however, the greatest values of longitudinal efforts can be received with a precision sufficient for application.

3. MODE OF TRAIN OPERATION

To determine rational mode of train operation with consideration of longitudinal dynamics, we shall examine the following system of equations [4]:

$$\left\{ \begin{array}{l} \dot{v}_j(t) = \left[S_j(t) - S_{j-1}(t) + F_j(t, v_j, x, u(x), U_c) \right] / m_j, \; (j = 1 \div n, \; S_1 = S_{n+1} = 0); \\ \dot{q}_1(t) = \dot{x}_1(t) = v_1(t); \\ \dot{q}_j(t) = v_{j-1}(t) - v_j(t), \; (j = 2 \div n); \\ \text{entry conditions } v_j(0) = v_{j0}; \; q_j(0) = q_{j0}; \text{ aditional conditions } S_j(0) = S_{j0}; \end{array} \right\}; \tag{10}$$

$$\left\{ \begin{array}{l} \dfrac{d(v^2)}{dx} = \dfrac{2\xi}{(P+Q)(1+\gamma)} \left[F_k(v, u(x), U_c) - W(x, v, u(x)) - B_\tau(v, k'(t), u(x), U_c \right] \; ; \\ \dfrac{d\tau}{dx} = \dfrac{\tau_\infty(I_a(v, u(x), U_c)) - \tau(x)}{v(x) \cdot T(I_a(v, u(x), U_c))}. \end{array} \right\}. \tag{11}$$

When calculating haulage, the state of train is traditionally characterized by a point in four-dimensional phase space (x, v, τ, U_c). However, such approach does not allow considering longitudinal dynamics, which significantly affects the mode of long train operation. We shall, therefore, characterize a state of the train by a point in five dimensional space $(x, v, \tau, U_c, \tilde{S})$, where the $\tilde{S}$ coordinate is entered as follows:

$$\tilde{S}(t) = \max_j S_j(t) \tag{12}$$

and is determined by solving the system of equations (10). The area of admissible phase trajectories shall be determined by the following restrictions:

$$\tilde{G}\left(x, v, \tau, \tilde{S}\right): \left\{\begin{matrix} x_n \le x \le x_k \\ 0 \le \tau \le \tau_a \\ 0 \le v \le v^{\max}(x) \end{matrix}\right\}; \tag{13}$$

$$\left\{S^{-\max}(v) \le \tilde{S} \le S^{+\max}(v)\right\}; \tag{14}$$

where: x_n, x_k are coordinates of the initial and the end points of the trajectory; τ_a is the maximum allowed temperature of electric engine overheat, $v^{\max}(x)$ is the maximum allowed train speed; $S^{-\max}(v)$, $S^{+\max}(v)$ are the maximum allowed contraction and stretching forces in the coupling devices [14] (Fig.4).

As to the vector of operation $u(x)=[u_1, u_2, \ldots, u_m]$, according to the rules for technical maintenance, all possible values of the vector of operation have restrictions that can be written down as follows:

$$\tilde{U}: \left\{\begin{matrix} I_a\left(v, U_c, u(x)\right) \le I_a^{\max} \\ F_k\left(v, u(x), U_c\right) \le \psi_k(v) \cdot P \\ K''\left(t, u(x)\right) \cdot \varphi_k(v) \le \psi_k(v) \cdot q_0 \\ B_r\left(v, u_j(x)\right) \le B_r^{\max}(v) \end{matrix}\right\}; \tag{15}$$

where: $\tilde{U}$ is the area of allowed operations; $I_a^{\max}$ is maximum allowed engine current; q_0 is the load per vehicle's axle; ψ_k is the coefficient of wheel-rail adhesion; K'' is the force of pressure of the brake blocks of a set of wheels; φ_k is the brake block-wheel friction coefficient; $B_r^{\max}$ maximum allowed force of recuperation (restriction in regards of the engine current to the excitation current).

Elements $u_j(j = 1 \div m)$ of the vector of operation, which characterize the number of control notches of the locomotive j, accept discrete values and are numbered as follows. For example, for a mode of traction $u_j = 1, 2, 3, \ldots, n_t$ of a no-load operation $u_j = 0$, of a mode of recuperation, $u_j = -1, -2, -3, \ldots, -n_r$, pneumatic braking $u_j = -(n_r + 1), -(n_r + 2), \ldots, -n_p$.

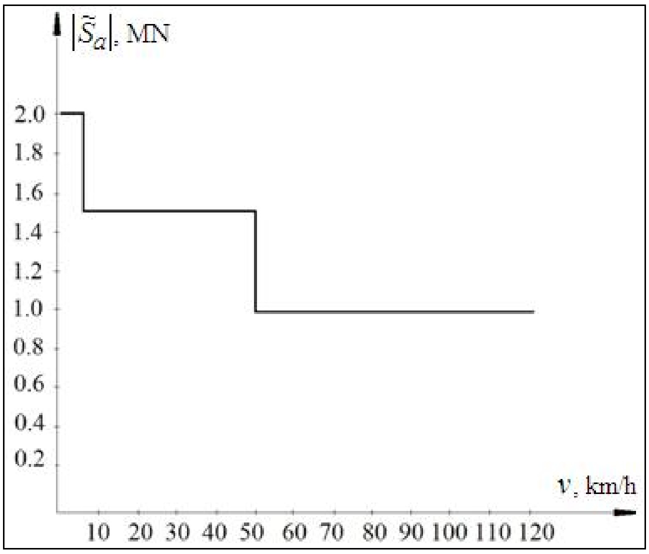

Fig. 4. The area of allowed contraction and stretching efforts
Рис. 4. Область допустимых сжимающих и растягивающих усилий

An optimum law of operation the train travel $u(x)$ shall satisfy the isoperimetric constraint (it is possible to compare the laws of operation only when the train's travel time taken is identical):

$$t_x[u] = \int_{x_n}^{x_k} \frac{dx}{v(x,u)} \equiv t_a \; ; \qquad (16)$$

where: t_x is travel time taken at the station-to-station block corresponding with the operation $u(x)$; t_a is the set travel time at the train's stop-to-stop section.

As a criterion for assessing the law of control $u(x)$ we shall take a composite function which represents the rate of fuel consumption for train traction at the stop-to-stop section

$$A[u] = \int_{x_n}^{x_k} \frac{g(v,u)\cdot \eta_e(v,u)dx}{v(x,u)} \; ; \qquad (17)$$

where: $g(v,u)$ is the rate of fuel consumption per unit of time; $\eta_e(v,u)$ is the locomotive's coefficient of efficiency.

Then, the problem of selecting a rational mode of operation long train can be formulated as follows: to find such law of operation $u(x) \in \tilde{U}$ that a corresponding solution of a system of equations (10, 11) would belonged to area $\tilde{G}$, constraint would be met (14), the criteria of optimality (15) would accept minimal value, whereas the initial and the end points of the trajectory would belong to the unspecified set Ω_0 and Ω_1, where Ω_0 is the set of initial values $\left(x_0, v_0, \tau_0, \tilde{S}_0\right)$; Ω_1 is the set of end values $\left(x_k, v_k, \tau_k, \tilde{S}_k\right)$.

The proposed methods is realized in the form of computer program (programming language C++ Builder, environment - Windows XP/2000/Vista/7) for optimization of long train operation with consideration of longitudinal dynamic efforts.

4. CONCLUSIONS

The developed methods of selecting the mode of long train operation with consideration of longitudinal dynamic efforts allows determining an optimum station-to-station travel times taken in regards of train timing and corresponding performance charts on the main directions of Lithuanian railways.

Bibliography

1. Belforte, P. & Cheli, F & Diana, G. & Melzi, S. Numerical and experimental approach for the evaluation of severe longitudinal dynamics of heavy freight trains. Vehicle System Dynamics. *International Journal of Vehicle Mechanics and Mobility*. 2008. Vol. 46. S1. P. 937-955.
2. Dailydka, S. & Lingaitis, L.P. & Myamlin, S., Prichodko, V. Modelling the interaction between railway wheel and rail. *Transport*. 2008. Vol. 23 (3). P. 236-239.
3. Horn, P. Experimentelle Simulations-studien zur energieoptim alen Zugsteuerung. *DET – Eisenbahntechn*. 1973. No. 11. P. 517-519. [In German: Experimental simulation studies for energy-optimal control]
4. Lingaitis, L.P. & Jastremskas, V. & Jevdomacha, G. On the issue on fuel-energy resources economy for train traction force. In: *Proceedings of the 7th International Scientific Conference „Transbaltica 2011"*. Vilnius. 2011. P. 87-90.
5. Liudvinavičius, L. & Lingaitis, L.P. New locomotive energy management systems. *Eksploatacja i niezawodność - Maintenance and reliability.* 2010. Vol. 1 (45). P. 35-41.
6. Liudvinavičius, L. & Lingaitis, L.P. Electrodynamic braking in high-speed rail transport. *Transport.* 2007. Vol. XXII. No. 3. P. 178-186.

7. Liudvinavičius, L. & Lingaitis, L.P. New technical solutions of using rolling stock electrodynamical braking. *Transport Problems.* 2009. Vol. 4, No. 2. P. 23-35.
8. Vaičiūnas, G. & Lingaitis L.P. Comparing the performance of diesel locomotives for passenger transportation. *Transport Problems.* 2008. Vol. 3. Iss. 1. P. 45-50.
9. Vaičiūnas, G. & Lingaitis, L.P. Investigating the Dynamics of Passenger Rolling Stock Deterioration. *Transport*. 2008. Vol. XXIII. No 1. P. 51-54.
10. Vaičiūnas, G. & Lingaitis, L.P. Investigating the Dynamics of Motor-Rail Car Elements. *Transport and Engineering. Railway Transport. Scientific Proceedings of Riga Technical University*. 2007. Series 6. P. 93-98.
11. Zhaohui, Qi & Zhihao, H. & Xianchao, K. Simulation of longitudinal dynamics of long freight trains in positioning operations. *Vehicle System Dynamics.* 2012. Vol. 50. No 9. P. 1409-1433.
12. Zhuan, X & Xia, X. Speed regulation with measured output feedback in the control of heavy haul trains. *Automatica*. 2008. Vol. 44. Iss.1. P. 242-247.
13. Беллман, Р. *Динамическое программирование*. Москва: Иностранная литература. 1960. 400 с. [In Russian: Bellman, R. *Dynamic programming.* Moscow: Foreign literature. 1960. 400 p.]
14. Блохин, Е.П. & Манашкин, Л.А. *Динамика поезда (нестационарные продольные колебания)*. Москва: Транспорт. 1982. 222 с. [In Russian: Blokhin, E. P. & Manashkin, L. A. *The dynamics of the train (non-stationary longitudinal oscillations).* Moscow: Transport. 1982. 222 p.]

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