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RAILWAY CROSSING WITH ADJUSTABLE PULLING FORCE AS FUNCTION OF LOADING

Summary. In this paper presents the concept of controlling the pulling force as a function of loading of railway crossing that was modelled as a discrete-continuous system. Two elements figure are described as continuous systems: elastomer support and pull-strings. Tension in pull-strings is adjusted in relation with railway crossing loading.

PRZEJAZD KOLEJOWY Z REGULOWANĄ SIŁĄ NACIĄGU W FUNKCJI OBCIĄŻENIA

Streszczenie. W pracy przedstawiono koncepcję regulowanej siły naciągu liny zastosowanej w wibroizolowanym przejeździe kolejowo–drogowym w funkcji obciążenia. Regulację oparto o model dyskretno ciągły systemu złożonego z elementu elastomerowego oraz liny traktowanej jako układ ciągły.

1. THE MODEL OF VIBROISOLATION SYSTEM

Vibroisolation system of the railway crossing can be described as discrete-continuous model presented in fig.1, in which the additional vibroisolating elements are pull-string 1, vibroisolating elastomer elements 3 and 2, and sensor of loading of railway crossing (scales).

Development and common usage of string systems in variety of constructions and also in vibroisolation systems, caused a necessity to consider dynamic loading of a string, along with a mass hung under it, in calculations and experiments. Dynamic loads cause vibrations of the system. As a result of these vibrations additional dynamic stress is observed in string. Those stresses must be taken into consideration especially in strength calculation, since they can reach values significantly exceeding the values of stresses resulted from static loads.

2. THEORETICAL ANALYSE

Given the dynamic load of a string, mathematical model can be created, describing vibrations in a string caused by this load. That description can be presented by means of partial differential equations or integro-differential equations. Such an analytical description of vibrations can only be done for certain model of a real object. This model is created after introduction of some simplifications, i.e. idealization of a real model. Thus created model, depending on the assumed simplifications, can be close or far away from the real model. Of course, the fewer simplifications are introduced, the more

accurately does the model present the vibration in a real object. Although, at the same time, its analytical description will be more complex and so will be its analysis.
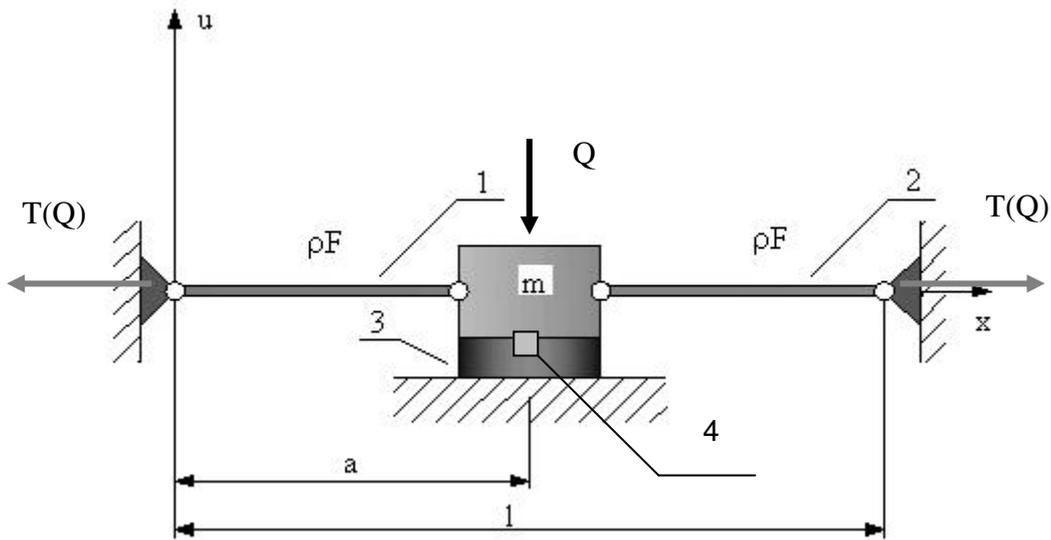
Fig. 1. The vibroisolation system of the railway crossing
Rys. 1. Model układu wibroizolacji przejazdu kolejowego

For the purpose of as accurate as possible description of a string is the mechanical system is assumed that features continuous mass loading with suitable boundary conditions. Of course, the solution of such an equation set is extremely difficult even in approximate approach, let alone the accurate solution. Basic dynamic element of the examined string system is eigenfrequency. The value of eigenfrequency often decides about values of dynamic stresses in a string. The characteristic feature of the string dynamics is the fact, that it is inextricably related to string statics. This circumstance permits carrying out the analysis of vibration independently of string statics.

Let us consider a string model vibrating only in a vertical plane (fig.2). Location of a given cross-section of the string during vibrations, more specifically – its geometric middle A_0 can be described by the coordinates:

$$x_e = x_e(x, y, t) \qquad y_e = y_e(x, y, t)$$

where: x, y – coordinates of the middle point of cross-section of string A in its static balance position, in relation to which we examine small vibrations in the string; t – time.
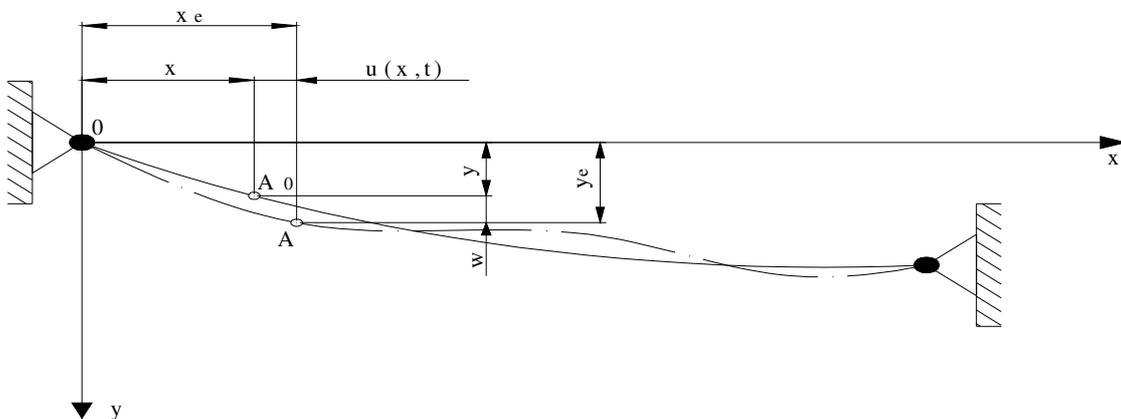
Fig. 2. Displacement of a point on tensile string during vibrations in vertical plane
Rys. 2. Przemieszczenie punktu cięgna rozciągliwego podczas drgań w płaszczyźnie zwisu

As it is depicted fig.2, it can be stated that the following relation take place between the coordinates x_e, y_e, x;

$$x_e(x,y,t) = x + u(x,t), \quad y_e(x,y,t) = y(x) + \omega(x,t)$$

where: $u(x,t)$ - displacement component of point A in the direction of x-axis during vibrations: $\omega(x,t)$ - displacement component of point A in the direction of y-axis during vibrations;

System of equations (1) describing vibrations in a tensile string in vertical plane can be set in the following form:

$$\frac{m}{\cos\alpha_0(x)}\frac{\partial^2\omega(x,t)}{\partial t^2} = \frac{\partial}{\partial x}\left[T(x)\frac{\frac{\partial\omega(x,t)}{\partial x}-\varepsilon(x,t)y'(x)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + \frac{\partial}{\partial x}\left[N(x,t)\frac{\frac{\partial\omega(x,t)}{\partial x}+y'(x)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + p_y(x,t)$$

(1)

$$\frac{m}{\cos\alpha_0(x)}\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial x}\left[T(x)\frac{\frac{\partial u(x,t)}{\partial x}-\varepsilon(x,t)}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + \frac{\partial}{\partial x}\left[N(x,t)\frac{1+\frac{\partial u(x,t)}{\partial x}}{1+\varepsilon(x,t)}\cos\alpha_0(x)\right] + p_x(x,t)$$

where: m – mass of a unit of string-length that was not distorted as a result of vibrations $[kNs^2/m^2]$; $p(x,t)$ - changed in time external load falling for a length unit measured along x axis, causing string vibrations $[kN/m]$; $T(x)$ - normal tension kN appearing in the string as an effect of static loading $q(x)$; $N(x,t)$ - total tension kN appearing in the string during vibrations; $\alpha_0(x)$ - inclination angle of tangent to the string at point $A_0(x,y)$; $\varepsilon(x,t)$ - relative elongation of the string occurring during vibrations.

Solution of this system of equations is, as we mentioned before, very complex, for this reason it is assumed that the equations describe the vibrations of string of small sag, which is most often encountered in practice. It is assumed that string sag is small if the maximum value of the sag f_0 does not exceed 1/8 of the distance between props. In case of low vibration amplitudes, when

$$\left(\frac{\partial\omega}{\partial x}\right)^2 << \frac{\partial u}{\partial x} \quad \text{and} \quad EF\frac{\partial}{\partial x}\left[\varepsilon(x,t)\frac{\partial\omega}{\partial x}\right] << H_{st}\frac{\partial^2\omega}{\partial x^2}$$

classic equation of string vibrations is yielded:

$$\left.\begin{array}{c} m\dfrac{\partial^2\omega}{\partial t^2} = H_{st}\dfrac{\partial^2\omega}{\partial x^2} \\[3mm] m\dfrac{\partial^2 u}{\partial t^2} = EF\dfrac{\partial^2 u}{\partial x^2} \end{array}\right\}$$

(2)

As it can be noticed, longitudinal and transverse vibrations are specified by independent equations so they can be examined separately. Mathematical model selected for dynamic analysis must be subject to significant simplification if the results in the analytical form are to be rendered simple. This model does not include many phenomena, among other things - non-linear, that occur in this kind of

technical solutions. Based on this analysis free vibrations of the model presented in fig. 3.8 can be specified. That permits to select the parameters of vibroisolation system of elastic railway crossings so that the dynamic effects on the environment should be minimized and, at the same time, periods between maintenance services should be extended, securing the same dynamic parameters of both automotive and railway vehicles.

Vibroisolation system of railway crossing presented in fig.1 assumes:

m - substitute mass corresponding to heavy truck mass,

ρ - line density (mass density),

F - line cross-section,

H_{st} - horizontal static component of string pulling-force – controlled value,

H_d - horizontal dynamic component of string pulling-force – controlled value,

Motion differential equations describing free vibrations of this system take the form of:

$$\left.\begin{array}{c} \rho_1 F_1 \dfrac{\partial^2 u_1}{\partial t^2} = H_{st} \dfrac{\partial^2 u_1}{\partial x^2} + y_1'' H_d \\[2em] \rho_1 F_1 \dfrac{\partial^2 u_2}{\partial t^2} = H_{st} \dfrac{\partial^2 u_2}{\partial x^2} + y_2'' H_d \\[2em] \dfrac{\partial^2 u_1(a,t)}{\partial t^2} = a^2 \dfrac{\partial^2 u_1(a,t)}{\partial x^2} \end{array}\right\} \qquad (3)$$

where: $a^2 = \sqrt{\dfrac{E^*}{\rho}}$, E^* - dynamic Young's modulus of rubber element, ρ - rubber element

density; $H_d = \dfrac{EF}{l}\left[\displaystyle\int_0^a \dfrac{\partial u_1}{\partial x} y_1'(x)dx + \displaystyle\int_a^l \dfrac{\partial u_2}{\partial x} y_2'(x)dx\right]$.

Solutions of the system of equations (3) must fulfil the following four boundary conditions:

$$u_1(0,t)=0, \quad u_2(l,t)=0, u_1(a,t)=u_2(a,t) \qquad (4)$$

$$m\dfrac{\partial^2 u_1(a,t)}{\partial t^2} + \dfrac{EF}{l}\dfrac{\partial u(a,t)}{\partial x} + H_d[y_1'(a)-y_2'(a)]+[H_{st}+H_d]\cdot[\dfrac{\partial u_2(a,t)}{\partial t}-\dfrac{\partial u_2(a,t)}{\partial x}]=0 \qquad (5)$$

After determining eigenfrequencies f_i (i=1,2,…υ) of the model of the system of railway crossing presented above it is possible to fulfil the vibroisolation requirement acquiring the form of by controlling the puling force in the string H

$$\dfrac{f}{f_0} \geq \sqrt{2}$$

In case of discrete-continuous system, as it was mentioned before, this technical vibroisolation requirement cannot be fulfilled at all times so it must be mitigated as follows:

$$f_{oi} < f_w < f_{oi+1} , \qquad (6)$$

where: i =1,2,3,…,n

3. THE NUMERICAL SIMULATION OF THE VIBROISOLATION SYSTEM

In order to obtain the information about vibroisolation system with variable pull-force controlled in relation to railway crossing loading, a mathematical model was introduced In a form of vibroisolation system of which scheme is enclosed in fig. 3.
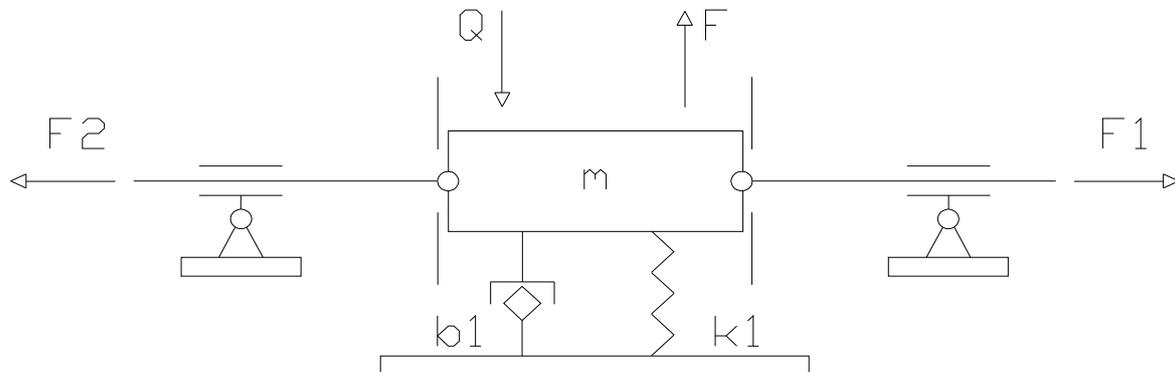
Fig. 3. Scheme of vibroisolation system
Rys. 3. Schemat układu wibroizolacji

For such a model the dynamic equation of motion for *y*-axis can be written in the following form:

$$m\ddot{y} + b_1\dot{y} + k_1 y = Q(y) \tag{7}$$

The equation above presents mathematical description of the model. Correlation between the influence of a vehicle mass *Q* and pulling forces *F1* and *F2* in the strings is modeled by the introduction of suitable strings of the length *l* connected to two actuators symmetrically settled on *x* axis. Stiffness k_1 and damping b_1 presented on the model specify the reaction of the bedding on vibroisolation system.

In order to correlate the two forces appearing in the system a system of coordinates was adapted oriented by *y*-axis along the direction of force *Q* and turn opposite to its action. *X*-axis was located on the direction of *F1* and *F2* forces. To associate a displacement along *y*-axis with suitable displacement along *x*-axis, it is assumed that the deflection of mass *m* is not significant and does not cause the increase in the *l* length of the string. The conducted of the system confirms that the optimum solution is application of contained regulation system, since only then it is possible to compensate for the displacement along y axis resulting as an effect of excitation which is the mass of an automotive vehicle driving over the crossing. Therefore, the essential purpose of the adjusting system is active influence on vibroisolation system. Based on a mathematical model the displacement of the adjusting system along y axis is determined for a given excitation, and then the undesired deflection is determined. This deflection should converge towards zero value. The control mode is determined by the way in which regulator specifies the regulation signal.

The general principle of control by changing vibroisolation system is presented in fig. 4.

z(t)

r(t)   —   model obiektu   c(t)   a(t)   regulator   u(t)   obiekt   y(t)
            Gm(s)   D(s)   G(s)

b(t)   czujnik   v(t)
                    H(s)
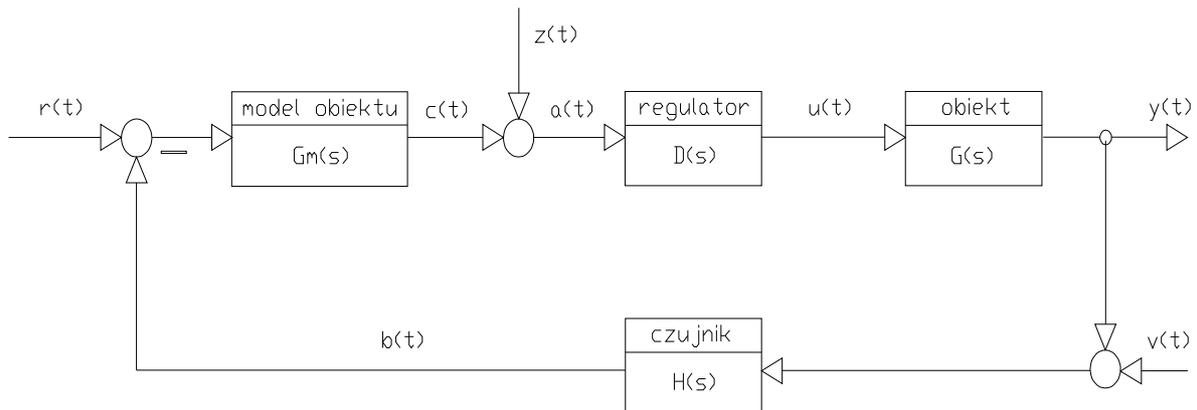
Fig. 4. Block diagram of analyzed regulation system with single feedback loop
Rys. 4. Schemat blokowy analizowanego układu regulacji z pojedynczą pętlą sprzężenia

Regulation system presented above encompasses the variety of signals and system elements. Their designations are given below:

Table 1

r(t)	-	reference signal
u(t)	-	regulation signal
y(t)	-	object output signal
e(t)	-	error signal r(t)-y(t)
a(t)	-	executive signal
b(t)	-	feedback loop output signal
c(t)	-	model of object output signal
Gm(s)	-	model of object transmittance
H(s)	-	sensor transmittance
D(s)	-	regulator transmittance
G(s)	-	object of regulation transmittance
z(t)	-	interference affecting an object
v(t)	-	sensor white noise

The explained structure of controlling vibroisolation system was, as we mentioned before, selected in order to minimize the displacement of system platform along y-axis. So r(t) can be understood as gravity force, expressed in [N], of an object that moves through the vibroisolation system. Gravity force of a moving vehicle should be determined by means of scales placed several metres in front of the crossing. Such a layout results from the fact that the signal on gravity force is passed onto the model of an object in order to theoretically determine the platform displacement $c(t)$.

The determined displacement *c(t),* expressed in metres, is compared with interference caused by the object. In this analyzed case our signal *z(t)* describes the assigned platform displacement, which by assumption equals 0. As a result, signal *a(t)* is obtained, which is difference between interference signal and model-of-object output signal. This signal is subsequently passed into the regulator. Thanks to the pre-selected settings of the regulator in the output of the regulator a regulation signal was yielded, represented by a force expressed in [N], which should be exerted onto the vibroisolation system to counteract the displacement of the platform along *y*-axis. In order to carry out a simulation of the adapted regulation system we modelled the analyzed vibroisolation system in Matlab-Simulink software environment. Fig. 5 below presents the model of vibroisolation system.
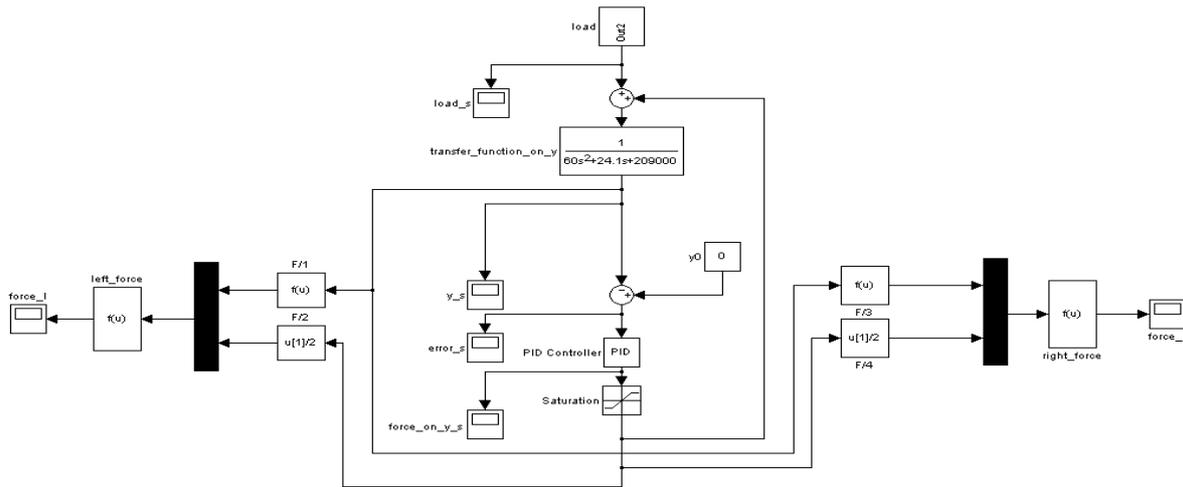
Fig. 5. Block diagram of the analyzed regulation system modelled in Matlab-Simulink environment
Rys. 5. Schemat blokowy analizowanego układu regulacji zamodelowany w środowisku programu
         Matlab-Simulink

One of the most crucial elements in the process of designing regulation systems is selecting suitable system structure and optimum regulator settings. In the analyzed case the regulation system should be provided with PID regulator. Among the requirements that are commonly applied in regulation systems are over-regulation in the range from 0 to 5% and minimum regulation time t_r.

Suggested PID regulator is an element of regulation system executing, in an ideal case, the following control principle:

$$\mathrm{u}(t) = K_p \, \mathrm{e}(t) + K_i \int \mathrm{e}(t) \, dt + K_d \left(\frac{d}{dt} \mathrm{e}(t) \right)$$

where:

Table 2

K_p	-	proportional-section coefficient
K_i	-	integral-section coefficient
K_d	-	differentiating-section coefficient

Selecting the structure of PID regulator was imposed by the fact that vibroisolation system requires a wide control over regulator settings.

The type of regulator that is commonly applied in practice is PID regulator with real differentiating section:

$$G_{PID}(s) = K_p + \frac{1}{T_i s} + \frac{T_d s}{\tau s + 1}$$

$$G_{PID}(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\tau s + 1}$$

Differentiating section implemented in practice contains insignificant inertia, so time constant of inertial filter most often equals $\tau = (0.05 \div 0.25)T_d$

Optimal regulator settings are in regulation systems generally different for delay error and the error associated with interference. In practical applications PID regulator is adjusted in a regulation system into which it is actually applied. Elementary rules of this adjustment are assembled below:

Table 3

1	Determining K_p value, in order to reach suitable response quickness. Increasing proportional amplification results in the increase in response quickness and reduces the error in a stable state.
2	Selection of integrating control $1/T_i$ in order to obtain suitable quality in a stable state (necessity of correcting amplification value K_p is likely) the increase in integral $1/T_i$ action deteriorates stability, yet it helps liquidate the error in stable state.
3	Addition of differentiating control in order to reduce over-regulations and improve regulation time. Increasing differential constant enhances stability and facilitates oscillation damping.

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In case of PID regulator control characteristics of individual section must be put forward. With proportional control with K_p setting there is a possibility of influencing the reduction of the increase time and reduction error in stable state. After analyzing the effect of integral control with K_i setting it can be noted that it eliminates the error in stable state but it deteriorates the response in transition time. Differentiating control with K_d setting causes the increase in system stability by decreasing over-regulation and improving transitional response of the system.

In order to carry out a computer simulation of vibroisolation system and to select suitable settings of PID regulator the model of the object of regulation was assumed and subsequently recorded in the form of transition function:

$$\frac{Y(s)}{U(s)} = \frac{1}{ms^2 + b_1 s + k_1} \tag{8}$$

The following values of mass, elasticity coefficient and damping coefficient were assumed in the simulations:

M	=	60	[N]	-	platform mass
b_1	=	24.1	[Ns/m]	-	damping coefficient
k_1	=	209000	[N/m]	-	elasticity coefficient

According to the assumed designation in the input of the system signals, determining gravity force Q of a vehicle moving over the crossing in t time will appear. The exemplary time characteristic of such a signal is presented in fig. 6.
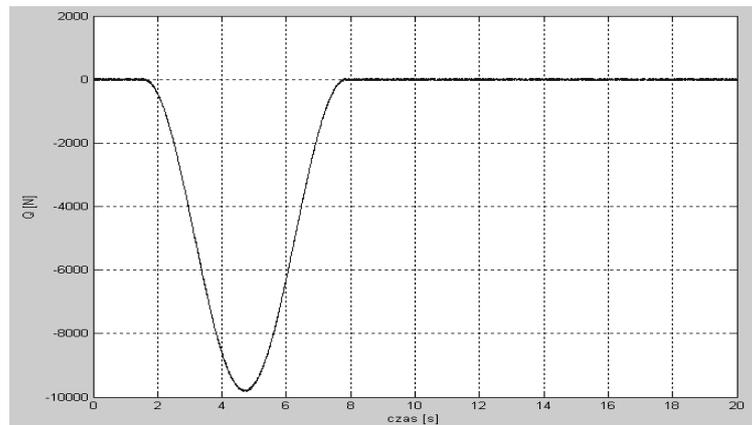
Fig. 6. Time curve determining gravity force Q of a vehicle
Rys. 6. Przebieg czasowy określający siłę ciężkości Q pojazdu

Time curve of gravity force presented in fig. 6 corresponds to car of a mass of 1000 [kg] passing over the railway crossing in 6.3 [s] time. Applying the model of the object we can specify the theoretical deflection of the crossing under the exertion of the measured gravity force. Fig. 7 presents the response of the object model in the form of displacement along y-axis.
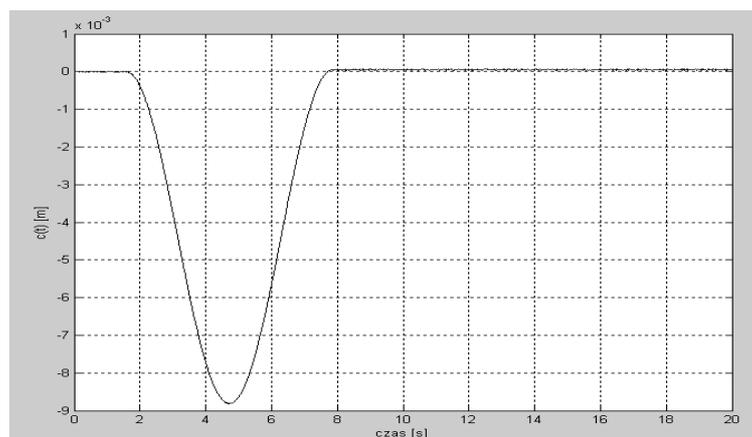
Fig. 7. Time curve determining platform displacement - c(t)
Rys. 7. Przebieg czasowy określający przemieszczenie platformy - c(t)

Based on time characteristic of c(t) signal and z(t) signal the difference of displacements was determined which was subsequently fed into the input of PID regulator. As a result of executed simulations the regulator settings were settled. They are contained in table 5.

Table 5

K_p	-	1000
K_i	-	2000
K_d	-	10000

Based on these settings, the regulator output produces time characteristic of the force that must counteract the gravity force of a vehicle to maintain the assigned platform position. Fig. 8 below presents force F [N] counteracting gravity force of a vehicle.
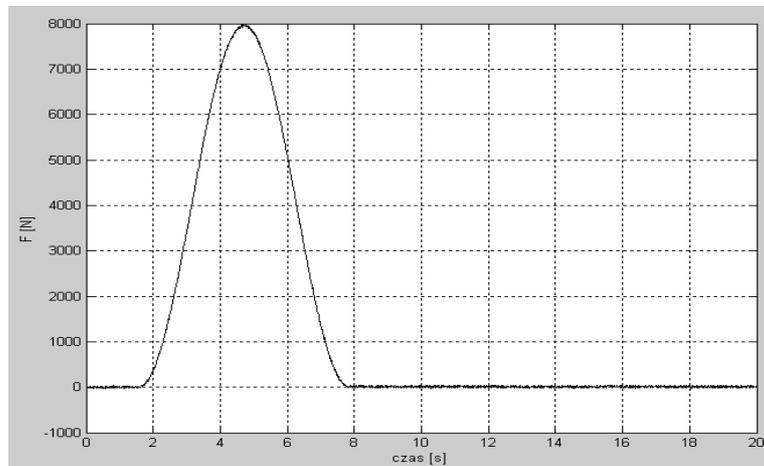
Fig. 8. Time curve determining gravity force F - u(t)
Rys. 8. Przebieg czasowy określający siłę F - u(t)

Resulting signal u(t) is a control signal that should be put onto the object. Owing to the type of distribution of force *F* into the actuators we carried out calculations of the necessary forces F1 and F2 in actuators 1 and 2 respectively. Fig. 9a and 9b present values that must be generated for the vibroisolation system to compensate the displacement along *y*-axis.

a   b
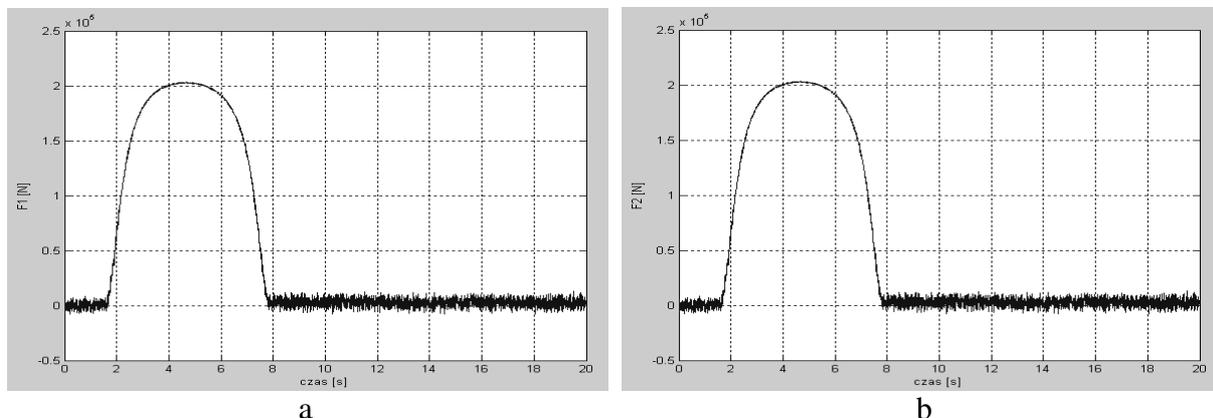
Fig. 9. Time characteristics determining forces F1 (a) and F2 (b) in actuators 1 and 2 - y(t)
Rys. 9. Przebiegi czasowe określające siły F1(a) i F2(b) w siłownikach 1 i 2 - y(t)

4. CONCLUSIONS

In case of complex vibroisolation objects, when the mass of vibroisolating element is significant – and such a condition occurs with transportation machines and vehicles, when geometric dimensions of the elements of vibroisolation system are plate-like – modelling of vibroisolation system as a discrete system entails immense risk, since we must not apply vibroisolation elements without taking their masses into consideration.

The most serious consequence of such an error is the occurrence of wave phenomenon in resilient-damping elements, since we cannot assume that the elements are weightless. Wave effect can cause that vibroisolation will bring the opposite effect to the intended goal of reducing dynamic influence on the environment. To avert such a possibility it is necessary to determine free vibration frequency of vibroisolation system based on the assumption that the model of this vibroisolation system is continuous or discrete-continuous. When vibroisolating elements (rubber or pull-strings) feature continuous distribution of mass, the frequencies for homogeneous prismatic systems e.g. rubbers of l height can be determined based on the methodology presented in this research.

Analysis of the suggested regulation system proves that there is a possibility to apply this type of control into stabilizing vibroisolation platform. Based on a series of simulations carried out for various weights of vehicles driving over the railway crossing in different times we conclude that the experimental method of determining PID regulator settings, for different mass and passage-time configurations, may prove effective in reducing dynamic influence of vibrations on the environment. In this method the settings are dynamically changed based on the information on speed and mass of a vehicle.

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