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TRENDS AND CORRELATION ANALYSIS IN DIAGNOSING TURBINE ROTOR BOW

Summary. Permanent rotor bow in a steam turbine is a serious failure which usually demands a time-consuming and costly repair. Its vibration-related symptoms are not specific and qualitative diagnosis typically has to employ results obtained during transients.

In a 230 MW power steam turbine, gradual dynamic behavior deterioration was observed, immediately after commissioning. Increase of the fundamental component of rear intermediate-pressure turbine bearing vertical vibration was detected, with the time constant of the order of months. Permanent rotor bow, exceeding 200 μm, turned out to be the cause. Rotor repair resulted in a dramatic improvement of dynamic behavior, which, however, soon began to deteriorate again. Vibration spectra had been detected in the off-line mode since commissioning, which allowed to determine vibration time histories.

Vibration trends analysis does not provide sufficient information to determine root cause, but allows for eliminating a number of possible malfunctions that give similar symptoms. In particular, the possibility of a sudden random-type damage due to human error is eliminated, which in fact is the most common cause of a permanent bow.

Analysis of vibration amplitude correlation between vertical and axial directions reveals very strong correlation between fundamental components in the turbine under consideration, as well in the other one, in which similar failure has been observed. Third unit of the same type, apart from qualitatively different vibration trends, is characterized by correlation factors lower by a few times.

This particular case is indicative of the importance of evolutionary symptoms (vibration amplitude time dependence and increase rate, as well as correlation factors) in qualitative diagnosis. Such symptoms can be very useful in distinguishing between possible failures which result in similar changes of machine vibration behavior.

ANALIZA TRENDÓW I KORELACJI W DIAGNOZOWANIU WYGIĘCIA WIRNIKA TURBINY

Streszczenie. Trwałe wygięcie wirnika turbozespołu jest poważnym uszkodzeniem, wymagającym na ogół długotrwałej i kosztownej naprawy. Jego symptomy drganiowe są niejednoznaczne i diagnozowanie jakościowe przeważnie musi uwzględniać wyniki pomiarów w stanach nieustalonych.

W turbozespole energetycznym 230 MW stwierdzono stopniowe pogarszanie się stanu dynamicznego bezpośrednio po oddaniu do eksploatacji. Obserwowano wzrost składowej podstawowej drgań pionowych tylnego łożyska części średnioprężnej, ze stałą czasową rzędu miesięcy. Przyczyną okazało się trwałe wygięcie wirnika, przekraczające 200 μm. Naprawa wirnika przyniosła zasadniczą poprawę stanu dynamicznego, choć po niej zaczął on ponownie się pogarszać. Przez cały okres eksploatacji były wykonywane okresowe rejestracje widm drgań, dzięki czemu możliwe było określenie trendów drgań.

Analiza trendów nie daje podstawy do jednoznacznego zidentyfikowania niesprawności, pozwala jednak na wyeliminowanie wielu uszkodzeń, dających zbliżone objawy. W szczególności wykluczyła ona nagłe uszkodzenie awaryjne, spowodowane błędem obsługi, które jest najczęstszą przyczyną trwałego wygięcia wirnika.

Analiza korelacji amplitud drgań w kierunkach pionowym i osiowym wykazuje bardzo wysokie współczynniki korelacji liniowej między składowymi podstawowymi dla omawianego turbozespołu oraz drugiego, tego samego typu, w którym wystąpiło podobne uszkodzenie. Trzeci turbozespół tego samego typu charakteryzuje się, oprócz jakościowo innych trendów drgań, kilkakrotnie słabszą korelacją.

Opisany przypadek wskazuje na dużą przydatność symptomów typu ewolucyjnego (kształt zależności amplitudy drgań od czasu i szybkość jej wzrostu oraz współczynnik korelacji) w diagnozowaniu jakościowym. Symptomy takie mogą być bardzo użyteczne w przypadku konieczności rozróżnienia między możliwymi niesprawnościami, którym odpowiadają podobne zmiany charakterystyk drganiowych maszyny.

1. INTRODUCTION

In rotating machines three types of rotor bow can be distinguished [1], namely:

- elastic bow due to static load,
- temporary elastic bow, caused by non-uniform temperature field or rotor material anisotropy,
- permanent bow, which takes place if rotor material yield strength has been exceeded.

Bow of the first type results directly from design features and operational conditions. In machines with a horizontal rotor this is principally the bow resulting from gravity load, but some other causes may also be present, e.g. non-uniform pressure distribution in the control stage of steam turbines. Temporary elastic bow is in most cases caused by uneven rotor cooling; in steam turbines, a failure to start the shaft rotating gear after shutdown is a typical cause. Such occurrences are quite frequent, but usually, once the temperature field has stabilized, the bow disappears, although this process can take a long time. Other typical cause is rotor rub in sealing glands, which can result in a local temperature increase and deformation, again usually a temporary one.

Causes of permanent rotor bow are not frequent, but usually should be regarded as serious ones. Even a small bow can result in a dynamic behavior deterioration to a point wherein further operation is not possible. If the permanent bow magnitude is within certain limits, the rotor can be repaired by machining and thermal treatment; otherwise, rotor replacement is the only solution. In both cases, high costs and long forced outages (in critical machines of a unique design the replacement rotor often has to be procured and manufactured), and hence considerable losses, are involved.

According to [1], response of a rotor rotating with angular velocity ω to a permanent bow can be expressed as a sum of two terms:

$$\mathbf{r} = r_e e^{j\delta} + \frac{M r_e \omega^2 e^{j\delta}}{[\, K - M\omega^2 + jD(1-\lambda)\omega \,]} \tag{1}$$

wherein it has been assumed that, as a result of rotor bow, mass M is shifted by r_e from the symmetry axis in the direction determined by the angle δ, K denotes elasticity coefficient, D – damping coefficient, while λ is the dimensionless rotational viscosity coefficient, defined as

$$\lambda = \nu_s / \omega \ , \tag{2}$$

where ν_s denotes average angular velocity of the fluid surrounding the rotor. First term in Eq.(1) determines the position vector imposed by the bow itself, while the second one results from the fact that bow causes an unbalance (it has been assumed that no other sources of unbalance are present). Note that at ω far above the resonance (which takes place at the angular velocity ω_r) K and D can be neglected, so that both terms become equal and have opposite signs, i.e. response becomes zero. Such rotor 'self-balancing' is in fact sometimes observed in practice, but often vibration resulting from the bow is so high that resonance cannot be passed and startup has to be terminated at $\omega < \omega_r$. In steam turbines operated at power plants at the nominal angular velocity ω_0 the condition $\omega_0 >> \omega_r$ is usually not met. Due to large rotor weight, even a minor bow can render turbine startup impossible.

Vibrational symptoms of rotor bow reminds that of a 'plain' unbalance [1-3] and thus additional diagnostic experiments are required for a correct and reliable diagnosis. As it can be seen from Eq.(1), rotor bow results in a pronounced reaction at slow rotational speed that decreases above resonance. Such behavior during startup is thus indicative of this type of failure, although presence of other unbalance sources or, more generally, of the synchronous $1 \times f_0$ signal (which is usually the case with more complex machines) can substantially distort this picture. Behavior of harmonic components during machine trips in various thermal conditions [4] and phase variations during startups [5] have also been pointed out as additional sources of diagnostic information. All these procedures, however, involve tests in transient conditions (startups and trips) which, with steam turbines operated by utility power plants, can be difficult or even impossible to perform.

2. CASE DESCRIPTION

The case discussed hereinafter refers to 230 MW condensing steam turbines, installed at a utility power plant. Three-section turbine (high-pressure WP, intermediate-pressure SP and two-stream low-pressure NP) with steam reheat drives a two-pole generator. Turbine-generator unit shaft is supported by seven journal bearings and measures about 23 m in length. Nominal rotational speed is 3000 rpm. Schematic diagram of the unit is shown in Fig. 1.

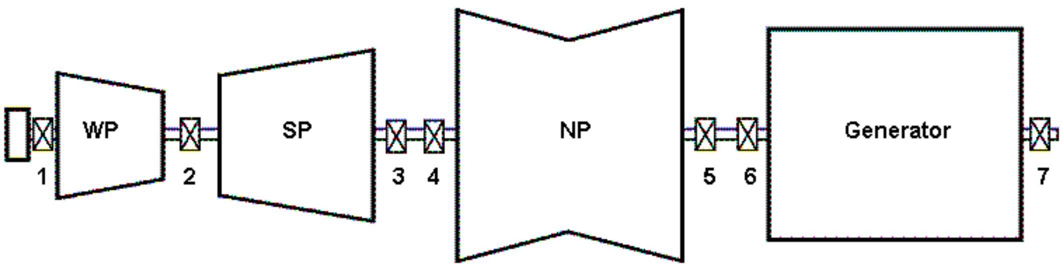

Fig. 1. Schematic diagram of the 230 MW turbine-generator unit (numbers denote bearings)
Rys. 1. Schemat turbozespołu 230 MW (liczby oznaczają łożyska)

First unit was commissioned in 1998, two more followed in 1999 and 2001, respectively. In the following they shall be referred to as T1, T2 and T3, in the order of commissioning. Immediately after first startup dynamic behavior of the T1 unit was good, but soon an increase of absolute vertical vibration at the rear SP bearing (No.3) was observed. A few weeks after commissioning vibration velocity amplitude at this point was slightly below 1 mm/s, which is a very good result; then it increased systematically, to reach about 5.8 mm/s in less than four years. At the same time, problems with cold startups were encountered. Passing through critical speeds repeatedly resulted in turbine trips due to excessive vibration. In early 2003 it was decided to perform SP shaft balancing. This was a routine action, intended to keep the turbine in operation until the overhaul scheduled in 2006. After a few attempts, vertical vibration velocity at the bearing No.3 was reduced to about 3.2 mm/s, but in two years it increased again, to reach 5.5 mm/s.

During the overhaul, a permanent SP rotor bow was found (over 200 μm), which had to be repaired by machining and thermal treatment. First startup after the overhaul revealed that vertical vibration velocity amplitude at the bearing No.3 had decreased to about 0.95 mm/s, i.e. almost exactly the value recorded immediately after commissioning. The unit remains in operation, but vertical vibration at the bearing No.3 still tend to increase; currently (early 2009) vibration velocity amplitude is about 3.9 mm/s. It is worth mentioning here that the unit T2 has displayed qualitatively very similar behavior, while dynamic condition of the unit T3 has been much better and its evolution very slow.

3. ROTOR BOW DIAGNOSING

3.1. Initial remarks

As mentioned above, in this particular case rotor bow has not been detected with vibration-based (and indeed any) diagnostic methods, but simply measured after disassembly. However, database is available from vibration measurements that allows for tracking relations between various vibrational symptoms and failure development, which may provide a valuable contribution to establishing relevant diagnostic relations.

Diagnosing may be referred to as reasoning on the object technical condition on the basis of signal features accessible by measurements, or diagnostic symptoms[1]. According to [7], as many as 21 symptom types can be distinguished for steam turbines. Even if we restrict our attention to vibration-based symptoms, it seems that – particularly in practical applications – attention is paid mainly to those physical quantities which are directly measurable and may be compared with corresponding critical values. This, however, means that the entire object history is in fact neglected. Parameters that describe time dependence of signal features, which can be referred to as evolutionary symptoms, are a valuable source of diagnostic information [8].

3.2. Vibration trends analysis

Importance of information provided by evolutionary symptoms is illustrated by time histories of the $1{\times}f_0$ component of vertical vibration, measured at the bearing No.3 in units T1, T2 and T3, which are shown in Fig. 2 (other vibration components have not displayed any noticeable increase tendency).

Before we proceed with an analysis of time histories shown in Fig. 2, a remark has to be made. It is known that for complex machines, operated in an industrial plant environment, diagnostic relation can, in the most general form, be expressed by [7]

$$\mathbf{S}(\theta) = \vartheta[\mathbf{X}(\theta), \mathbf{R}(\theta), \mathbf{Z}(\theta)] \ , \tag{3}$$

where: **S**, **X**, **R** and **Z** are vectors of symptoms, condition parameters, control and interference, respectively, all changing with time θ; ϑ denotes an operator. Components of the control and interference vectors are generally random variables, which can be described by relevant statistic parameters, in the simplest approach by their expected values and standard deviations. Time dependence of a given symptom $S(\theta)$ can thus be considered a superposition of the 'pure' $S = F[\mathbf{X}(\theta)]$ dependence and fluctuations resulting from the influence of the **R** and **Z** vectors. For many machines, including turbine-generator units, it is reasonable to assume that components of these vectors have no monotonic time evolution tendencies, which can be expressed as [9]

$$\bigwedge_i \Delta\theta \to \infty \Rightarrow \frac{\Delta R_i(\theta)}{\Delta\theta} \to 0 \, , \tag{4}$$

[1] We recall that symptom is treated here as a variable, usually a continuous one, rather than as an event. This corresponds to the symptom definition as a 'physical quantity covariant with object condition' (see e.g. [6]) and is related to distinguishing between a 'failure symptom'and a 'condition symptom' [7].

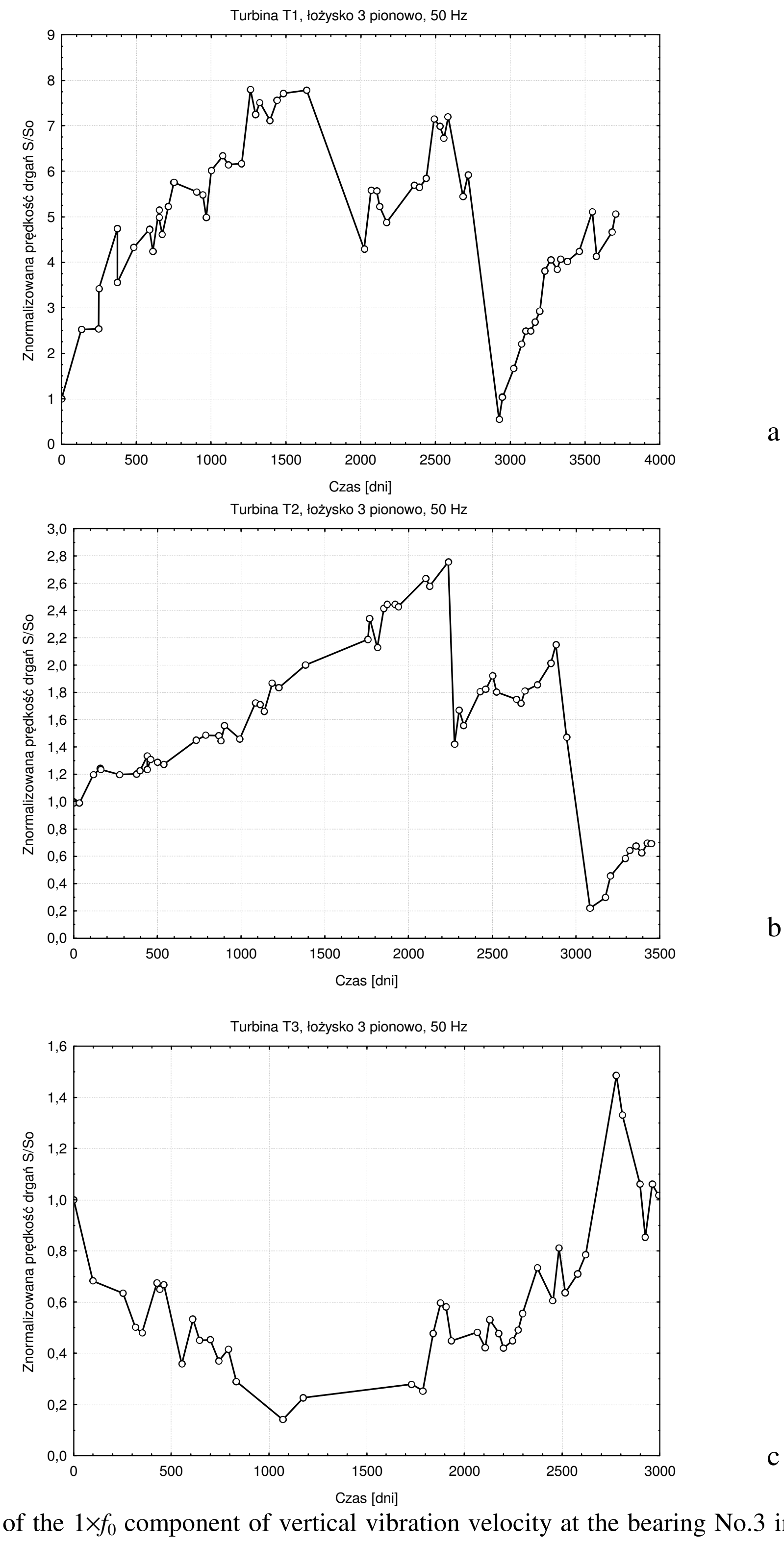

Fig. 2. Time histories of the $1\times f_0$ component of vertical vibration velocity at the bearing No.3 in units T1 (a), T2 (b) and T3 (c)

Rys. 2. Trendy składowej podstawowej prędkości drgań pionowych łożyska 3 turbozespołów T1 (a), T2 (b), T3 (c)

$$\bigwedge_i \Delta\theta \to \infty \Rightarrow \frac{\Delta Z_i(\theta)}{\Delta\theta} \to 0\,, \tag{5}$$

where:

$$\Delta Z_i(\theta) = Z_i(\theta + \Delta\theta) - Z_i(\theta),\ \Delta R_i(\theta) = R_i(\theta + \Delta\theta) - R_i(\theta). \tag{6}$$

This is equivalent to stating that results of time histories analysis can be considered meaningful if the period covered by this analysis ($\Delta\theta$ in Eqs.(4 ÷ 6)) is sufficiently long. What does ‘sufficiently’ in fact mean depends on relative values of statistic parameters that describe **R** and **Z** vectors influence and of ΔS, which is the increase of S during $\Delta\theta$. More detailed analysis has revealed that, for vibration-based symptoms, this strongly depends on frequency [10]. As it is seen from an example shown in Fig. 3, in the harmonic components range relative standard deviation can be estimated at a few percent, while in the blade frequency range it is much larger, at a few dozen percent. Qualitatively and even quantitatively similar results have been obtained for other turbine types and other measuring points [10,11]. It can be estimated that in the particular case discussed in this paper, due to a pronounced increasing tendency of a component from the harmonic frequency range, a period of about two years is sufficient for a meaningful reasoning from vibration time histories.

Fig. 2 shows that for units T1 and T2 time histories of the component under consideration are qualitatively very similar. Maximum amplitude (normalized with respect to its initial value in order to facilitate comparison) peaks at 7.8 in the first case and at 2.8 in the second one, the increase being close to linear. Rotor balancing results in a stepwise drop, but increasing tendency is not removed. Effect of the major overhaul is similar, which indicates that the root cause has not been eradicated. If we assume that balancing attempts determine object life cycles and apply a linear fit to each, we get for the normalized symptom $s = S/S(\theta = 0)$ the $ds/d\theta$ values in consecutive cycles of 3.7×10^{-3}, 2.4×10^{-3} and 5.2×10^{-3} /day for T1 and 0.7×10^{-3}, 0.4×10^{-3} and 1.4×10^{-3} /day for T2. It can thus be seen that for T2 the increase is much slower, but for each unit the values in individual life cycles are of the same order of magnitude; for this frequency range they should be considered as high [8]. As described in reference [8], a measure of the vibration amplitude evolution intensity is an important diagnostic symptom. As for T3, the $s(\theta)$ time history is qualitatively different: maximum value of s is about 1.5, relation is far from linear and an initial section can be seen that reminds the well-known ‘bathtub curve’.

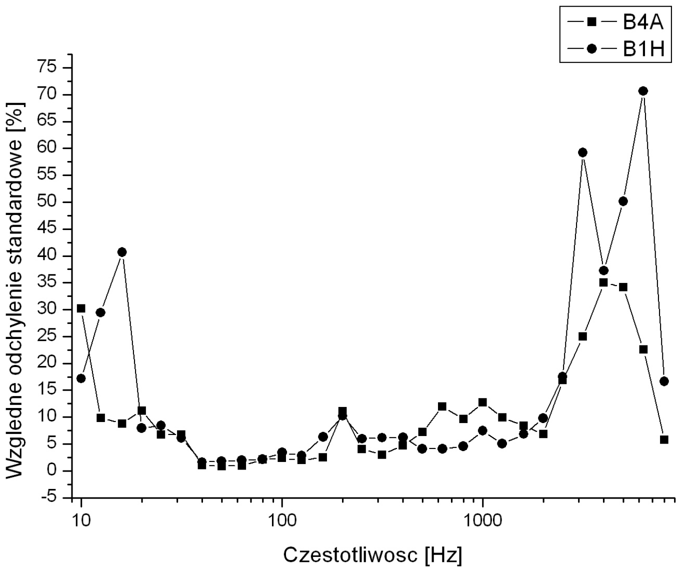

Fig. 3. Relative standard deviation σ/S_{av} vs frequency, estimated from 23% CPB vibration velocity spectra for a K-200 turbine (the same layout as shown in Fig.1); ● – bearing 1 horizontal, 55 measurements; ■ – bearing 4 axial, 58 measurements

Rys. 3. Zależność względnego odchylenia standardowego σ/S_{av} od częstotliwości, oszacowana na podstawie tercjowych widm prędkości drgań dla turbozespołu typu K-200 (układ konstrukcyjny analogiczny jak na rys.1); ● – łożysko 1 poziomo, 55 pomiarów; ■ – łożysko 4 osiowo, 58 pomiarów

Time histories recorded for T1 and T2 do not allow for an unequivocal qualitative diagnosis, but nonetheless on their basis some possible causes of dynamic behavior deterioration can be rejected, primarily those of the sudden type (which in fact are usually the rotor bow causes). Observed processes are characterized by time constants of the order of months. On this basis, foundation deformation was suggested as the root cause [3,7]. This seemed possible, as the unit had been settled on an old foundation that had remained after decommissioning of an old unit from the early 1960s. Geodetic measurements, however, excluded such possibility. Similarly, turbine fluid-flow system failure suggestion was rejected, as no increase of blade components amplitude had been recorded.

Vibration time histories analysis is in this case useful primarily due to the fact that several possible root causes can be rejected. Apart from facilitating diagnostic reasoning, this was important for plant staff, as a human error possibility has been virtually eliminated.

3.3. Correlation analysis

In complex machines, as a rule, each symptom S_i depends on a number of condition parameters X_j. Thus, even if influences of control and interference are neglected, we cannot speak in terms of the $S_i = f(X_j)$ relations and symptoms are characterized by probability distribution rather than deterministic values. Stochastic relations [12] can, however, be expected, which implies that a change of X_j shall change a probability distribution. Moreover, it is justified to assume that (with the influences of **R** and **Z** vectors neglected for a moment) if

$$S_i = F(X_1, X_2, ..., X_j. ..., X_N) \tag{6}$$

and condition parameter X_j changes considerably, then the value of S_i will, from the statistical point of view, fluctuate around an expected value $\hat{S}_i = f(X_j)$. This means a correlative relation. The above may be considered a form of generalization, or broadening, of the reasoning pertaining to the control and interference parameters.

If two symptoms are correlated, we may infer they are dependent, i.e. their changes have been caused by the same condition parameters. The reverse is not correct [13]: lack of correlation is not equivalent to independence. Strength of correlation between two random variables is often described by the Pearson linear correlation coefficient, which is a normalized covariance defined by [13]:

$$\rho = \frac{E\{(S_1-\eta_1)(S_2-\eta_2)\}}{\sqrt{E\{(S_1-\eta_1)^2\}E\{(S_2-\eta_2)^2\}}} \quad , \tag{7}$$

where E denotes expected value and

$$\eta_1 = E(S_1),\ \eta_2 = E(S_2) \quad . \tag{8}$$

It can be easily shown that $|\rho| \leq 1$; $\rho = 0$ means no correlative relation between S_1 and S_2, while $|\rho| = 1$ means functional (deterministic) relation.

In order to employ correlation analysis in this particular case, let us consider a very simple (yet sufficient) approach, shown schematically in Fig. 4. With no permanent bow (only rotor sag due to gravity load is present) the shape shown in Fig. 4a is maintained during a full rotation and angle α is constant (assuming ideal matching between disk and shaft). If, however, a permanent bow is present, after rotating by $\pi/2$ rotor will take the shape shown in Fig. 4b. In such case, $\alpha = \alpha(t)$ and, neglecting interaction between the disk and the medium (steam), we get

$$\alpha \approx \alpha_s + \alpha_b \sin 2\pi f_0 t \quad , \tag{9}$$

where α_s denotes the angle between disk and a plane perpendicular to the rotor axis due to elastic sag, while α_b is the amplitude of the angle resulting from permanent bow. The disk will thus exhibit a time-dependent tilt in the plane parallel to the rotor axis, so that an additional periodic axial force with the period of $T = 1/f_0$ should be expected. In such conditions we may expect additional component of axial vibration, with the frequency f_0 and amplitude increasing with permanent bow magnitude. At the same time, increasing bow induces increasing unbalance and hence increasing $1{\times}f_0$ component of vertical vibration. This can be easily seen in Fig. 2, linear dependence being a direct consequence of Eq.(1).

Strong correlation should thus be expected between $1 \times f_0$ components of vertical vibration at bearing No.3 and axial vibration in this point, and in fact also in adjacent points[2].

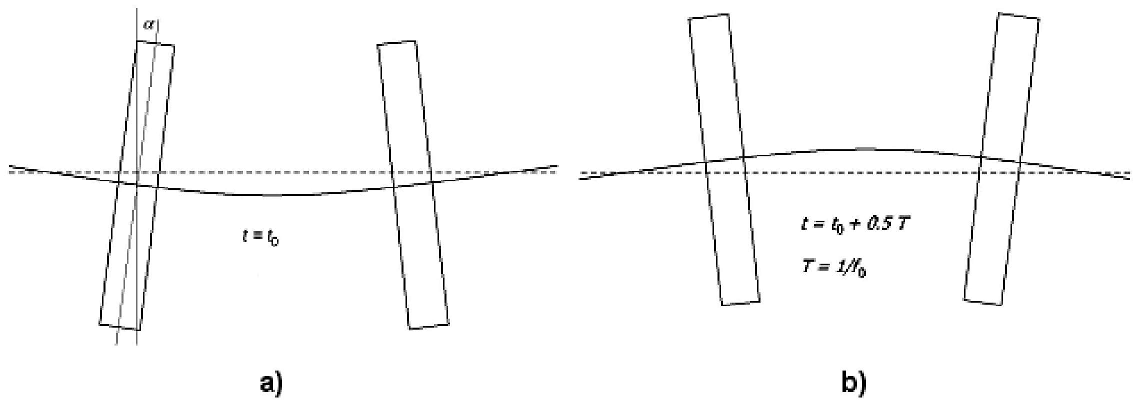

Fig. 4. Schematic illustration of a rotor without (a) and with (b) permanent bow
Rys. 4. Schematyczna ilustracja kształtu wirnika bez (a) i z wygięciem trwałym (b)

Pearson linear correlation coefficient values for three units and individual life cycles are given in Table 1 (cycle 3 for T2 has been too short for a meaningful analysis). It can be seen that for units T1 and T2 correlation of components under consideration is strong: all values are positive and in seven cases of fifteen we have $|\rho| \geq 0.9$. On the other hand, values for T3 are lower by a few times and in two cases negative, which means that an increase of one symptom is accompanied by a decrease of the other.

Table 1

Linear correlation coefficients for the $1 \times f_0$ vibration velocity components in units T1, T2 and T3

Unit and cycle	Coefficient ρ of linear correlation between $1 \times f_0$ components of vertical vibration velocity at bearing No.3 and axial at		
	bearing No. 2	bearing No. 3	bearing No. 4
T1, cycle 1	0.69	0.90	0.91
T1, cycle 2	0.94	0.79	0.73
T1, cycle 3	0.87	0.91	0.88
T2, cycle 1	0.97	0.97	0.95
T2, cycle 2	0.65	0.60	0.51
T3	-0.15	0.14	-0.04

The above comparison justifies a conclusion that the linear correlation coefficient is in this case a very good symptom that facilitates a meaningful diagnosis. Approach shown in Fig. 4 is a very simplified one and, in the author's opinion, this issue deserves further research, aimed at developing a more precise model. It is of interest, however, that with permanent bow of modest magnitude (causing problems during startups, but still acceptable in operation – dynamic behavior in worst moments about the lower limit of the C vibration assessment zone according to ISO 10816-1, no damage of labyrinth seals) the phenomenon has been so strongly manifested. Correlative symptoms can thus be very useful in diagnosing this rare, but serious malfunction.

The root cause question obviously remains to be found, but here vibrodiagnostic methods can only provide a basis for conjectures. As mentioned earlier, the malfunction has not been a sudden event, which suggests that the problem is with the rotor itself. Some material anisotropy seems

[2] In this turbine, pedestals of bearings Nos. 3 to 6 are mounted on a common plate which comprises a common element with lower part of the NP casing. This facilitates vibration propagation along shaft line.

possible, which has led to an anisotropic response to the high-cycle fatigue; this hypothesis, however, can be confirmed or rejected only on the basis of detailed material tests.

4. CONCLUSION

Analysis of vibration trends and correlation should be considered a valuable tool in diagnosing a slowly developing fault with a strong, yet unspecific impact on the turbine-generator unit vibration patterns. In particular, evolutionary symptoms have proven their usefulness, namely:

- type of time dependence of vibration components,
- slope of the linear fit of vibration component time history,
- coefficients of vibration trends linear correlation.

It should be kept in mind that, in such cases, problems are encountered at the qualitative diagnosis stage.

It is also worth mentioning that, even with comparatively unsophisticated vibration measurements and recording systems, acquisition of data that allow for such analyses to be performed is simple and does not require any upgrading of existing monitoring equipment.

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References

1. Bently D.E., Hatch C.T.: *Fundamentals of Rotating Machinery Diagnostics*. Bently Pressurized Bearing Press, Minden, USA, 2002.
2. Cempel C.: *Vibroacoustic Condition Monitoring*. Ellis Horwood, Chichester, UK, 1991.
3. Morel J.: *Vibrations des machines et diagnostic de leur état mécanique*. Eyrolles, Paris, 1992.
4. Lapini G.L., Zippo M., Bachschmid N., Collina A., Vallini A.: *Experimental tests and model based calculations for the diagnosis of a crack in a 320 MW generator*. Proceeding of the CISM Symposium *Diagnostics of Rotating Machines in Power Plants*, Springer, Wien-New York, 1994.
5. Rao J.S., Sharma M.: *Dynamic analysis of bowed rotors*. Proceedings of VETOMAC-1 Conference, Bangalore, India, 2000.
6. Cempel C.: *Multidimensional condition monitoring of mechanical systems in operation*. Mechanical Systems and Signal Processing, vol.17, No.6, 2003, pp. 1291-1303.
7. Orłowski Z.: *Diagnostics in the Life of Steam Turbines*. WNT, Warszawa (in Polish), 2001.
8. Gałka T.: *Representation of machine technical condition in evolutionary diagnostic symptoms*. Eksploatacja i Niezawodność, Nr 1(37), 2008, pp. 23-29.
9. Gałka T.: *Statistical vibration-based symptoms in rotating machinery diagnostics*. Diagnostyka, Nr 2(46), 2008, str. 25-32.
10. Gałka T: *Problem of accuracy in rotating machines vibration trends analysis*. Proceedings of the XXXIIIth Symposium 'Diagnostyka Maszyn', Węgierska Górka (CD-ROM edition) (in Polish), 2006.
11. Gałka T.: Ponikiewski T.: *Assessment of vibration-based statistical symptoms suitability in turbine fluid-flow system condition diagnosis*. Report of the Institute of Power Engineering (unpublished) (in Polish) ,2007.
12. Gałka T.: *Correlation-based symptoms in rotating machines diagnostics*. Proceedings of the 21st International Congress COMADEM 2008, Praha, pp.213-226.
13. Papoulis, A.: *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, New York, 1964.

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