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ASSESSMENT OF THE POSSIBILITY OF INCREASING A LOAD - CARRYING STRENGTH OF TRUSS BRIDGES OF OVERHEAD TRAVELLING CRANES

Summary. A possibility of increasing the load-carrying strength of truss bridges of overhead travelling cranes by transferring a part of a useful load from the main truss (which transfers directly the load originated from road wheels of the crane carriage) on the parallel auxiliary truss by means of increasing rigidity of truss members joining both trusses - was presented in the paper. The verification of a static relief of the main truss by the auxiliary one as a function of rigidity of transverse and skew space trusses was performed. The A. Blum method was used in analytical calculations. The obtained results were verified by means of the ANSYS 12.1 program utilising the finite element method (FEM).

OCENA MOŻLIWOŚCI ZWIĘKSZENIA NOŚNOŚCI WYTRZYMAŁOŚCIOWEJ KRATOWNICOWYCH MOSTÓW SUWNIC POMOSTOWYCH

Streszczenie. W artykule przedstawiono możliwość zwiększenia nośności wytrzymałościowej kratowych dźwigarów mostów suwnicowych poprzez przeniesienie części obciążenia użytkowego z kraty głównej, przenoszącej bezpośrednio obciążenie pochodzące od nacisku kół biegowych wózka suwnicowego, na równoległą kratę pomocniczą, drogą zwiększenia sztywności prętów wykratowania łączących obie kratownice. Sprawdzono statyczne odciążenie kraty głównej przez kratę pomocniczą w funkcji sztywności prętów wykratowania poprzecznego i ukośnego kratownicy przestrzennej. Do obliczeń analitycznych wykorzystano metodę A. Bluma. Otrzymane wyniki zweryfikowano przy użyciu oprogramowania ANSYS 12.1 wykorzystującego metodę elementów skończonych (MES).

1. INTRODUCTION

Truss bridges constitute a part of a railway, road and industrial infrastructure. They find applications in materials handling, at building overhead cranes and normal cranes. Objects built after 1945 were made out of steel which, due to the technical level and production technologies of the time, contained significant amounts of non-metallic inclusions. As a result of many years of an intensive exploitation, structural material discontinuities were joining together forming internal cracks called lameral cracks. Those cracks significantly decreased a carrying capacity of structures. This is one of the reasons which justifies endeavours to increase the carrying capacity of structures during repair works. The paper concerns problems attributed to space trusses in the system: main truss – auxiliary

truss constituting the basic carrying structure of overhead cranes, built in the sixtieth of the previous century and being still currently exploited. The example of such structural solution is the truss box girder shown in Fig. 1. It is characterised by the main truss (2), to which the external load is applied, and the parallel to it auxiliary truss (3). Both trusses are connected by transverse bars called wind beams (5), and by skew bars called struts (4).

The aim of the paper is to indicate in which way the carrying capacity of the space truss, which constitutes the box girder, is changing with an increase of transverse and skew bars stiffness. Analytical methods as well as numerical ones will be applied for calculations.
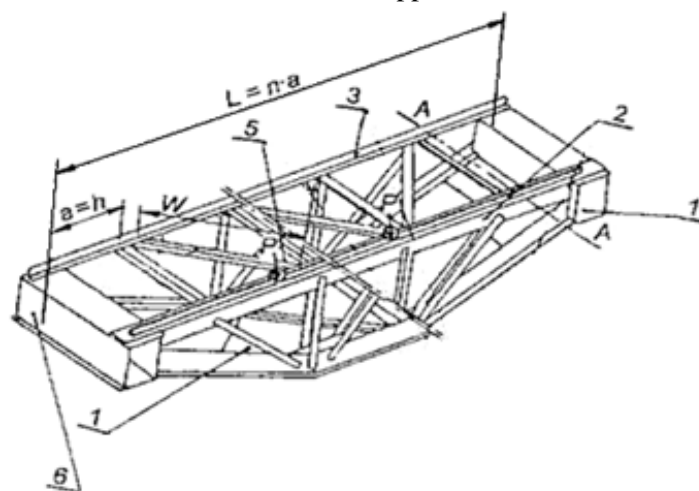

Fig. 1. Truss box girder: 1 – truss box girder, 2 – main truss, 3 – auxiliary truss, 4 – skew bars, 5 – transverse bars, 6 – frontal beams

Rys. 1. Kratowy dźwigar skrzynkowy: 1 – kratowy dźwigar skrzynkowy, 2 – krata główna, 3 – krata pomocnicza, 4 – wykratowanie ukośne, 5 – wykratowanie poprzeczne, 6 – czołownice

2. REVIEW OF THE APPLIED METHODS

The idea lying behind the research concerning space trusses, in the previous century, was the will to simplify the necessary calculations while retaining satisfactory accuracy. Attempts to develop the simplified analytical methods for the space trusses in systems: main truss-auxiliary truss were undertaken by many outstanding researchers. Among them A. Blum [1], as the developer of the computational method, should be mentioned. Presently numerical methods utilising the calculation power of computers are more and more often used. The most often the finite element method (FEM) is applied. Both methods will be presented in this paper.

2.1. A. Blum's method

A. Blum's method was selected for the analytical calculations since it considers problems related to the space work of the box girder in a comprehensive way. A point of departure for the method was assuming such static structure diagram, which in an accurate way will resemble its space work character. A real transverse deformation of the structure under an influence of the vertically applied force P is presented in Figure 2a. On this basis the simplified diagram used in this method was assumed (Fig. 2b). Then, it was assumed that both carrying trusses will be considered as beams freely supported but connected with each other by crosswise and skew truss members. The articulated fixing of transverse and skew beams with the auxiliary truss and the rigid junction with the main truss – were assumed. The bar structure of the box girder presented in Fig. 3 was assumed for the theoretical considerations. As the result of the bar structure deformation caused by the vertical force P, applied in the main truss plane, the interaction of both carrying structures (main and auxiliary trusses), which are connected by crosswise and skew trusses, occurs. The interaction moment of the main truss transfers itself on the connecting trusses in a form of unknown moments x_i. The auxiliary truss is loaded by a

system of transverse forces: $\sum_i \frac{x_i}{b_i}$ (b_i is the length of the bar, which joins the carrying trusses) applied in joints. The main truss is loaded by external force P and by transverse forces of the auxiliary truss interaction with the main truss: $\sum_i \frac{x_i}{b_i}$. The considered dependencies are presented in Fig.4.
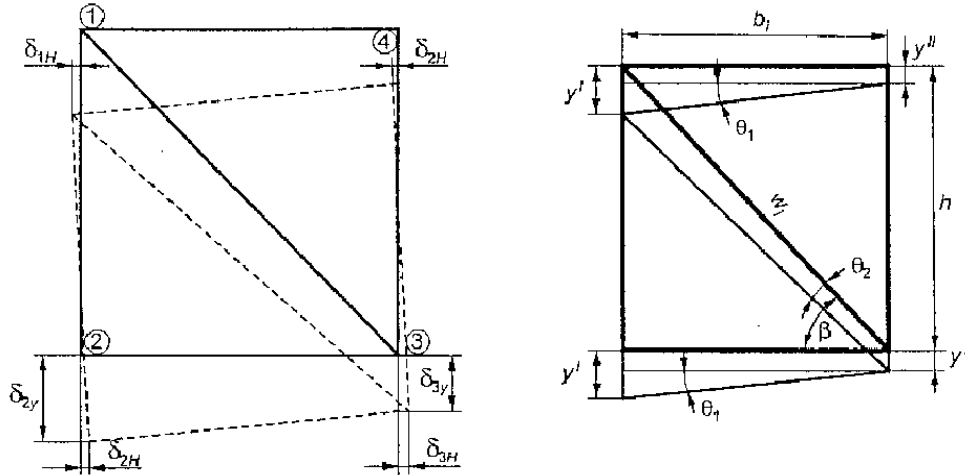

Fig. 2. a) Real deformation diagram of a box girder, b) Computational diagram of A. Blum's method
Rys. 2. a) Schemat deformacji rzeczywistej dźwigara, b) Schemat obliczeniowy metody A. Bluza

where: y^I - vertical displacement of the main truss, caused by a vertical load applied in the main truss
Plane, y^{II} - vertical displacement of the auxiliary truss caused by a vertical load applied in the main truss plane, b_i, w_i - length of transverse and skew beams
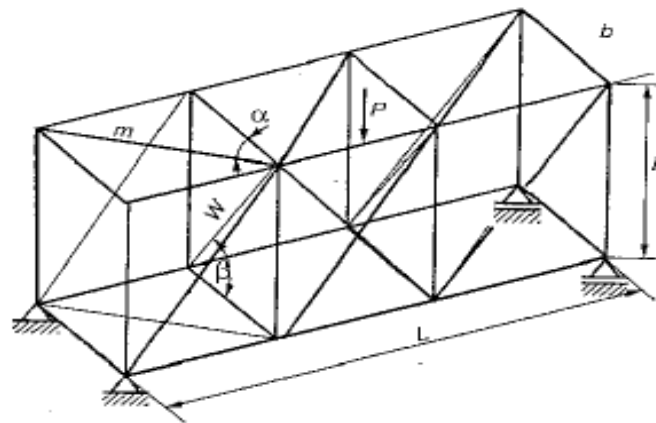

Fig. 3. Theoretical bar structure of the box girder
Rys. 3. Teoretyczna konstrukcja prętowa dźwigara skrzynkowego

Each bar of the transverse coupling sustains rotation resulting from unequal displacements of both load carrying trusses. The angle of rotation of the chord of the deformed bar axis, being the result of unequal displacement of its fixing in both trusses, can be estimated, on the grounds of the displacement method, by equation 1 (see Fig.2.b).

$$\theta_{ik} = \frac{y_i - y_k}{b_{ik}} \tag{1}$$

where: b_{ik} - bar length ik, y_i, y_k - vertical displacements of points: i,k, θ_{ik} - angle of rotation of the chord of the deformed transverse bar axis.
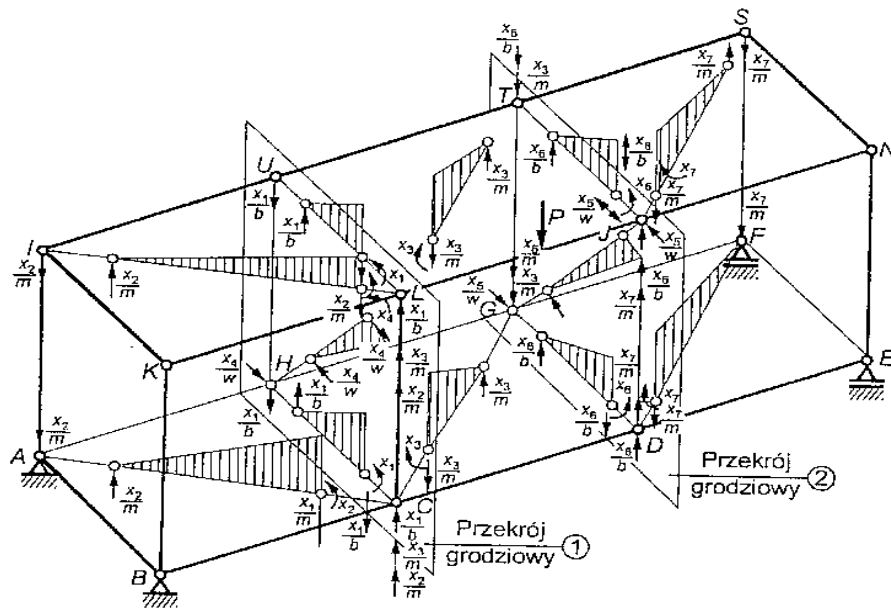
Fig. 4. Static computational diagram of A. Blum's method
Rys. 4. Statyczny schemat obliczeniowy metody A. Bluma

In the case of the considered structure of the truss box girder, each bar of transverse and skew trusses is loaded with an unknown supernumerary moment x_i (Fig. 5), being the result of not the same displacement of both carrying trusses.
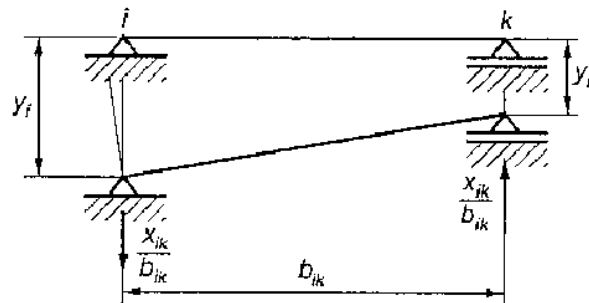

Fig. 5. Truss member loaded by the interaction moment
Rys. 5. Pręt wykratowania obciążony momentem oddziaływania

On the basis of the method of displacements for the considered structure the general equation for the interaction moment x_{ik} obtains the following form:

$$x_{ik} = \frac{3EJ_R}{b_{ik}}\theta_{ik} = \frac{3EJ_R}{b_{ik}^2}\left(y_i - y_k\right) \tag{2}$$

where: x_{ik} - interaction moment in joint i of bar ik, J_R - inertial moment of the cross-section of the
transverse or skew trusses versus neutral axis

The successive step constitutes the determination of equations describing interaction moments occurring in the structure. This is being done by a separate analysis of blocking cross-sections of the truss, in taking into account their displacements. The following equations system was obtained for the model used in calculations:

$$x_1 = \frac{3EJ_R}{b^2}\left(y_{1P} - y_1^I - y_1^{II}\right); \qquad x_2 = \frac{3EJ_R}{m^2}\left(y_{1P} - y_1^I\right); \qquad x_3 = \frac{3EJ_R}{m^2}\left(y_{1P} - y_1^I - y_2^{II}\right);$$

$$x_4 = \frac{3EJ_R}{w^2}\left(y_{1P} - y_1^I - y_1^{II}\right); \qquad x_5 = \frac{3EJ_R}{w^2}\left(y_{2P} - y_2^I - y_2^{II}\right); \qquad x_6 = \frac{3EJ_R}{b^2}\left(y_{2P} - y_2^I - y_2^{II}\right);$$

$$x_7 = \frac{3EJ_R}{m^2}\left(y_{2P} - y_2^I\right); \tag{3}$$

where: $y_1^{'}$, $y_2^{'}$ are displacements of points 1 and 2 of the main truss under an influence of internal forces; y_{1P}, y_{2P} are displacements of points 1 and 2 of the main truss under an influence of external force P; $y_1^{''}$, $y_2^{''}$ are displacements of points 1 and 2 of the auxiliary truss under an influence of internal forces. The positions of points 1 and 2 in the structure are seen in Fig. 6.
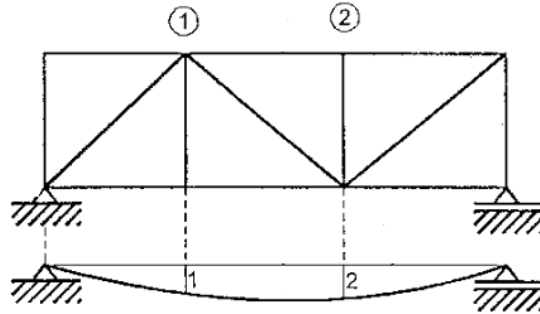

Fig. 6. Points 1 and 2, for which the displacements are calculated
Rys. 6. Punkty, dla których obliczane są przemieszczenia

The further analysis requires calculations of those displacements. The way of their detailed definition can be found in the reference [1]. Displacements y_{ik} were substituted by products of elementary displacements δ_{ik} and the relevant forces. This leads to a significant increase of the volume of the mathematic notation. Therefore, it is more convenient to present the final notation in the matrix form:

$$[A] \cdot \{y\} = \{w\} \tag{4}$$

Individual components of equation (4) assume a form:

$$A = \begin{bmatrix} \delta_{11}+k\delta_{11}+\frac{b^3k_r}{6} & \delta_{11} & \delta_{11}+k\delta_{12} & \delta_{11} & \delta_{12}+k\delta_{12} & \delta_{12}+k\delta_{12} & \delta_{12} \\ \delta_{11} & \delta_{11}+\frac{m^3k_r}{6} & \delta_{11} & \delta_{11} & \delta_{12} & \delta_{12} & \delta_{12} \\ \delta_{11}+k\delta_{11} & \delta_{11} & \delta_{11}+k\delta_{22}+\frac{m^3k_r}{6} & \delta_{11}+k\delta_{11} & \delta_{12}+k\delta_{22} & \delta_{12}+k\delta_{22} & \delta_{12} \\ \delta_{11}+k\delta_{11} & \delta_{11} & \delta_{11}+k\delta_{12} & \delta_{11}+k\delta_{11}+\frac{w^3k_r}{3\cos\beta} & \delta_{12}+k\delta_{22} & \delta_{12}+k\delta_{22} & \delta_{12} \\ \delta_{21}+k\delta_{21} & \delta_{21} & \delta_{21}+k\delta_{22} & \delta_{21}+k\delta_{21} & \delta_{22}+k\delta_{22}+\frac{w^3k_r}{3\cos\beta} & \delta_{22}+k\delta_{22} & \delta_{22} \\ \delta_{21}+k\delta_{21} & \delta_{21} & \delta_{21}+k\delta_{22} & \delta_{21}+k\delta_{21} & \delta_{22}+k\delta_{22} & \delta_{22}+k\delta_{22}+\frac{b^3k_r}{6} & \delta_{22} \\ \delta_{21} & \delta_{21} & \delta_{21} & \delta_{21} & \delta_{22} & \delta_{22} & \delta_{22}+\frac{m^3k_r}{6} \end{bmatrix} \cdot \begin{bmatrix} 2\frac{x_1}{b} \\ 2\frac{x_2}{b} \\ 2\frac{x_3}{b} \\ 2\frac{x_4}{w}\cos\beta \\ 2\frac{x_5}{w}\cos\beta \\ 2\frac{x_6}{w} \\ 2\frac{x_7}{m} \end{bmatrix} = \begin{bmatrix} \delta_{1i}P \\ \delta_{1i}P \\ \delta_{1i}P \\ \delta_{1i}P \\ \delta_{2i}P \\ \delta_{2i}P \\ \delta_{2i}P \end{bmatrix} \tag{5}$$

where: δ_{ki} - displacement of point k under an elementary load applied in point I k - ratio of the equivalent moment of inertia of the main truss to the equivalent moment of inertia of the

auxiliary truss, k_r – ratio of the equivalent moment of inertia of the main truss to the moment of inertia of individual bar of transverse truss versus its neutral axis, P - external load w,b,m – structure dimensions

The rearrangement details leading to obtaining the matrix form can be found in the reference [1].

2.2. Finite Element Method

The Finite Element Method (FEM) has its origin in the structure analysis. An undoubted merit of this method is its universality in solving problems of a complex geometry. A short explanation of the method is given below.

The analysed zone is divided into a certain, finite number of simple geometrical elements, the so-called *finite elements*. It is assumed that they are interconnected in the finite number of points being in the perimeters. The most often these are corner points (nodes).
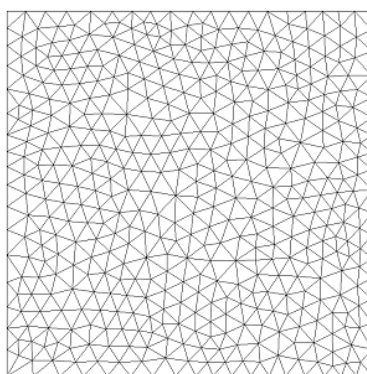

Fig. 7. Discretised zone (divided). Visible nodes
Rys. 7. Obszar poddany dyskretyzacji (podziałowi)

Then certain functions explicitly determining the distribution of the analysed physical quantity inside finite elements, dependent on values of these physical quantities, are chosen. These functions are called nodal functions or *shape functions*. Differential equations describing the investigated effect are rearranged by means the so-called *weight functions* to equations of the finite element method (algebraic equations). On the basis of equations of the finite elements method, values of coefficients being at unknowns and the corresponding values of the right members are calculated. If the task being solved is unstable, then in the calculation of right members the initial conditions are additionally utilized. The number of equations in the system is equal to the number of nodes multiplied by the number of degrees of freedom occurring in the individual node, it means the number of unknowns occurring in the individual node. The boundary conditions are introduced into the system of equations by performing the appropriate modifications of the coefficients matrix of this system and the right vector members.

Then the system of equations is solved and values of the needed physical quantities in nodes obtained. In dependence on the type of the problem being solved the additional values are calculated (energy, forces). If the task is not stable these activities are repeated, starting from the calculations of coefficients at unknowns up to the moment of fulfilling the condition of ending the calculations. This can be e.g. the determined value of the physical quantity in one of the nodes.

3. ANALYTICAL CALCULATIONS OF SPATIAL BAR SYSTEMS WITH THE APPLICATION OF THE SIMPLIFIED CALCULATION METHOD

In order to verify in which way, the rigidity of truss members joining the main and the auxiliary truss of the box girder, influences the main truss unloading (it means which part of a load is

transferred from the main truss into the auxiliary one), the representative truss group, of similar dimensions, the same structural sections forming the main and auxiliary trusses and shape elements of various cross-sections for the transverse and skew trusses, should be analysed. The above assumptions warrant, that different results obtained by calculations for individual cases will be only dependent on the transverse and skew trusses.

Five various trusses were selected for the analysis. Such number of analysed objects allows to draw - with a good accuracy - the dependence between the truss members ridigity, and unloading the main truss. Trusses are marked: K_1, K_2, K_3, K_4, K_5. The dimensions of trusses comply to the real ones with structures which can be met in industry. The shaped elements were selected in such a way that - at the assumed external load 100 [kN] - the stresses occurring in structures would not exceed the permissible values. Overall dimensions of the analysed trusses as well as notations of shaped elements were presented in a simplified way in Fig. 8. The structural sections dimensions, common for all five structures, are in Table 1.

Table 1

Structural sections of the analysed trusses

A_{GG}	A_{GD}	A_{GW}	A_{GS}	A_{PG}	A_{PD}	A_{PW}	A_{PS}
T-bar I 400	T-bar I 400	Double T-bar L60x60x6	T-bar L80x80x86	T-bar L75x75x8	T-bar L75x75x8	Double T-bar L25x25x3	T-bar L30x30x5

Table 2 contains the dimensions of the structural sections of transverse and skew trusses, which are changing for individual structures.

Table 2

Structural sections of individual trusses

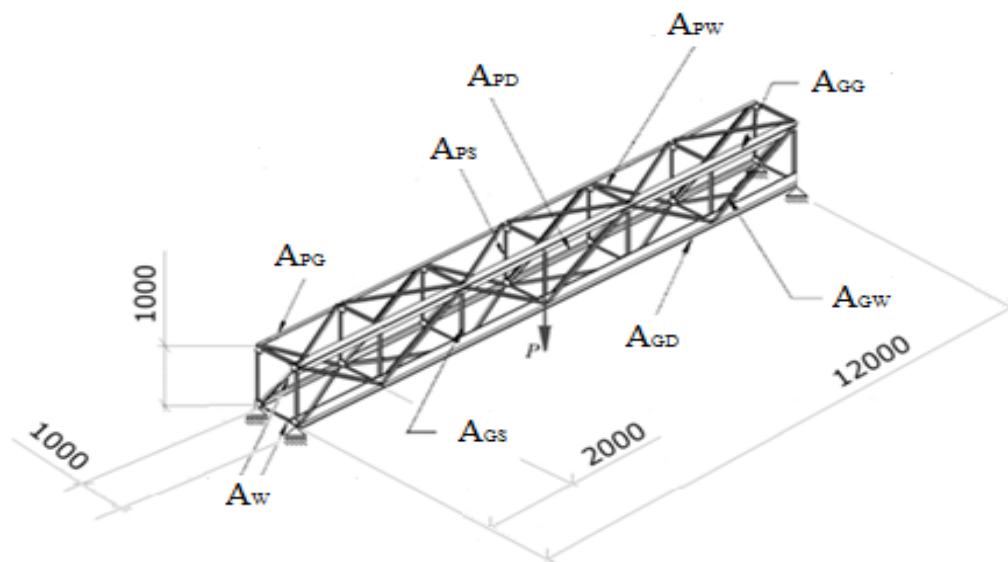| Symbol	Truss K_1	Truss K_2	Truss K_3	Truss K_4	Truss K_5
A_W	T-bar L50x50x6	T-bar L60x60x5	T-bar L65x65x7	T-bar L90x90x8	T-bar L100x100x8

Fig. 8. Analysed truss together with structural sections, markings and dimensions
Rys. 8. Analizowana kratownica wraz z oznaczeniami kształtowników i wymiarami

3.1. Analytical calculations on the example of truss K$_1$

Analytical calculations were performed on the example of truss K$_1$. The kind of the crosswise and skew truss members were taken from Table 2. Dimensions of the remaining structural sections are given in Table 1. The remaining dimensions, necessary for calculations, are given in Table 3.

Table 3

Data necessary for calculations. Truss K$_1$

Load of the truss	Transverse bar length (wind beam)	Skew bar length (strut)	Transverse bar length	Cosinus of the angle between skew and transverse bars
P=100 [kN]	w=2,2361 [m]	m= 1,4142 [m]	b=1,0 [m]	$\cos\beta$=0,707

The equation system derivation method, which allows to calculate the interaction moments was presented in Chapter 2.1. For the analysed case the equation system obtains the following form:

$$
\begin{aligned}
&\left(\frac{2\delta_{22}}{b}+\frac{2k\delta_{22}}{b}+\frac{b^2k_r}{3}\right)X_1 &&+\left(\frac{2\delta_{23}}{w}+\frac{2k\delta_{22}}{w}\right)X_2 &&+\left(\frac{2\delta_{23}}{b}+\frac{2k\delta_{23}}{b}\right)X_3 &&+\left(\frac{2\delta_{23}}{w}+\frac{2k\delta_{24}}{w}\right)X_4 &&\left(\frac{\delta_{23}}{m}+\frac{k\delta_{23}}{m}\right)\cos\beta\cdot X_5 &&\left(\frac{2\delta_{24}}{b}+\frac{2k\delta_{24}}{b}\right)X_6 &&=\delta_{23}P\\[4pt]
&\left(\frac{2\delta_{32}}{b}+\frac{2k\delta_{22}}{b}\right)X_1 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{22}}{w}+\frac{w^2k_r}{3}\right)X_2 &&+\left(\frac{2\delta_{33}}{b}+\frac{2k\delta_{23}}{b}\right)X_3 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{24}}{w}\right)X_4 &&\left(\frac{\delta_{33}}{m}+\frac{k\delta_{23}}{m}\right)\cos\beta\cdot X_5 &&\left(\frac{2\delta_{24}}{b}+\frac{2k\delta_{24}}{b}\right)X_6 &&=\delta_{33}P\\[4pt]
&\left(\frac{2\delta_{32}}{b}+\frac{2k\delta_{32}}{b}\right)X_1 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{32}}{w}\right)X_2 &&+\left(\frac{2\delta_{33}}{b}+\frac{2k\delta_{33}}{b}+\frac{b^2k_r}{3}\right)X_3 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{34}}{w}\right)X_4 &&\left(\frac{\delta_{33}}{m}+\frac{k\delta_{33}}{m}\right)\cos\beta X_5 &&\left(\frac{2\delta_{34}}{b}+\frac{2k\delta_{34}}{b}\right)X_6 &&=\delta_{33}P\\[4pt]
&\left(\frac{2\delta_{32}}{b}+\frac{2k\delta_{42}}{b}\right)X_1 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{42}}{w}\right)X_2 &&+\left(\frac{2\delta_{33}}{b}+\frac{2k\delta_{43}}{b}\right)X_3 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{44}}{w}+\frac{w^2k_r}{3}\right)X_4 &&\left(\frac{\delta_{33}}{m}+\frac{k\delta_{43}}{m}\right)\cos\beta\cdot X_5 &&\left(\frac{2\delta_{34}}{b}+\frac{2k\delta_{44}}{b}\right)X_6 &&=\delta_{33}P\\[4pt]
&\left(\frac{2\delta_{32}}{b}+\frac{2k\delta_{32}}{b}\right)X_1 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{43}}{w}\right)X_2 &&+\left(\frac{2\delta_{33}}{b}+\frac{2k\delta_{33}}{b}\right)X_3 &&+\left(\frac{2\delta_{33}}{w}+\frac{2k\delta_{34}}{w}\right)X_4 &&\left(\frac{2\delta_{33}}{m}+\frac{2k\delta_{33}}{m}+\frac{m^2k_r}{3}\right)\cos\beta\cdot X_5 &&\left(\frac{2\delta_{34}}{b}+\frac{2k\delta_{34}}{b}\right)X_6 &&=\delta_{33}P\\[4pt]
&\left(\frac{2\delta_{42}}{b}+\frac{2k\delta_{42}}{b}\right)X_1 &&+\left(\frac{2\delta_{43}}{w}+\frac{2k\delta_{42}}{w}\right)X_2 &&+\left(\frac{2\delta_{43}}{b}+\frac{2k\delta_{43}}{b}\right)X_3 &&+\left(\frac{2\delta_{43}}{w}+\frac{2k\delta_{44}}{w}\right)X_4 &&\left(\frac{\delta_{43}}{m}+\frac{k\delta_{43}}{m}\right)\cos\beta\cdot X_5 &&\left(\frac{2\delta_{44}}{b}+\frac{2k\delta_{44}}{b}+\frac{b^2k_r}{3}\right)X_6 &&=\delta_{43}P
\end{aligned}
$$

(6)

The equation system (8) was reduced to the matrix:

$$
\begin{bmatrix}
\frac{2\delta_{22}}{b}+\frac{2k\delta_{22}}{b}+\frac{b^2k_r}{3} & \frac{2\delta_{23}}{w}+\frac{2k\delta_{22}}{w} & \frac{2\delta_{23}}{b}+\frac{2k\delta_{23}}{b} & \frac{2\delta_{23}}{w}+\frac{2k\delta_{24}}{w} & \left(\frac{\delta_{23}}{m}+\frac{k\delta_{23}}{m}\right)\cos\beta & \frac{2\delta_{24}}{b}+\frac{2k\delta_{24}}{b}\\[6pt]
\frac{2\delta_{32}}{b}+\frac{2k\delta_{22}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{22}}{w}+\frac{w^2k_r}{3} & \frac{2\delta_{33}}{b}+\frac{2k\delta_{23}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{24}}{w} & \left(\frac{\delta_{33}}{m}+\frac{k\delta_{23}}{m}\right)\cos\beta & \frac{2\delta_{24}}{b}+\frac{2k\delta_{24}}{b}\\[6pt]
\frac{2\delta_{32}}{b}+\frac{2k\delta_{32}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{32}}{w} & \frac{2\delta_{33}}{b}+\frac{2k\delta_{33}}{b}+\frac{b^2k_r}{3} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{34}}{w} & \left(\frac{\delta_{33}}{m}+\frac{k\delta_{33}}{m}\right)\cos\beta & \frac{2\delta_{34}}{b}+\frac{2k\delta_{34}}{b}\\[6pt]
\frac{2\delta_{32}}{b}+\frac{2k\delta_{42}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{42}}{w} & \frac{2\delta_{33}}{b}+\frac{2k\delta_{43}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{44}}{w}+\frac{w^2k_r}{3} & \left(\frac{\delta_{33}}{m}+\frac{k\delta_{43}}{m}\right)\cos\beta & \frac{2\delta_{34}}{b}+\frac{2k\delta_{44}}{b}\\[6pt]
\frac{2\delta_{32}}{b}+\frac{2k\delta_{32}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{43}}{w} & \frac{2\delta_{33}}{b}+\frac{2k\delta_{33}}{b} & \frac{2\delta_{33}}{w}+\frac{2k\delta_{34}}{w} & \left(\frac{2\delta_{33}}{m}+\frac{2k\delta_{33}}{m}+\frac{m^2k_r}{3}\right)\cos\beta & \frac{2\delta_{34}}{b}+\frac{2k\delta_{34}}{b}\\[6pt]
\frac{2\delta_{42}}{b}+\frac{2k\delta_{42}}{b} & \frac{2\delta_{43}}{w}+\frac{2k\delta_{42}}{w} & \frac{2\delta_{43}}{b}+\frac{2k\delta_{43}}{b} & \frac{2\delta_{43}}{b}+\frac{2k\delta_{44}}{b} & \left(\frac{\delta_{43}}{m}+\frac{k\delta_{43}}{m}\right)\cos\beta & \frac{2\delta_{44}}{b}+\frac{2k\delta_{44}}{b}+\frac{b^2k_r}{3}
\end{bmatrix}
\cdot
\begin{bmatrix}X_1\\X_2\\X_3\\X_4\\X_5\\X_6\end{bmatrix}
=
\begin{bmatrix}\delta_{23}P\\\delta_{33}P\\\delta_{33}P\\\delta_{33}P\\\delta_{33}P\\\delta_{34}P\end{bmatrix}
$$

(7)

where: δ_{ki}- displacements of point k under an elementary load applied in point i, calculated by a simple software, k - ratio of the equivalent moment of inertia of the main truss, to the equivalent moment of inertia of the auxiliary truss, k_r - ratio of the equivalent moment of inertia of the main truss to the moment of inertia of individual bar of transverse truss versus its neutral axis, P - external load of a value of 100 [kN], w,b,m - structure dimensions from Table 3.

Introducing the numerical values into the matrix notation (7) we obtain:

$$\begin{bmatrix} 1,548 & 0,092 & 0,220 & 0,083 & 0,055 & 0,178 \\ 0,206 & 6,818 & 0,228 & 0,086 & 0,056 & 0,185 \\ 0,219 & 0,101 & 1,602 & 0,101 & 0,064 & 0,220 \\ 0,185 & 0,085 & 0,226 & 6,818 & 0,056 & 0,206 \\ 0,219 & 0,101 & 0,258 & 0,101 & 2,753 & 0,220 \\ 0,178 & 0,082 & 0,220 & 0,092 & 0,055 & 1,548 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} 1,7747 \\ 2,0833 \\ 2,0833 \\ 2,0833 \\ 2,0833 \\ 1,7747 \end{bmatrix} \quad (8)$$

Applying dependence: $\{y\} = [A]^{-1} \cdot \{w\}$ (where y is a columnar vector containing the looked for values of interaction moments) matrix (8) was transformed and solved by means of the MATLAB software. The calculated values of the interaction moments of the main truss K_1 on the auxiliary one are given in Table 4.

Table 4

Interaction moments values.

X_1	X_2	X_3	X_4	X_5	X_6
0,8605 [kNm]	0,2156 [kNm]	1,0165 [kNm]	0.2156 [kNm]	0,5083 [kNm]	0,8605 [kNm]

The total force relieving the main girder of truss K_1 was calculated as a sum of all components of force of internal influences:

$$\sum \frac{X_i}{b_i} = \frac{2X_1}{b} + \frac{2X_2}{w} + \frac{2X_3}{b} + \frac{2X_4}{w} + \frac{X_5}{m}\cos\beta + \frac{2X_6}{b} \quad (9)$$

After substitution into equation (9) the results from Table 4 and the data from Table 3:

$$\sum \frac{X_i}{b_i} = \frac{2 \cdot 0,8605}{1,0} + \frac{2 \cdot 0,2156}{2,2361} + \frac{2 \cdot 1,0165}{1,0} + \frac{2 \cdot 0,2156}{2,2361} + \frac{0,5083}{1,4142} + \frac{2 \cdot 0,8605}{1,0} = 6,21992[kN]$$

Relieving of the main truss (percent):

$$\frac{P_{odc}}{P} \cdot 100 = \frac{6,21992}{100,0} \cdot 100 = 6,22\% \quad (10)$$

where: P_{odc} - resultant relief force.

For the applied transverse and skew truss members L50x50x6 the total unloading force constitutes 6,22% of the external load applied to the main truss of truss K$_1$. Knowing values of the external force and transverse forces it is possible to calculate axial forces occurring in the analysed structure. This is the necessary step allowing to compare the results obtained by the analytical method with the results of the numerical one.

4. NUMERICAL ANALYSIS

The aim of the numerical analysis is the comparison of the obtained results with the result of the ANSYS 12.1 program utilising the finite element method. The creation of the structure space model in the ANSYS program is necessary for the calculations.

Two types of elements were chosen from the ANSYS library: BEAM 188 – which was used for representing beam elements joint rigidly and LINK 180 – for modelling elements with articulated joints (in accordance with the assumptions of the analytical method). Then the structural material was defined by the Young's modulus and Poisson ratio for steel. By means of the window of the „*Common Sections*" program, the structural sections out of which the structure was built, were defined by choosing the shape type and determining its all transverse dimensions. The three-dimensional structure model was formed by depositing characteristic truss points on the plane. They were joined by lines

into the structure representing the analysed object structure. The finished elements network for the analysed model was generated by means of the tool: *„MeshTool"*. In accordance with the assumed in Chapter 3 guidelines, the truss was loaded by a force of 100 [kN], fixed in four supports. In accordance with the A. Blum's method it was deprived from the displacement possibility in *x* direction perpendicular to the external load (this displacement was not taken into account in the analytical method due to its small influence on the calculation result).
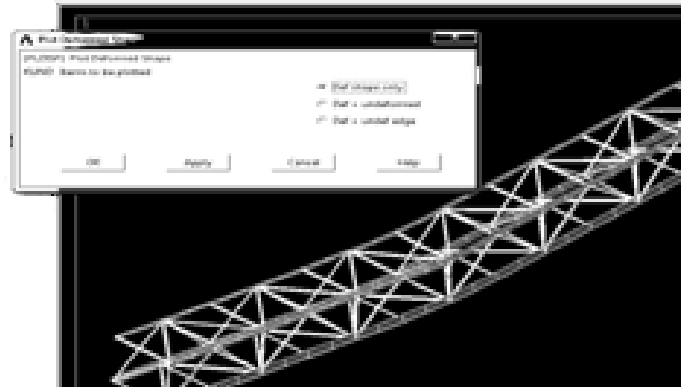

Fig. 9. Numerical truss model. Taking readings of the analytical results
Rys. 9. Numeryczny model kratownicy. Odczytywanie wyników analizy

5. ANALYSIS OF THE OBTAINED RESULTS

The results obtained by the numerical analysis are listed together with the ones obtained by the analytical method. Numerations of individual bar elements are presented in Fig. 10 and Fig. 11. An introduction of these numbers will allow to associate the obtained results with an individual truss member.
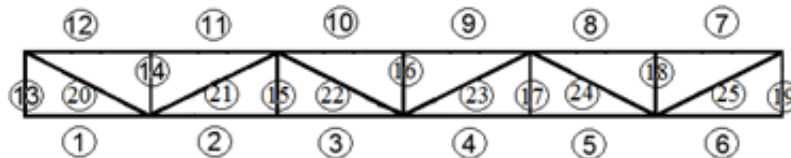

Fig. 10. Numeration of bar elements of the main truss
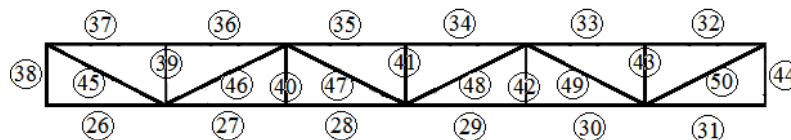Rys. 10. Oznaczenia elementów prętowych dla kraty głównej kratownicy

Fig. 11. Numeration of bar elements of the auxiliary truss
Rys. 11. Oznaczenia elementów prętowych dla kraty pomocniczej kratownicy

Table 5 contains lists of axial forces occurring in all truss K_1 bars calculated by the analytical and numerical method.

The obtained numerical calculation results indicate that the relieving (that is also the load carrying strength) of the main truss increases non-linearly with the rigidity of the crosswise and skew bars. This dependency in a diagram form is seen in Figure 11. For bars of the truss L50x50x6 (k_r=22809) of a low rigidity (in relation to the main truss rigidity) the analytical method indicates the main truss relief (which means unloading) at a level of 6,22%. For bars L100x100x8 (k_r=2023) this reliew increases to 15,5%. For bars of the truss L50x50x6 (k_r=22809) of a low rigidity (in relation to the main truss

rigidity) the numerical method indicates the main truss relief at a level of 3,5%. For bars L100x100x8 (k_r=2023) this relief increases to 9,6%.

Table 5

Calculation results obtained for truss K_1 by the numerical and analytical method

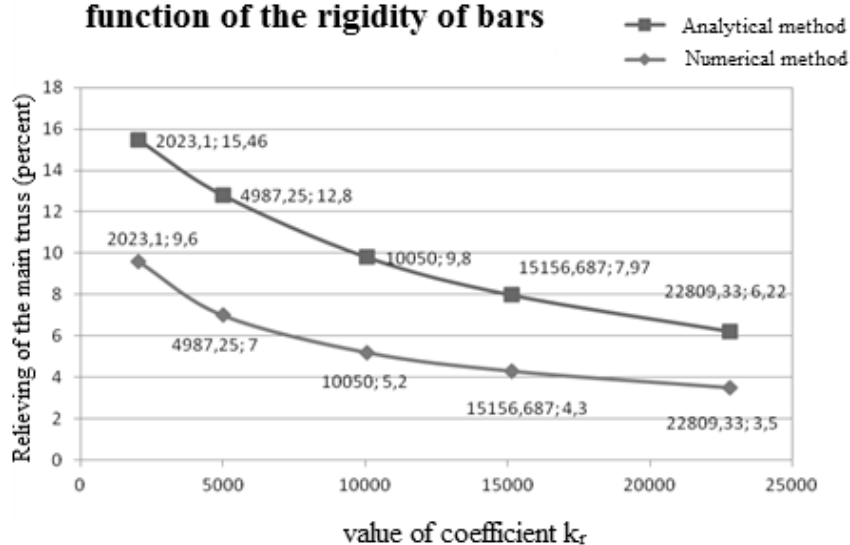| Element number	Numerical method [kN]	Analytical method [kN]	Element number	Numerical method [kN]	Analytical method [kN]
1	1,128	0	26	2,782	0
2	172,965	187,56	27	14,351	12,44
3	173,984	187,56	28	34,061	12,44
4	172,124	187,56	29	34,075	12,44
5	169,624	187,56	30	19,152	12,44
6	1,681	0	31	8,769	0
7	-95,771	-93,78	32	-1,556	-6,22
8	-95,738	-93,78	33	-1,561	-6,22
9	-251,509	-284,79	34	-38,343	-14,831
10	-250,779	-284,79	35	-38,416	-14,832
11	-84,523	-93,78	36	-3,273	-6,22
12	-95,179	-93,78	37	-3,266	-6,22
13	-48,270	-46,890	38	-0,602	-3,11
14	2,27	0	39	0,065	0
15	-3,585	-0,860	40	-0,008	-0,9569
16	-80,616	-98,43	41	-0,006	-1,016
17	-3,849	-0,8605	42	-0,070	-0,9569
18	3,180	0	43	0,138	0
19	-48,776	-46,890	44	-0,028	-3,11
20	104,751	104,85	45	2,134	6,954
21	-102,604	-104,85	46	-10,689	-6,954
22	82,468	108,7	47	10,552	2,675
23	81,282	108,7	48	-10,531	2,677
24	-101,173	-104,85	49	-10,779	-6,954
25	105,797	104,85	50	0,75	6,954

Fig. 12. Relieving of the main truss as a function of the rigidity of bars of the analysed trusses (a higher value of coefficient k_r corresponds to a smaller cross-section of the bar)

Rys. 12. Odciążenia kraty głównej w funkcji sztywności prętów wykratowania analizowanych kratownic (większa wartość współczynnika kr odpowiada mniejszemu przekrojowi poprzecznemu pręta)

6. CONCLUSIONS

Non-linear increase of the structure load-carrying capacity can be utilised in industry. An example of the practical application of the results could be an exchange of truss bars of over-head crane bridge or crane beams increasing significantly the structure load-carrying strength.

Differences between the results obtained by two considered methods can be explained by simplifications arising from the analytical method assumptions, as well as from simplifications in building the model used in the FEM. The method of increasing the load-carrying strength of truss bridges of overhead cranes by transferring a part of a useful load from the main truss on the auxiliary one was illustrated by means of the diagram in Fig. 12. The essential feature of this method is a significant relief of the main truss which allows to increase the load-carrying strength of the crane. In the case of the truss bridges, made of rolled sections containing non-metallic inclusions and exploited for many years, the method allows to limit significantly the development of lamellar cracks in the main truss by decreasing the stress gradient acting on it.

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