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MODELLING OF AN IRREGULAR SHAPE CARGO ALLOCATED IN OPEN WAGON

Summary. The article gives the account of the results of calculation of stretches in flexible resilient elements of fastenings on the lower, inaccessible for pretwisting, irregular shape cargo section transported in an open wagon, depending upon initial value of wire pretwisting between excessive bends in cargo upper sections.

МОДЕЛИРОВАНИЕ ГРУЗА СЛОЖНОЙ КОНФИГУРАЦИИ, РАЗМЕЩЁННОГО В ПОЛУВАГОНЕ

Аннотация. В статье изложены результаты решения задачи по определению натяжений в гибких упругих элементах креплений на нижнем не доступном для предварительной скрутки участке груза сложной конфигурации, перевозимого в полувагоне, в зависимости от значения предварительной скрутки проволоки между точками перегибов, расположенных в верхних участках груза.

1. FORMULATION OF A PROBLEM

An irregular shape cargo in an open wagon causes some difficulties in pretwisting of a fastening wire due to limited space between a cargo lateral surface and an open wagon lateral wall. As a result, it is impossible to twist the lower fastening branch, not going beyond twisting of its upper branches, though it is lower cargo branches that experience the main load during transportation or wagons blow in the hump yard. In calculations of stretches in irregular cargo fastening flexible elements technical requirements of cargo allocation and fastening in wagons and containers [1, 2] do not take into account their excessive bend, and stretch redistribution of wire pretwisting between branches. In this paper [3] we have calculated stretches of cargo fastening wire pretwisting in the presence of a single excessive bend. However, in reality there may be some cases when cargoes are fastened by flexible elements having several excessive bends. In this connection, it is quite pressing and important for rail transport and transport science to find the character of stretch distribution of wire pretwisting between branches and to calculate stretches in such fastening flexible elements.

1.1. Man-made assumption

Configuration of cargo edges may be different, therefore the direction of constraint reaction and friction forces at contact points of flexible elements with cargo edges are unknown. For analytic derivation of formulas which make it possible to calculate stretches in fastening flexible elements, having excessive bends between branches according to pretwisting values, we take the following assumptions [3, 4]:

the slope angle of tangent to the flexible element axis to horizon at the excessive bend point (for example, α_{12}) equals a half-sum of slope angles relative to the horizon of both branches of the flexible element (for example, $(\alpha_1+\alpha_2)/2$);

the friction force reaches the maximum value (utmost equilibrium is considered);

the forces applied to the flexible element lie in the same plane, as the flexible element is sliding along the rib during its elastic behaviour. We assume that there are no physical impediments (except but friction forces) for sliding of this element at the point of contact between a flexible element and a cargo edge.

We think that an excessive bend of some flexible elements results from cargo fastening and transition of ribs separating cargo vertical and horizontal surfaces. Prestretching may occur at lower or upper branches of a flexible element.

It is essential to obtain the formulas which make it possible to ascertain the character of stretch distribution of wire pre-twisting between lower and middle branch of the cargo fastening flexible element having some excessive bends.

2. METHODS OF SOLUTION

Evaluation of stretches in wagon (truck, open wagon) cargo fastening as a mechanical system “cargo – fastening – wagon” is known to be a problem of a statically indeterminate system. Methods of indeterminate system problem solution are of common knowledge and include formation of additional equations which consider deformation of elastic constraints. A more practicable interpretation of solving the problems of statically indeterminate systems is proposed by the research [4]. To solve the selected problem for evaluation of unknown stretches of fastening pre-twisting and normal constraint reactions on cargo edges, a method of sequential exclusion of unknown quantity, well established in mathematics, was applied. To check accuracy of analytical solution results, a numeric method was used which is based on iteration procedure including a function *Given* и *Find* [6], included into MathCAD calculating environment.

3. SOLUTION RESULTS

We shall consider a case of prestretching of cargo fastening flexible element lower branch in the presence of several excessive bends on cargo edges. Analytical model in cargo horizontal rib rounding is shown in Fig.1.

Fig. 1. A - point of fastening of a flexible element to a wagon linking device; B, C and D - flexible element excessive bend points; E - fastening point for cargo erection loop; l_1, l_2, l_3 and l_4 - lengths in OB, BC, CD and DE fastening flexible elements respectively, m;

a_1, a_2, a_3 and a_4 - lower, middle and upper branch slopes of the flexible element to X axis (in its plane); $\overline{R}_{01}$ and $\overline{R}_{04}$ - stretches of pre-twisting in OB and DE fastening flexible element branches , кН; α_{12}, α_{23} и α_{34} - tangent slopes to flexible element axis in relation to X axis at excessive bend points; $\overline{N}_{12}$, $\overline{N}_{23}$ и $\overline{N}_{34}$ - perfectly smooth surface normal reactions at the point of tangency of fastening with cargo rib, кН; $\overline{F}_{\tau12}$, $\overline{F}_{\tau23}$ and $\overline{F}_{\tau34}$ - frictional forces at points of tangency of flexible element with cargo ribs, кН; a - b - BC flexible element section, where wire pretwisting occurs.

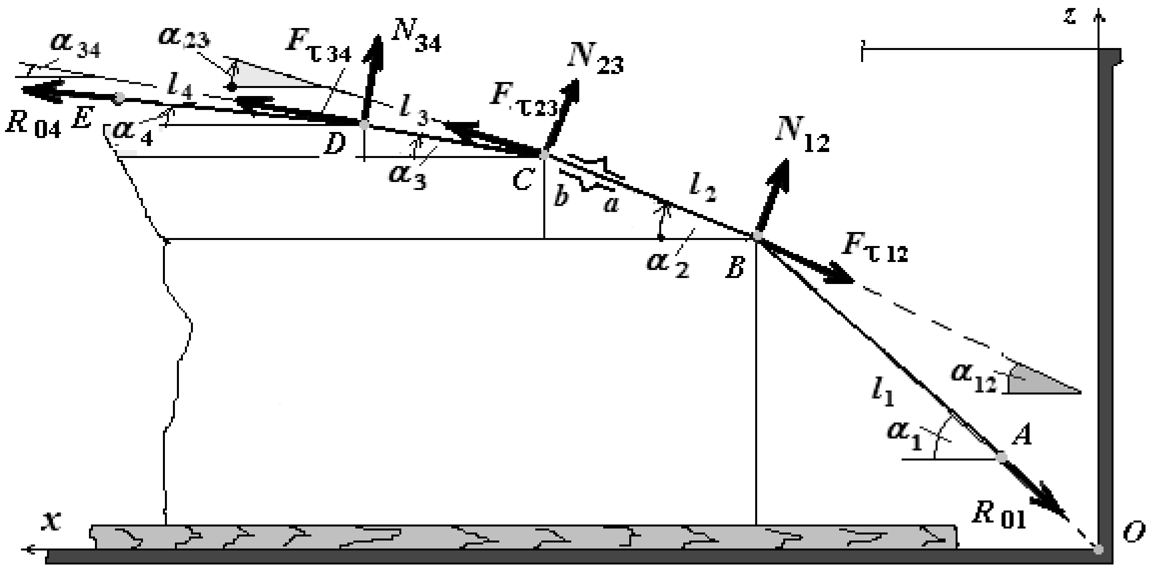

Fig. 1. Analytical modeling of forces exposure to flexible resilient element with excessive bends on cargo edges in stretching of lower middle branch

Рис. 1. Расчётная модель действия сил на гибкий упругий элемент с перегибами на кромках груза при натяжении нижней средней ветви

To derive the formulas, sectioning method is used [3, 4], implying cleaving of the flexible element with subsequent discarding its dissected parts and replacement of the neglected part effect by wire pretwisting stretches $\overline{R}_{0i}$.

The forces applied to a flexible element (Fig. 1), with introduced assumptions, are a general coplanar system in utmost equilibrium. The number of equilibrium equations for this system is 6, the number of unknown forces is 8.

If we use Coulomb’s law for dry friction at the flexible element excessive bend point (physical (second) part of a statically indeterminate problem)

$$F_{\tau12}^{\max} = fN_{12}, \quad F_{\tau23}^{\max} = fN_{23}, \quad F_{\tau34}^{\max} = fN_{34} \tag{1}$$

the number of unknown forces reduces to 7.

Therefore, the selected problem is once statically indeterminate, as it is also for a flexible element with a single excessive bend. In Formula (1) f is friction coefficient of sliding motion between rubbing surfaces, $f < f_0$ at that, f_0 is static friction coefficient, its value being taken according to Specifications.

To determine stretches in the flexible element middle part (between excessive bend points) it is necessary to split the system into two parts. As a result, we have two new stretches $\overline{R}_{02}$ и $\overline{R}'_{02}$, modulo equal (law of action and reaction) and applied, for example, to points B и C, for which we can form two equilibrium equations per each, corresponding to plane converging force systems.

The number of equilibrium equations is six (first part of solution of a statically indeterminate problem):

$$\sum_{k=1}^{n} F_{kx} = 0 : -R_{01}\cos\alpha_1 + R'_{02}\cos\alpha_2 - F_{\tau12}\cos\alpha_{12} - N_{12}\sin\alpha_{12} = 0;$$

$$\sum_{k=1}^{n} F_{kz} = 0: -R_{01}\sin\alpha_1 + R'_{02}\sin\alpha_2 - F_{\tau 12}\sin\alpha_{12} + N_{12}\cos\alpha_{12} = 0;$$
$$\sum_{k=1}^{n} F_{kx} = 0: -R_{02}\cos\alpha_2 + R_{03}\cos\alpha_3 + F_{\tau 23}\cos\alpha_{23} - N_{23}\sin\alpha_{23} = 0;$$
$$\sum_{k=1}^{n} F_{kz} = 0: -R_{02}\sin\alpha_2 + R_{03}\sin\alpha_3 + F_{\tau 23}\sin\alpha_{23} - N_{23}\cos\alpha_{23} = 0; \quad (2)$$
$$\sum_{k=1}^{n} F_{kx} = 0: -R_{03}\cos\alpha_3 + R_{04}\cos\alpha_4 + F_{\tau 34}\cos\alpha_{34} - N_{34}\sin\alpha_{34} = 0;$$
$$\sum_{k=1}^{n} F_{kz} = 0: -R_{03}\sin\alpha_3 + R_{04}\sin\alpha_4 + F_{\tau 34}\sin\alpha_{34} - N_{34}\cos\alpha_{34} = 0.$$

Here the number of unknown forces is 8.

Considering the equality R_{02} и R'_{02} and Coulomb's law (1) the selected problem will once become statically indeterminate (the number of unknown forces will equal 7). After simple mathematical transformations the equations (2) will turn into

$$\begin{aligned}
&-R_{01}\cos\alpha_1 + R_{02}\cos\alpha_2 + N_{12}\cos\alpha_{12}A_{12} = 0;\\
&-R_{01}\sin\alpha_1 + R_{02}\sin\alpha_2 + N_{12}\cos\alpha_{12}B_{12} = 0;\\
&-R_{02}\cos\alpha_2 + R_{03}\cos\alpha_3 + N_{23}\cos\alpha_{23}A_{23} = 0;\\
&-R_{02}\sin\alpha_2 + R_{03}\sin\alpha_3 + N_{23}\cos\alpha_{23}B_{23} = 0;\\
&-R_{03}\cos\alpha_3 + R_{04}\cos\alpha_4 + N_{34}\cos\alpha_{34}A_{34} = 0;\\
&-R_{04}\sin\alpha_3 + R_{04}\sin\alpha_4 + N_{34}\cos\alpha_{34}B_{34} = 0,
\end{aligned} \quad (3)$$

where: A_{12}, B_{12}, A_{23}, B_{23} and A_{34}, B_{34} - dimentionless quantities:

$A_{12} = f + tg\alpha_{12}$; $B_{12} = ftg\alpha_{12} - 1$; $A_{23} = f - tg\alpha_{23}$;

$B_{23} = ftg\alpha_{23} + 1$; $A_{34} = f - tg\alpha_{34}$; $B_{34} = ftg\alpha_{34} + 1$.

Deformation compatibility equations (geometrical (third) part of a statically indeterminate problem) of all four flexible element branches including known elastic Hookes ratios can be finally written as:

$$\frac{R_{01}l_1}{EA_1} + \frac{R_{02}l_2}{EA_2} + \frac{R_{03}l_3}{EA_3} + \frac{R_{04}l_4}{EA_4} = \Delta. \quad (4)$$

ing the latter ratio, it is easy to formulate dependence of a single stretch, for example, R_{04}, on other stretches as:

$$R_{04} = \frac{EA\Delta - R_{01}l_1 - R_{02}l_2 - R_{03}l_3}{l_4}. \quad (5)$$

Here E - modulus of elasticity for stretching of a flexible element wire material, кПа ($E = 1\cdot 10^7$ кПа); A – wire cross-section area, м^2, determined for a flexible element from n wire threads with each thread diameter d.

Substituting the equality (5) into the system (3) we shall have

$$\begin{aligned}
&-R_{01}\cos\alpha_1 + R_{02}\cos\alpha_2 - N_{12}\cos\alpha_{12}A_{12} = 0;\\
&-R_{01}\sin\alpha_1 + R_{02}\sin\alpha_2 + N_{12}\cos\alpha_{12}B_{12} = 0;\\
&-R_{02}\cos\alpha_2 + R_{03}\cos\alpha_3 + N_{23}\cos\alpha_{23}A_{23} = 0;
\end{aligned} \quad (6)$$

$$-R_{02}\sin\alpha_2+R_{03}\sin\alpha_3+N_{23}\cos\alpha_{23}B_{23}=0;$$

$$R_{01}l_1\cos\alpha_4+R_{02}l_2\cos\alpha_4+R_{03}D_{34}-N_{34}l_4\sin\alpha_{34}B_{34}=EA\Delta\cos\alpha_4,$$

$$R_{01}l_1\sin\alpha_4+R_{02}l_2\sin\alpha_4+R_{03}D_{34}-N_{34}l_4\cos\alpha_{34}B_{34}=EA\Delta\sin\alpha_4,$$

where: C_{34} and D_{34} - dimensional quantities, m:

$$C_{34}=l_4\cos\alpha_3+l_3\cos\alpha_4;\quad D_{34}=l_4\sin\alpha_3+l_3\sin\alpha_4.$$

To determine unknown quantity of the system (6) we use the sequential exclusion of unknown quantity, well-known in mathematics.

From the first equation of the system (6) we shall derive the analytical formula for evaluation of the constraint normal reaction between OB and BC branches (at point B) of the flexible fastening

$$N_{12}=\frac{-R_{01}\cos\alpha_1+R_{02}\cos\alpha_2}{A_{12}\cos\alpha_{12}}. \tag{7}$$

Substituting formula (7) into the second equation of the system (6), we have:

$$-R_{01}\sin\alpha_1+R_{02}\sin\alpha_2+\frac{-R_{01}\cos\alpha_1+R_{02}\cos\alpha_2}{A_{12}\cos\alpha_{12}}\cos\alpha_{12}B_{12}=0,$$

or

$$-R_{01}K_1+R_{02}K_2=0, \tag{8}$$

where: K_1 и K_2 - nondimentional quantities:

$$K_1=A_{12}\sin\alpha_1+B_{12}\cos\alpha_1;\quad K_2=A_{12}\sin\alpha_2+B_{12}\cos\alpha_2.$$

From equation (8) we shall derive the formula for determination of wire pretwisting stretches in the flexible fastening branch (section BC):

$$R_{02}=\frac{K_1}{K_2}R_{01}. \tag{9}$$

From the third equation of the system (6) we shall derive the formula for evaluation of the constraint normal reaction between branches BC and CD (at point C) of the flexible fastening.

$$N_{23}=\frac{R_{02}\cos\alpha_2-R_{03}\cos\alpha_3}{A_{23}\cos\alpha_{23}}. \tag{10}$$

Substituting the formula (10) into the fourth equation of the system (6) we have:

$$-R_{02}\sin\alpha_2+R_{03}\sin\alpha_3+\frac{R_{02}\cos\alpha_2-R_{03}\cos\alpha_3}{A_{23}\cos\alpha_{23}}\cos\alpha_{23}B_{23}=0$$

or

$$R_{03}K_3-R_{02}K_4=0, \tag{11}$$

where: K_3 and K_4 - nondimentional quantities:

$$K_3=A_{23}\sin\alpha_3-B_{23}\cos\alpha_3;\quad K_4=A_{23}\sin\alpha_2-B_{23}\cos\alpha_2.$$

From the equation (11) we derive the formula for determination of wire pretwisting stretches in the flexible fastening branch BC

$$R_{03}=\frac{K_4}{K_3}R_{02}\ \text{or}\ R_{03}=\frac{K_1}{K_2}\frac{K_4}{K_3}R_{01}. \tag{12}$$

Substituting formulas (9) and (12) into the fifth and sixth equations of the system (6), we have

$$R_{01}l_1\cos\alpha_4+\frac{K_1}{K_2}R_{01}l_2\cos\alpha_4+\frac{K_1}{K_2}\frac{K_4}{K_3}R_{01}C_{34}-N_{34}l_4\cos\alpha_{34}A_{34}=EA\Delta\cos\alpha_4;$$

$$R_{01}l_1\sin\alpha_4+\frac{K_1}{K_2}R_{01}l_2\sin\alpha_4+\frac{K_1}{K_2}\frac{K_4}{K_3}R_{01}D_{34}-N_{34}l_4\cos\alpha_{34}B_{34}=EA\Delta\sin\alpha_4,$$

or

$$R_{01}E_{34}-N_{34}l_4\cos\alpha_{34}A_{34}=EA\Delta\cos\alpha_4;$$

$$R_{01}F_{34}-N_{34}l_4\cos\alpha_{34}B_{34}=EA\Delta\sin\alpha_4, \qquad (13)$$

where E_{34}, F_{34} - dimensional quantities, m:

$$E_{34}=l_1\cos\alpha_4+\frac{K_1}{K_2}l_2\cos\alpha_4+\frac{K_1}{K_2}\frac{K_4}{K_3}C_{34};$$

$$F_{34}=l_1\sin\alpha_4+\frac{K_1}{K_2}l_2\sin\alpha_4+\frac{K_1}{K_2}\frac{K_4}{K_3}D_{34}.$$

Using Kramer’s rule [5], from the system (13), we shall assess unknown stretches of wire pretwisting in the wire lower branch (section OB) and constraint normal reaction between flexible element branches CD и DE (at point D):

$$R_{01}=\frac{\Delta_1}{\Delta_0};\quad N_{34}=\frac{\Delta_2}{\Delta_0}, \qquad (14)$$

where: D and Δ_2 - dimensional quantities, кН·m$^{2:}$

$$\Delta_1=EA\Delta l_4\cos\alpha_{34}(A_{34}\sin\alpha_4-B_{34}\cos\alpha_4);$$
$$\Delta_2=EA\Delta(E_{34}\sin\alpha_4-F_{34}\cos\alpha_4);$$

Δ_0 - dimensional determinant of the system (13), m^2:

$$\Delta 0=l_4\cos\alpha_{34}(A_{34}F_{34}-B_{34}E_{34}).$$

The latter ratios which use introduced signs give us the following formulas for determination of stretches of wire pretwisting in the flexible element branch as far as the excessive bend (section OB) and constraint normal reaction between branches CD and DE (at point D):

$$R_{01}=\frac{A_{34}\sin\alpha_4-B_{34}\cos\alpha_4}{A_{34}F_{34}-B_{34}E_{34}}EA\Delta;$$

$$N_{34}=\frac{E_{34}\sin\alpha_4-F_{34}\cos\alpha_4}{l_4\cos\alpha_{34}(A_{34}F_{34}-B_{34}E_{34})}EA\Delta. \qquad (15)$$

Thus, the formulas have been derived to determine stretches of pretwisting in the flexible element branch as far as an excessive bend according to linear dimensions l_1, l_2, l_3 and l_4, physical and geometric parameter EA and its selected length variation Δ, as well as to constraint normal reaction between upper middle and upper branches of that element. The obtained formulas are necessary for calculation of stretches in fastening flexible resilient elements on the lower section inaccessible for pretwisting of an irregular shape cargo. These are peculiarities of calculations of an irregular shape cargo in the open wagon.

4. CALCULATION

These are the results of computing experiments for calculation of pretwisting stretches of a flexible resilient element in case of several excessive bends on cargo edges based on MathCAD [6] computing environment. Precision of analytical solutions to determine stretches in flexible elements and constraint normal reactions on cargo edges is performed with the help of numerical method, based on iteration procedure with application of *Given* и *Find* function included into MathCAD computing environment. The unknown quantities are preset to a first approximation, *Given* function is introduced and then the equation system (2) and (4) is written with an equals sign as Boolean functions followed by the solution of this system on the basis of *Find* function.

Computing experiments using specific analytical formulas (9), (10, (12) and (15) were performed with application of the following initial data: lengths and slope angles of the flexible element left, middle and right branches respectively - l_1 = 1,0, l_2 =0,8, l_3 = 0,6 и l_4 = 0,4 m и α_1= 75^0, α_2 = 40^0, α_3 =30^0 и α_4 =10^0; frictional coefficient between flexible element and cargo materials - f = 0.55 (steel all along reinforced concrete), fastening prestretching R_0= 20 кН; flexible elements rigidity tension EA = 2,262·10^3 кН, that corresponds to their modulus of elasticity E = 1·10^7 кPa, the number of threads n = 8 pcs., wire diameter d = 0,006 m and cross-section area A = 2,262·10^{-4} m^2.

The results of calculating experiments showed that:

- stretches in the fastening flexible element lower (section OB), lower (section BC) and upper (section CD), middle and upper (section DE) branches (Fig.1) are equal to, кН, R_{01} = 6,41, R_{02} = 8,26, R_{03} = 7,17, R_{04} = 6,69;
- constraint reactions are equal to, кН, N_{12} = 4,41, N_{23} = 2,68, N_{34} = 1,21;
- frictional forces are equal to, кН, $F_{\tau 12}$ = 1,76, $F_{\tau 23}$ = 1,07, $F_{\tau 34}$ = 0,48.

Comparison of calculation results by analytical and numeric techniques showed their precise coincidence.

The analysis of the obtained data made it possible to note that stretching is a maximum in that branch (lower middle), where flexible element pretwisting occurs, and a minimum – on the lower side. Comparison of these results with the flexible element lower branch data enable us to point out that the constraint normal reaction (and consequently frictional force) on cargo edges reaches its maximum in that flexible element branch with several excessive bends that has the greatest excessive bend.

Fig. 2 shows the results of fastening power calculation R_{01}, R_{02}, R_{03}, R_{04} and constraint reactions N_{12}, N_{23}, N_{34} depending on slope angle variation α_2 of the fastening flexible element lower middle branch in case of constant rigidity in tension EA and prestretching Δ.

The analysis of computing experiments enabled to find that:

- with augmentation of the flexible element branch BC slope (Fig. 1) under constant slopes in other branches, stretching in that side remains practically constant, but highest possible as compared to stretches in other branches. In OB branch stretching increases by 1,18 times, but in CD and DE branches decrease (for example, in CD - by 1,28 times). Constraint reactions in excessive bend points of lower (OB) and upper (DE) branches increase abruptly (for example, by 19,14 times);

— in case of decrease of the tangent angle to a flexible element by 0,8 times stretching in branch OB increases by 1,14 times, stretching in BC branch remains practically permanent, but in other branches decreases (e.g., up to 1,16 times). Constraint reactions in excessive bend points of OB and DE branches decrease (e.g., in OB up to 1,41 times), but between sides DE and BC increase (e.g., up to 19,1 times). In comparison with the first case of stretching in all branches (except for OB branch) constraint reactions at excessive bend points decrease.

- when the tangent angle to a flexible element increases by 1,2 times stretching in BC branch remains practically constant, but stretches in other sides of the element decreases (e.g., by 1,12 times).

Constraint reactions at OB and DE excessive bend points decrease (e.g., in OB up to 1,39 times) but between BC and DE increases abruptly (e.g., up to 18,2 times). In comparison with the first stretching case in all branches (except for OB) constraint reactions at excessive bend points also increase.

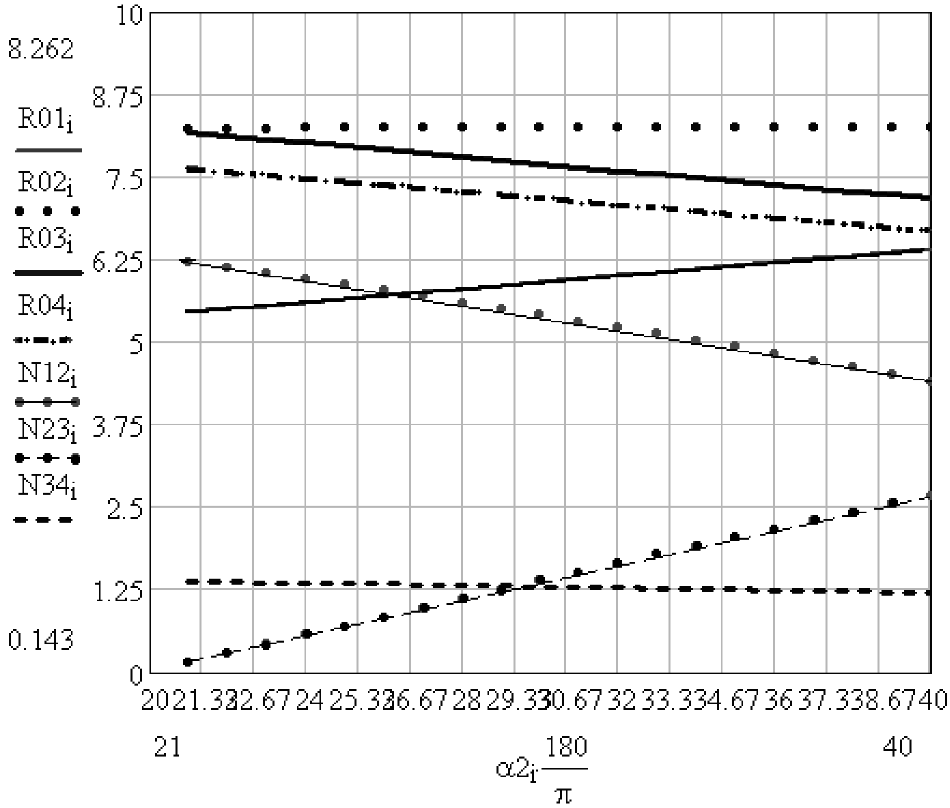

Fig. 2. Changes of stretches in flexible element branches R_{01}, R_{02}, R_{03}, R_{04} and constraint reactions N_{12}, N_{23}, N_{34}

Рис. 2. Рис. 2. Изменения натяжений в гибких элементах R_{01}, R_{02}, R_{03}, R_{04} и реакции связей N_{12}, N_{23}, N_{34}

5. CONCLUSIONS

1. Analytic formulas previously unknown in the theory of cargo allocation and fastening have been derived to determine character of stretches distribution of fastening wire pretwisting in the presence of several excessive bends on irregular shape cargo edges and constraint normal reactions on edges subject to a branch being exposed to pretwisting.
2. Stretches of wire pretwisting reach to a maximum in that branch (BC), which is being exposed to twisting, and a minimum – on OB branch. Constraint normal reaction (consequently, frictional force) on cargo edges reaches a maximum in that branch of a flexible element with several excessive bends which has the greatest excessive bend.
3. The obtained analytic formulas of constraint normal reactions on irregular shape cargo edges in the open wagon, depending on wire pretwisting on the section between excessive bend points in upper cargo areas, are necessary for determination of stretches in flexible resilient elements on lower, inaccessible for pretwisting section.

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