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AN IMPACT OF SOME PARAMETERS ON CORRECTNESS OF THE LOCATION PROBLEM SOLUTION

Summary. This paper deals with the modifications of customer demands and with the impact of modifications of customer demands on the location of supply centers. The solution of the location problem with exactly *p* location is sought. During the computation the penalty constant is used very often. Accumulation of the penalty values in the same variable causes the memory overflow. To eliminate this defect, we use the coefficient of the proportionality in some parameters of the location problem. We find how the coefficient of the proportionality influences the accuracy of the final result. The obtained results can help to solve the problem of the distribution systems design.

WPŁYW NIEKTÓRYCH PARAMETRÓW NA POPRAWNOŚĆ ROZWIĄZANIA PROBLEMU LOKALIZACJI

Steszczenie. Artykuł zajmuje się modyfikacjami popytu klientów i wpływem modyfikacji popytu klientów na lokalizację centrów zaopatrzenia. Poszukiwane jest rozwiązanie problemu lokalizacji z lokalizacją dokładnie *p*. W czasie obliczeń bardzo często używana jest stała karna. Akumulacja wartości karnych w tej samej zmiennej powoduje przepełnienie pamięci. W celu wyeliminowania tej wady dla niektórych parametrów problemu lokalizacji używamy współczynnika proporcjonalności. Stwierdzamy, jak współczynnik proporcjonalności wpływa na dokładność ostatecznego wyniku. Uzyskane wyniki mogą pomóc w rozwiązaniu problemu projektowania systemów dystrybucji.

1. INTRODUCTION

Distribution systems are sometimes designed on networks. The distribution system design could be formulated and solved as a problem of mathematical programming. We can solve exactly the basic form of uncapacitated location problem in real time for large network. Troubles appear after some additional conditions are added to the basic problem. For example it could be a demand to locate an exact number of possible terminals from a given set of candidates [1, 4].

In some cases, it is possible to integrate the conditions into a mathematical model and to obtain basic form of the uncapacitated location problem after its rearrangement. However, the structure of the input data will usually become deformed. We can solve the rearranged problems by exact algorithms [2, 3], but when we use the deformed data, the algorithms perform less effectively. In these cases the solution of a large problem takes much more time or the computation stops incorrectly due to memory overflow. To eliminate such defects we tried to round the input data, before the associated location problem with specified number of locations (*p*-median) is solved.

This article deals with the influence of this data adjustment on the values of the objective function.

2. MODEL

Let J is a set of customers and let I is the set of candidates for possible locations of centers. We also know $\{d_{ij}\}$ - the matrix of the shortest distances among the nodes of the network and $\{b_j\}$ – the matrix of customers' demands. The number p of required location is given. The task is to select exactly p locations from the set of candidates in such a way that all customers will be served at minimal costs. The costs are proportional to the size of the demands and to the distance between customer and center that is allocated to him. The capacities of centers are not limited and we assume that the costs of their location are equal to zero. Each customer can be served exactly from one center.

The model of the problem follows:

Minimize
$$U = \sum_{i \in I} \sum_{j \in J} b_j d_{ij} z_{ij} \tag{1}$$

Subject to
$$\sum_{i \in I} z_{ij} = 1 \qquad \forall j \in J \tag{2}$$

$$z_{ij} \le y_i \qquad \forall i \in I, \forall j \in J \tag{3}$$

$$\sum_{i \in I} y_i = p \tag{4}$$

$$z_{ij} \in \{0, 1\} \qquad \forall i \in I, \forall j \in J \tag{5}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I \tag{6}$$

where the used coefficients have the following meaning:

b_j ... the demand of customer j,

d_{ij} ... the distance between center i and customer j,

I ... the set of possible center locations,

J ... the set of customers,

The decision zero-one variable z_{ij} takes the value of 1, if customer j is satisfied from centre i and takes the value of 0 otherwise.

The zero-one variable y_i takes the value of 1, if the centre is located in place i and takes the value of 0 otherwise.

The constraints (2) guarantee that customer $j \in J$ is assigned to exactly one of possible locations $i \in I$. The constraints (3) ensure that variable y_i takes the value 1, when at least one customer is assigned to location i. The constraint (4) puts limit p on the number of locations.

The objective function contains demands b_j and, if the network is large and demand values are big, then the objective function value is very high. To prevent the memory overflow during the computation we used modified values of demands, proportionally lower than original ones. The objective function *U* (1) is linear in consideration of demands b_j therefore its value should be proportionally reduced. However, the used software works with integral variables. So, we used not only the coefficient of the proportionality but we also constructed new demands $\underline{b_j}$. These new values are estimations of the value b_j/q and they are defined as follows:

$\underline{b_j} = [b_j/q]$ for $q \le b_j$

$\underline{b_j} = 1$ for $q > b_j$.

Demands b_j which correspond with values $q>b_j$ will cause some quantitative shifts opposite to other customers and this might change the optimal solution. We will study the range of possible error, which is caused by rounding of demands b_j for $q>1$, using numerical experiments. The same troubles will occur when the uncapacitated location problem with maximal service distance is solved [5, 6].

3. EXPERIMENTS

The experiments were run with the intention to find how the rounding of the demands influences the consequent value of the objective function and the computational time. All instances were solved on the real network of the Slovak Republic which consists of 2916 localities. The computational time were measured in seconds for all instances. We used a software product, which was implemented in the University of Zilina. This software repeats the algorithm *BBDual* several times. The algorithm *BBDual* is based on the branch and bound methods and it works with the real-sized networks very well. A dichotomy algorithm is used to adjust Lagrangean multiplier at such value that resulting number of locations equals to the given value of *p*.

We solved several groups of tasks. They differed from each other in number *p* of required locations and in the size of the sets of candidates. We created the sets of candidates of localities with the highest number of inhabitants.

In the first group of instances, we studied the influence of the proportionality coefficient on the computational time and the objective function *U* (1). In this case, the experiments were run with the set of 103 possible locations, each of them had more then 8000 inhabitants. Demands b_j were divided by coefficient *q*, which step-by-step took the values of 1, 50, 200, 350, … , 1400. The results obtained for the required number of locations *p*=70 are shown in Table 1.

Table 1

Impact of q on Solution Inaccuracy and Time (*I*=103 candidates, *J*=2916 customers)

q	U_q	Difference	%	P	Time [s]
1	30 656 295	0	0,00	70	21
50	30 656 295	0	0,00	70	15
200	30 656 295	0	0,00	70	15
350	30 656 295	0	0,00	70	14
500	30 656 295	0	0,00	70	14
650	30 775 399	119 104	0,39	70	14
800	30 775 399	119 104	0,39	70	14
950	30 865 720	209 425	0,68	70	16
1 100	30 865 720	209 425	0,68	70	17
1 250	30 865 720	209 425	0,68	71	17
1 400	30 951 644	295 349	0,96	70	17

In Table 1 we show not only the absolute differences of the objective function values but also the relative measure that is the percentage proportion of the change.

In the next step we changed the values of parameter *p* and studied how the change of coefficient *q* and of parameter *p* influences the optimal solution. Because the set of candidates had 103 localities, parameter *p* changed its value from 1 to 90. Coefficient *q* took the same values as in the first group that is *q*=1 (without the rearrangement of demands) and from 50 to 1400 with step 150. The computational time did not change with increasing coefficient *q* in no-one instance and we do not mention it in the results. The absolute and relative differences of the objective function values for *p*=10, 30, 50, 70 and 90 are placed in Table 2.

Table 2

The absolute and relative differences of the objective function values
(*I*=103 candidates, *J*=2916 customers)

P	10		30		50		70		90	
	Value	.%	Value	.%	Value	.%	Value	.%	Value	%
U_1	113 615 831		56 594 911		39 049 861		30 656 295		26 888 775	
$U_{50} - U_1$	0	0,00	0	0,00	0	0,00	0	0,00	0	0,00
$U_{200} - U_1$	0	0,00	0	0,00	0	0,00	0	0,00	0	0,00
$U_{350} - U_1$	142 662	0,13	0	0,00	35 774	0,09	0	0,00	0	0,00
$U_{500} - U_1$	142 662	0,13	598 604	1,06	35 774	0,09	0	0,00	0	0,00
$U_{650} - U_1$	279 974	0,25	598 604	1,06	35 774	0,09	119 104	0,39	0	0,00
$U_{800} - U_1$	279 974	0,25	598 604	1,06	35 774	0,09	119 104	0,39	0	0,00
$U_{950} - U_1$	279 974	0,25	822 112	1,45	254 469	0,65	209 425	0,68	24 133	0,09
$U_{1100} - U_1$	279 974	0,25	822 112	1,45	254 469	0,65	209 425	0,68	20 169	0,08
$U_{1250} - U_1$	435 088	0,38	1 486 775	2,63	254 469	0,65	209 425	0,68	24 133	0,09
$U_{1400} - U_1$	435 088	0,38	1 486 775	2,63	328 868	0,84	295 349	0,96	145 788	0,54

The differences for the different values of parameter *p* diverged. It might be caused by influence of parameter *p* on the number of the combinations of possible locations. Furthermore, we compared the optimal solutions i.e. the values of location variables y_i and allocations variables z_{ij}. There were only smaller differences among the locations. Some localities were included in all solutions (locations). They were completed with different combinations of a small number of other localities.

In Table 3 we show the optimal locations of the problem, the set which consists of 103 candidates and the parameter *p*=10.

Table 3

Optimal locations for 103 candidates and parameter p=10

q=1		**q=500**		**q=800**		**q=1400**	
Demands	**Localities**	**Demands**	**Localities**	**Demands**	**Localities**	**Demands**	**Localities**
37418	Bratislava	37418	Bratislava	37418	Bratislava	21327	**Nové Mesto**
87285	Nitra	87285	Nitra	87285	Nitra	87285	Nitra
57854	**Trenčín**	57854	**Trenčín**	57854	**Trenčín**	70004	Bratislava
85400	Žilina	85400	Žilina	85400	Žilina	85400	Žilina
43789	Zvolen	43789	Zvolen	43789	Zvolen	43789	Zvolen
56157	Poprad	25088	**Rim. Sobota**	25088	**Rim. Sobota**	25088	**Rim. Sobota**
40870	**Košice**	56157	Poprad	56157	Poprad	56157	Poprad
92786	Prešov	92786	Prešov	92786	Prešov	92786	Prešov
39948	Michalovce	39948	Michalovce	39948	Michalovce	39948	Michalovce
19948	**Dolný Kubín**	19948	**Dolný Kubín**	30417	**Ružomberok**	30417	**Ružomberok**

The number of possible location combinations depends on the candidate set size too. Therefore we compared the changes of objective function for the listed values of q and for the different size of the set of candidates in the next group. The sets of candidates were formed according to the number of the inhabitants. Number of candidates ranges from 50 to 1000 localities. The differences among the values of the objective function for this group of instances and the number of requested locations p=40 are given in Table 4.

Table 4

Optimal locations for 103 candidates and parameter p=10

U_q \ Num	100		200		300	
	Value	%	Value	%	Value	%
U_1	46 033 396		46 033 396		46 006 935	
U_{50}-U_1	0	0.00	0	0.00	0	0.00
U_{200}-U_1	0	0.00	0	0.00	0	0.00
U_{350}-U_1	0	0.00	0	0.00	0	0.00
U_{500}-U_1	0	0.00	0	0.00	4 487	0.01
U_{650}-U_1	0	0.00	0	0.00	4 487	0.01
U_{800}-U_1	5 884	0.01	5 884	0.01	29 079	0.06
U_{950}-U_1	5 884	0.01	5 884	0.01	29 079	0.06
U_{1100}-U_1	265 404	0.58	278 052	0.60	391 996	0.85
U_{1250}-U_1	265 404	0.58	278 052	0.60	636 932	1.38
U_{1400}-U_1	519 026	1.13	519 026	1.13	624 284	1.36

U_q \ Num	400		500		1 000	
	Value	%	Value	%	Value	%
U_1	46 006 935		45 950 432		45 870 814	
U_{50}-U_1	0	0.00	0	0.00	0	0.00
U_{200}-U_1	0	0.00	0	0.00	0	0.00
U_{350}-U_1	0	0.00	0	0.00	1 862	0.00
U_{500}-U_1	4 487	0.01	4 487	0.01	51 197	0.11
U_{650}-U_1	4 487	0.01	4 487	0.01	38 322	0.08
U_{800}-U_1	59 629	0.13	35 037	0.08	38 322	0.08
U_{950}-U_1	59 629	0.13	35 037	0.08	44 313	0.10
U_{1100}-U_1	391 996	0.85	367 404	0.80	373 395	0.81
U_{1250}-U_1	636 932	1.38	613 588	1.34	582 383	1.27
U_{1400}-U_1	624 284	1.36	1 056 077	2.30	1 024 872	2.23

4. CONCLUSION

Obtained results shows that the mentioned deformation demand has no impact on computation time. Some deviations in objective function values appeared. These deviations partly grow parameter q which is used for the rounding. The specific way of rounding might change the proportionality of rounded values of demands. Note that a demand value b_j is rounded up, if b_j is smaller then q and it is rounded down otherwise. This is the source of deformations. It follows from the results that most differences are cumulated in the centre of tables. On the contrary, there is less differences if *p* is near to the value of 1 or if *p i*s near to the value of the candidate set cardinality. The worst situation is represented by the case, in which the value of *p* enables big number of feasible location combinations. The differences grow depending on increasing cardinality of candidate set. Differences took the value of 2.3% for a large set of candidates (from 500 to 1000). These results were obtained for p=40. Our hypothesis is that the differences could increase for such *p*'s, which are near to half of candidate set cardinality.

Computational time necessary for solution of associated problems reaches a value of hundreds of seconds. For these we found problems that the computational time increases depending on increasing coefficient *q* (on the contrary to the problems with 103 candidates). Nevertheless, this increase was not regular and furthermost the computational time did not exceed quintuple of the time necessary for the case with q=1.

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