passenger car bodies' vibration test, nonrecursive filter
with Fourier transformation, linear regression models

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ESTIMATION OF PARAMETERS OF FORMS OF BENDING OSCILLATIONS OF A BODY BY RESULTS OF VIBRATING TESTS WITH APPLICATION OF THE THEORY OF OPTIMUM FILTRATION AND LINEAR REGRESSION MODELS

Summary. Three different passenger car bodies' vibration tests were carried out. During the tests simultaneously were register accelerations, body middle displacements and strength from vibrator to a body. On processing of testing signal was decrease its sampling frequency and used nonrecursive filter with Fourier transformation. Signal parameter estimation for first self frequency of bending vibration determination was carried out with four linear regression models according to quadratic mean criteria with pass from discrete to continuous model. Was carried out each model estimation results analyses and did conclusions about processing methods error and noise attend in measurement system.

ОЦЕНКА ПАРАМЕТРОВ ФОРМ ИЗГИБНЫХ КОЛЕБАНИЙ КУЗОВА ПО РЕЗУЛЬТАТАМ ВИБРАЦИОННЫХ ИСПЫТАНИЙ С ПРИМЕНЕНИЕМ ТЕОРИИ ОПТИМАЛЬНОЙ ФИЛЬТРАЦИИ И ЛИНЕЙНЫХ РЕГРЕССИОННЫХ МОДЕЛЕЙ

Аннотация. Проведены вибрационные испытания трех различных кузовов. Во время проведения испытаний одновременно регистрировались ускорения, перемещения в середине кузова и сила от вибратора на кузов. При обработке сигнала уменьшена его частота дискретизации и применен нерекурсивный фильтр с использованием обратного преобразования Фурье. После прореживания и фильтрации оценка параметров сигнала для определения первой собственной частоты изгибных колебаний произведена четырьмя моделями линейной регрессии по средним квадратическим критериям с переходом от дискретной к непрерывной модели. Выполнен анализ результатов оценки каждой модели, и сделан вывод об ошибке методики обработки и наличии шумов в измерительной системе.

1. BASIC PROVISIONS

Processing of signals – force operating from the oscillator on a body, displacements or accelerations of bodies of passenger carriages, in particular the modernised carriages of electric trains

ER2, after accomplish of vibrating tests it is possible to carried out with application of the theory of optimum filtration and linear regressive models in Matlab program.

The signals of measurements which are subject to processing, contains a useful signal against any disturbances (noises) operating to measuring system, thus a spectrum of disturbances generally not determined and is to some extent presented on all interval of a working frequency range. In other words, the spectrum of a useful signal is imposed on a spectrum of disturbances. In this case start the task of realisation of so-called optimum filters which, depending on the concrete purpose, allows to make reliably enough signal detection, in the best way detect a signal against disturbances or in the maximum degree to exclude disturbances from a signal without its essential distortion.

One of the problems of exception of disturbances of a signal is the exclusion of noise of the measuring equipment approximate to white noise on the spectral structure.

White noise is a stationary casual process which autocorrelation function is described by delta function of Dirac and, accordingly, the spectral density of power does not depend on frequency and has constant value equal to dispersion of values of stationary casual process. In other words, all spectral components of white noise have identical power. It is possible to present model of white noise, as casual on time (argument) sequence delta - impulses with casual amplitude values, which satisfies to conditions of statistical uniformity: a constant average of impulses in unit of time and statistical independence of occurrence of each impulse of the previous.

Synthesis of optimum filters is made with the maximum use of the known aprioristic information about signals which must be selected and about disturbances. As a rule, the information about nature of a useful signal and noise, about their spectral structure, about correlation and mutually correlation characteristics is used. The presence of features (differences) in signal and noise characteristics allow realise the filter in general and the optimum filter in particular. Definition of characteristics of operating disturbances is a more complicated problem, but even at full uncertainty it is possible to admit that the disturbance is a normal stationary process with zero average value.

For analysis and synthesis of filters additive model of input signal is used, where useful signal and disturbance are assumed by independent stationary casual processes with known correlation functions or spectral density of power.

At processing of signals three basic criteria of making of optimum filters are used: a minimum of mean quadratic deviation of filtered signal from its valid, a maximum of amplitude and power signal/noise relation on filter output. A main criterion at making of optimum filters usually is minimization of root-mean-square error. [1]

The problem of making of optimum system, basically, can be solved in two ways. In the first of them, as such system the systems are examined, structure and the parameters of which are completely unknown. Presently there are no general methods of the decision of this problem. Therefore its decision is related with considerable difficulties and, as a rule, can be executed only in some special cases.

In the second, suppose initially, that the system structure is set, and optimisation is carried out by choice of its basic parameters. In this case, by set of structure of optimised system, it is possible to take into account the features of means which are supposed to use for its realisation.

Among all linear systems, to condition of simplicity of technical realisation most fully satisfies the systems which functioning is described by the linear non-stationary differential equations. This class of systems forms the linear subspace, which included in space of all linear systems. Therefore making of optimum system for this subclass can be carried out on the basis of optimality condition.

The structure of optimum system in the form of the linear differential equation of system, which describes bending oscillations of body, is accepted:

$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = f(t), \tag{1}$$

where: t – length of continuous record at resonance pass; m – reduced mass of bending form of body oscillation; b – reduced resistance of body; c – reduced bending stiffness.

2. PROCESSING METHODS

As the filter at processing of signals of oscillation tests for suppression of disturbances was used four-stage filter realised by function (2) with three-stage decimation (3). In this case at decrease of sampling frequency in n time's frequency of Nyquist (half of sampling frequency) became in n time lower, i.e. the frequency range was narrowed. Therefore for prevention of overlapping of spectrum was applied low frequency filter, which decrease all frequency constituents of higher future frequency of Nyquist. After filtration signal was decimate in n times that spectrum of signal lower new frequency of Nyquist not be deformed.

The filter described by function (2) is linear not recursive filter, with use inverse Fourier transform (method of overlapping with summation):

$$y = fftfilt(b, x, n), \tag{2}$$

where: b - vector of filter pulse characteristics, x - vector of input signal, n - fast Fourier transform dimension.

Function (2) carries out filtration of the data from vector x by a filter, described by vector of coefficients b. A result is vector of counting out of output signal y. Function fft from (2) is used for realisation of a method of overlapping with summation at which the input signal is divided by separate blocks, each of which is filtered in frequency area, and then results of a filtration of separate blocks combine. The input signal x divided into blocks. Then convolution of each block with the pulse characteristic of the filter b is calculated. Further function (2) dispose the blocks with overlapping got after filtration on $n-1$ counting out (n - length of filter) and adds up the overlapping fragments.

For decrease of signal sampling frequency (decimation) was used function (3):

$$y = decimate(x, r, n, 'fir') \tag{3}$$

At decimation by function (3) calculation of low frequency filter is made, it decrease signal sampling frequency x in r times. Length of the decimated vector y in r times less, than length of initial vector. Here instead of recursive filter of Chebyshev not recursive filter of 30th order with normalized cut $1/r$ frequency is used, which calculated by function $fir1$. Decimation of filtered signal made by selection of each r counting out. In this case the filtration is made only in one way, because such filter does not phase distortions [2].

The estimation of coefficients of equation (1) by signals of accelerations can be making in two ways. Firstly can twice differentiate (1) and enter new variable $y = \ddot{x}$, in this case equation (1) will be:

$$m\ddot{y} + b\dot{y} + cy = \ddot{F}$$

In this case it is necessary estimate $\ddot{x}$ and $\ddot{F}$ measurements.

Secondly if measured $y = \ddot{x}$, then x it is possible to get with double integration of $\ddot{x}$ and apply estimation of coefficients of the equation (1).

Estimation of coefficients of equation (1) by displacements or accelerations can be processed by four models of linear regressions ARX, ARMAX, OE and IV4 using root mean square criteria, with transition from discrete to continuous model.

The force operating from the oscillator on a body, was considered, how a system input, and displacement or acceleration, as output.

A general input-output linear model for a single-output system with input u and output y can be written:

$$A(q)y(t) = \sum_{i=1}^{nu} [B_i(q)/F_i(q)]u_i(t \quad nk_i) + [C(q)/D(q)]e(t) \tag{4}$$

Here u_i denotes input i, and A, B_i, C, D, and F_i, are polynomials in the shift operator q. The general structure is defined by giving the time-delays n_k and the orders of these polynomials (i.e., the number of poles and zeros of the dynamic model from u to y, as well as of the disturbance model from e to y).

Most often the choices are confined to one of the following special cases:

$$A(q)y(t) = B(q)u(t \quad nk) + e(t) \tag{5}$$

$$A(q)y(t) = B(q)u(t \quad nk) + C(q)e(t) \tag{6}$$

$$y(t) = [B(q)/F(q)]u(t \quad nk) + e(t) \tag{7}$$

Note that A(q) corresponds to poles that are common between the dynamic model and the disturbance model (useful if disturbances enter the system “close to” the input). Likewise determines the poles that are unique for the dynamics from input i, and D(q) the poles that are unique for the disturbances.

Model, which described by (5) estimate the parameters a_i and b_i of an ARX or AR model by using the least squares method. It is a commonly used parametric model where B(q) and A(q) are polynomials in the delay operator q^{-1}:

$$A(q) = 1 + a_1 q^{\ 1} + ... + a_{na} q^{\ na} \tag{8}$$

$$B(q) = b_1 + b_2 z^{\ 1} + ... + b_{nb} z^{\ nb+1} \tag{9}$$

Here, the numbers n_a and n_b are the orders of the respective polynomials.

The least squares estimation problem is over determined set of linear equations that are solved using QR factorization. The regression matrix is formed so that only measured quantities are used (no fill-out with zeros). When the regression matrix is larger than Maxsize, the QR factorization is performed in a for loop.

From the physical point of view model (5), probable not the most natural; here it is supposed that before the signal will develop on a system output, the signal of white noise will dynamically transformed through system denominator. Nevertheless, the great number of error models has one important feature which defines its primary value for many applications: the predictor leads to linear regression. The basic lack of simple model (5) consists of absence of sufficient freedom in choosing in the description of properties of disturbance.

Another very common, and more general, model structure is the ARMAX, which described by (6) expression here B(q) and A(q) are in (8) and (9), while:

$$C(q) = 1 + c_1 q^{\ 1} + ... + c_{nc} q^{\ nc} \tag{10}$$

It is estimate the parameters of an ARMAX or ARMA model using prediction error method, here a robustified quadratic prediction error criterion is minimized using aniterative search algorithm. The initial parameter values for the iterative search, if not specified in orders, are constructed in a special four-stage LS-IV algorithm. A stability test of the predictor is performed to ensure that only models corresponding to stable predictors are tested. Generally, both C(q) and F_i(q) (if applicable) must have all their zeros inside the unit circle.

Output-error (OE) model structure is obtained by (7), model estimate the parameters of an output-error model using a prediction error method. Polynomial F(q) is described:

$$F(q) = 1 + f_1 q^{\ 1} + ... + c_{nf} q^{\ nf} \tag{11}$$

Models use essentially the same algorithm as ARMAX, with modifications to the computation of prediction errors and gradients.

Model IV4 estimate the parameters of an ARX model using an approximately optimal four-stage instrumental variable procedure. The first stage uses the arx function. The resulting model generates the instruments for a second-stage IV estimate. The residuals obtained from this model are modeled as a high-order AR model. At the fourth stage, the input-output data is filtered through this AR model and then subjected to the IV function with the same instrument filters as in the second stage. The main difference from ARX model is that the procedure is not sensitive to the color of the noise term e(t) in the model equation [3].

3. PROCESSING RESULTS

Results of estimation of signals parameters by functions ARX, ARMAX, OE and IV4 at initial frequency of quantization 2400 Hz and decimated to frequency of quantization 200 Hz are resulted in table 1, table 2 and at initial frequency of quantization 200 Hz in table 3:

Table 1

Result of estimation of test signals of renovated body carcass (tare) of trailing carriage of electric train ER2 No.989-05 at initial frequency of quantization 2400 Hz and decimated to frequency of quantization 200 Hz

Procedures	$m_{red.}$ [kg]	$C_{red.}$ [N/m]	F [Hz]
ARX*	$0.82 \cdot 10^3$	$6948 \cdot 10^3$	14.70
ARMAX*	$0.81 \cdot 10^3$	$6945 \cdot 10^3$	14.74
IV4*	$0.62 \cdot 10^3$	$5198 \cdot 10^3$	14.58
OE*	$0.61 \cdot 10^3$	$5170 \cdot 10^3$	14.64

Table 2

Result of estimation of test signals of renovated body of motor carriage (tare) of electric train ER2 No. №991-06 at initial frequency of quantization 2400 Hz and decimated to frequency of quantization 200 Hz

Procedures	mred. [kg]	Cred. [N/m]	Fpaš. [Hz]
ARX*	$2.91 \cdot 10^3$	$10451 \cdot 10^3$	9.54
ARMAX*	$2.34 \cdot 10^3$	$8273 \cdot 10^3$	9.47
IV4*	$3.44 \cdot 10^3$	$12082 \cdot 10^3$	9.44
OE*	$3.98 \cdot 10^3$	$13869 \cdot 10^3$	9.40

Table 3

Result of estimation of test signals of renovated body of motor carriage (tare) of electric train ER2 at initial frequency of quantization 200 Hz

Procedures	$m_{red.}$ [kg]	$C_{red.}$ [N/m]	$F_{paš.}$ [Hz]
ARX*	$2.7 \cdot 10^3$	$10160 \cdot 10^3$	9.77
ARMAX*	$2.1 \cdot 10^3$	$8003 \cdot 10^3$	9.83
IV4*	$3.4 \cdot 10^3$	$12228 \cdot 10^3$	9.51
OE*	$3.75 \cdot 10^3$	$13297 \cdot 10^3$	9.48

* the second order of polynomial was accepted in a calculation.

The first self frequency of bending oscillations is defined by formula:

$$F = \frac{1}{2\pi} \sqrt{\frac{c_{red.}}{m_{red.}}} \tag{12}$$

Vibration tests were carrying out in Ltd “Baltic Testing Centre” Testing centre. During testing excitation of vibrations of body was carried out by electromechanical oscillator. The oscillator was connected to oscillator frame through strain force measuring device which was hardly fastened to carriage body. The force to body was regulated by loads of oscillator. Oscillator was set in the middle of examine body, but sensors of displacements and accelerations on body ends in middle cross-section. For the vertical displacement or acceleration was accepted half-sum of values of left and right body ends in middle cross section of body. At body test the maximum size of balance weight was set. Total mass of unbalanced bodies was 10.4 kg.

The force measurement device was designed of beams with pasted resistive strain sensors, which from one side was executed for a single whole with a base plate with screw-bolts, were fasten oscillator, and from other side was welded to oscillator frame.

Oscillator was drive by DC engine, which has adjustable number of turns. It produced oscillating action in working range of frequencies from 5 Hz to 15 Hz. Length of records at resonance pass was more than 60 sek. at uniform change of frequency in specified range.

For measurements were used two piezoelectric sensors of accelerations Brüel & Kjær type 4507, two optical sensors of displacements type optoNCDT 1300 and multichannel measuring amplifier MGCplus.

Amplitude frequency characteristics, coherence functions and noise spectra (difference of input and prediction signal), for renovated body carcass (tare) of trailed carriage of electric train ER2 at frequency of quantization 50 Hz are presented on fig. 1 and for renovated body (tare) of motor carriage of electric train ER2 at frequency of quantization 50 Hz are presented on fig. 2.

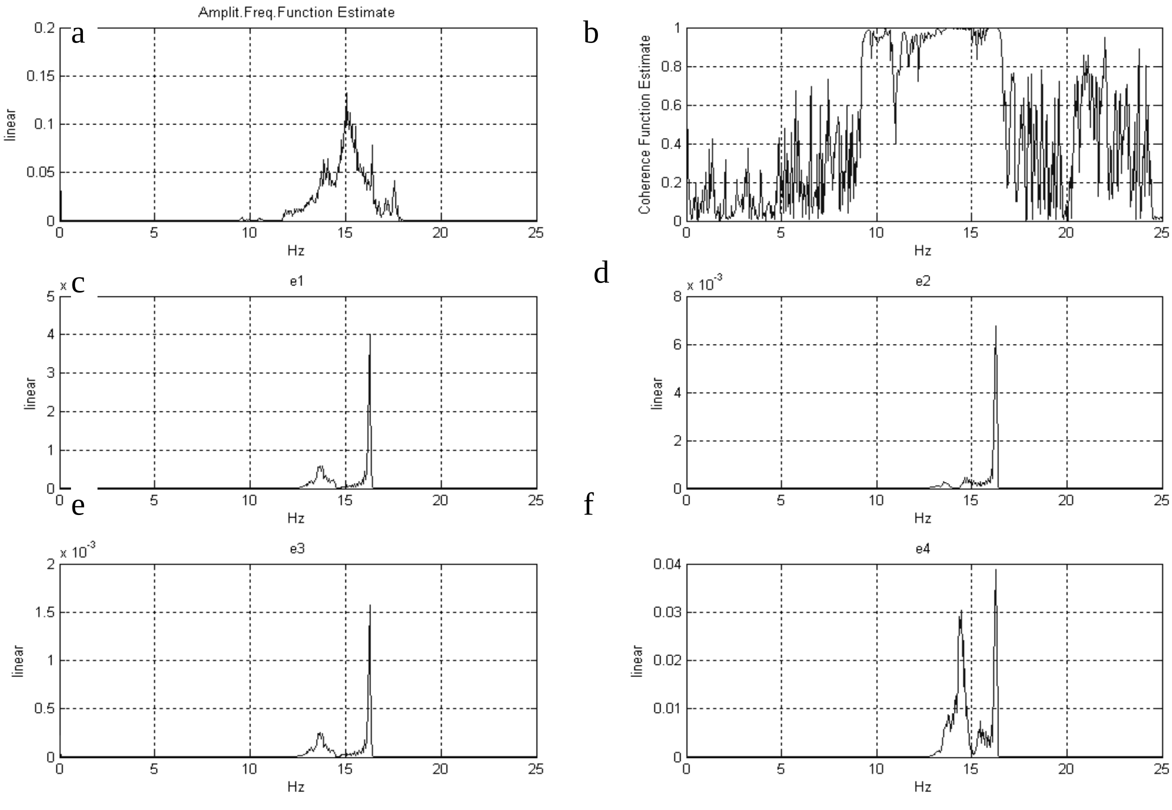

Fig. 1. Renovate electric train ER2 No.989-05 body carcass (tare) amplitude frequency response, coherence function and noise spectrum a - amplitude frequency response; b - coherence function; c - noise spectrum ARX model; d - noise spectrum IV4 model; e - noise spectrum ARMAX model; f - noise spectrum OE model

Рис. 1. Амплитудно-частотная характеристика, функция когерентности и спектры шумов модернизированного корпуса кузова (тара) электропоезда ЭР2 № 989-05 a - амплитудно-частотная характеристика; b - функция когерентности; c - спектр шума ARX модели; d - спектр шума IV4 модели; e - спектр шума ARMAX модели; f - спектр шума OE модели

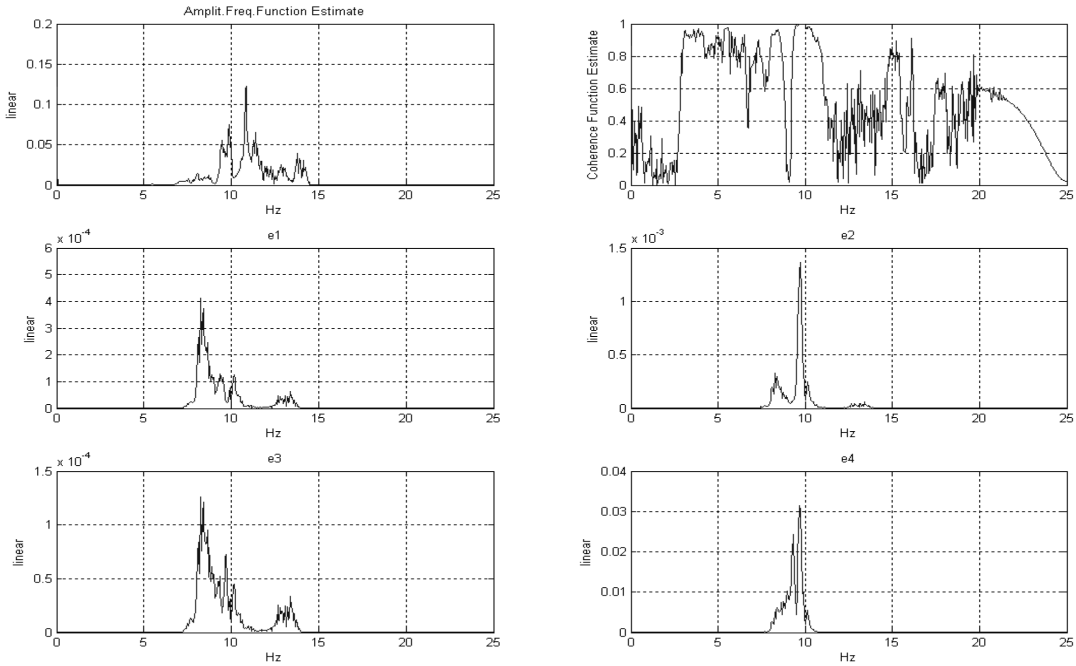

Fig. 2. Renovate electric train ER2 motor carriage body (tare) amplitude frequency response, coherence function and noise spectrum a - amplitude frequency response; b - coherence function; c - noise spectrum ARX model; d - noise spectrum IV4 model; e - noise spectrum ARMAX model; f - noise spectrum OE model

Рис. 2. Амплитудно-частотная характеристика, функция когерентности и спектры шумов модернизированного кузова моторного вагона (тара) электропоезда ЭР2 a - амплитудно-частотная характеристика; b - функция когерентности; c - спектр шума ARX модели; d - спектр шума IV4 модели; e - спектр шума ARMAX модели; f - спектр шума OE модели

The estimation of spectral density of power of signal noise for each model of parametrical estimation was processed by function pwelch, which realize method of average modified periodogram of Welch.

4. CONCLUSIONS

Comparison of calculation data of first self frequency of bending oscillations of passenger carriage of electric train ER2 with using different autoregressive models shows good coincidence of results which got with different models, namely difference of first self frequency of bending oscillations does not exceed 0.36 Hz.

It is possible to conclude from analysis of spectrum of noises, that initial frequency area has very much disturbances and in it there are noises which cannot be range to white noises.

The error of this method of processing of data does not exceed 3.6%.

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