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PROPOSITION OF DELAY MODEL FOR SIGNALIZED INTERSECTIONS WITH QUEUEING THEORY ANALYTICAL MODELS USAGE

Summary. Time delay on intersections is a very important transport problem. The article includes a proposition of time delay model. Variance of service times is considered by used average waiting time in queue for queuing system $M_{+\Delta} / G_{+\Delta} / 1$ with compressed queuing processes usage as a part of proposed time delays model.

PROPOZYCJA MODELU STRAT CZASU DLA SKRZYŻOWAŃ Z SYGNALIZACJĄ ŚWIETLNĄ Z ZASTOSOWANIEM ANALITYCZNYCH MODELI KOLEJKOWYCH

Streszczenie. Straty czasu stanowią istotny problem transportowy na skrzyżowaniach. W artykule przedstawiono propozycję modelu strat czasu. Model uwzględnia wariancję czasów obsługi, co zrealizowano poprzez zastosowanie – jako elementu modelu strat czasu – średniego czasu oczekiwania w kolejce dla systemu kolejkowego $M_{+\Delta} / G_{+\Delta} / 1$ z wykorzystaniem zlepionych procesów kolejek.

1. INTRODUCTION

Traffic flow models give a wide spectrum of possibilities for testing of complicated road traffic states and processes, as well as for proper representation of the transportation systems dynamics. Computer models of traffic flows allow receiving results in a very short time and they map the real traffic processes well enough. What is more, empiric methods are time-consuming and very often impossible to apply. Queuing theory has a wide range of applications. The queuing models are used in the following fields of life: traffic flow, scheduling, facility design and employee management. Implementation of the queuing theory for description of a signalized intersection simplifies creating the mathematical model of a real system. The article presents a proposed delay model.

2. PROPOSED DELAY MODEL

Proposed model, as most of delay models, contains two elements: a constant part and a random part. And this model includes:

- deterministic model (described by Clayton);

- average waiting time from $M_{+\Delta}/G_{+\Delta}/1$ queueing model with usage of the compressed queueing processes theory (described by Woch [17, 18, 19]).

2.1 Deterministic model

Deterministic model (first described by Clayton) is being estimated according to several assumptions [1, 3, 4]:
- uniform arrivals at the arrival rate during the cycle;
- uniform departures at the saturation flow rate.

At the beginning of the green phase given is a maximal value of the queue length (figure 1). Vehicles do not wait when the queue clears out.

The formula describing the average waiting time in the deterministic model is given as follows:

$$\overline{W} = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} \tag{1}$$

where: $\overline{W}$ – average waiting time per vehicle [s]; T_c – length of cycle [s]; G_e – effective green signal duration [s]; ρ – flow ratio [-].

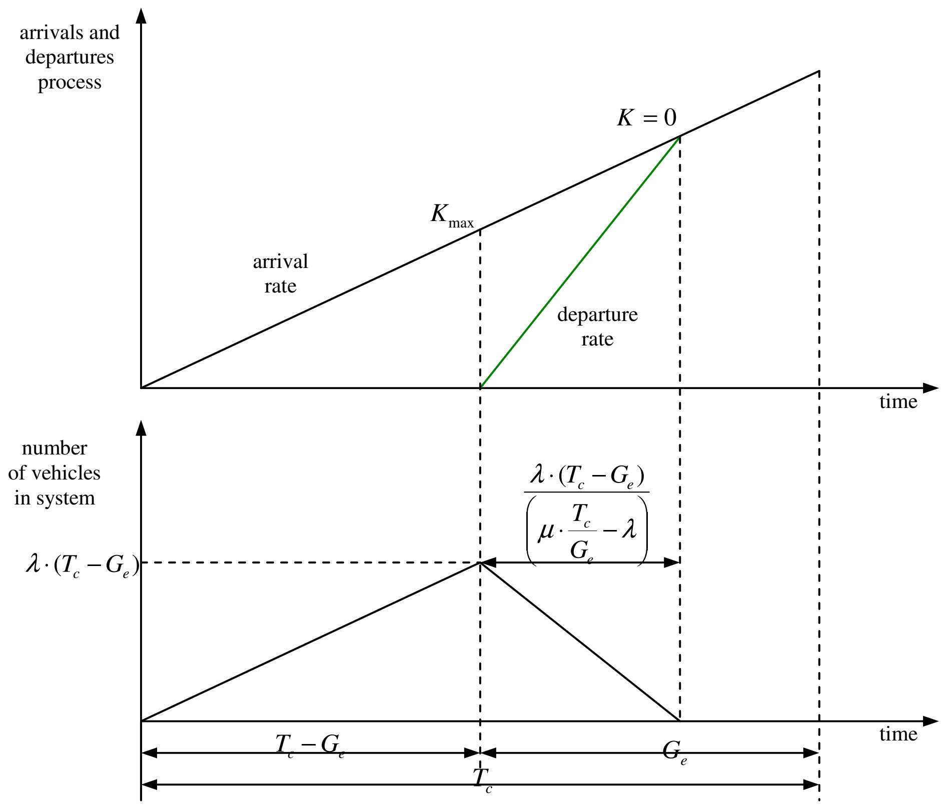

Fig. 1. Clayton's model
Rys. 1. Model Claytona

2.2 Model $M_{+\Delta}/G_{+\Delta}/1$ and compressed model M/G/1

The service time on signalized intersection has different distributions. That means that one distribution of the service time in model cannot be accepted. $M_{+\Delta}/G_{+\Delta}/1$ queueing system has a single server (only one service channel). The interarrival times have the exponential shifted distribution. $G_{+\Delta}$ means the general shifted distribution of the service times. The average service time (t_s) and the variance of the service time (σ^2) are used to describe service in this system. Compressed queueing processes are based on two assumptions (Woch [17]):

– average service time for a compressed model t_s', that equals:

$$t_s' = t_s - \Delta \tag{2}$$

where: t_s – average service time for a usual model [s]; Δ – minimal time distance between vehicles [s]; - reverse of arrival rate for a usual model $\frac{1}{\lambda}$, that equals:

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \Delta \text{; it means that } \lambda' = \frac{\lambda}{1 - \lambda \cdot \Delta} \tag{3}$$

where: λ' – arrival rate for a compressed model [veh/s]; Δ – minimal time distance between vehicles [s].

Graphic interpretation of compress process was shown on figure 2.

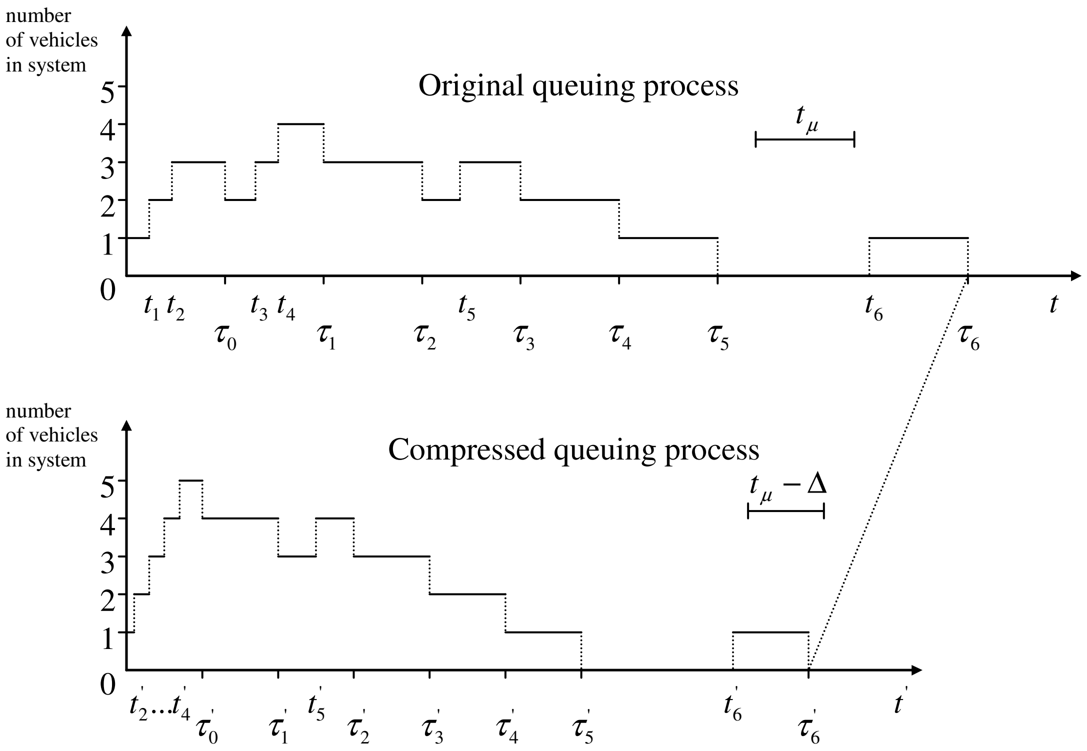

Fig. 2. Examples of original and compressed queueing processes [17, 18, 19]
Rys. 2. Przykładowe realizacje procesów oryginalnego i zlepionego [17, 18, 19]

To described waiting time in M/G/1 model the imbedded Markov chain can be used [5, 7]. The behaviour of the system is being observed in discrete moments. That is, when successive number of the vehicles leaves the system.

The Pollaczek-Khintchin formula is being obtained as follows (the average number of the vehicles in the system):

$$\overline{L} = \rho + \frac{\rho^2 + \lambda^2 \cdot \sigma^2}{2 \cdot (1-\rho)} \tag{4}$$

where: $\overline{L}$ – average number of the vehicles in the system [veh]; λ – arrival rate [veh/s]; ρ – flow ratio [-]; σ^2 - variance of service time [s].

The average waiting time in the queue can be obtained by using the formula (4) and Little's equations (5).

$$\overline{L} = \lambda \cdot \overline{W}; \qquad \overline{L} = \overline{L_q} + \frac{\lambda}{\mu}; \qquad \overline{L_q} = \lambda \cdot \overline{W_q}; \qquad \overline{W} = \overline{W_q} + \frac{1}{\mu}. \tag{5}$$

where: $\overline{L}$ – average number of the vehicles in the system [veh]; $\overline{L_q}$ – average number of the vehicles in the queue [veh]; $\overline{W}$ - average waiting time in the system [s/veh]; $\overline{W_q}$ - average waiting time in the queue [s/veh]; λ – arrival rate [veh/s]; μ – service rate [veh/s].

The sought equation receives now a form as follows:

$$\overline{W_q} = \frac{\lambda \cdot (\mu^{-2} + \sigma^2)}{2 \cdot (1-\rho)} = \frac{\rho^2 \cdot (1 + \sigma^2 \cdot \mu^2)}{2 \cdot \lambda \cdot (1-\rho)}. \tag{6}$$

where: $\overline{W_q}$ - average waiting time in the queue [s/veh] λ – arrival rate [veh/s]; μ – service rate [veh/s]; ρ – flow ratio [-]; σ^2 - variance of service time [s].

Finally, when the assumptions of compressed queueing processes (2) and (3) are being used, the formula describing average waiting time in the queue for original $M_{+\Delta}/G_{+\Delta}/1$ has a form as follows:

$$\overline{W_q} = \frac{\lambda \cdot \sigma^2 + \lambda \cdot \left(\frac{1}{\mu} - \Delta\right)^2}{2 \cdot (1-\rho)} \cdot (1 - \mu \cdot \Delta) \tag{7}$$

where: $\overline{W_q}$ - average waiting time in the queue [s/veh] λ – arrival rate [veh/s]; μ – service rate [veh/s]; ρ – flow ratio [-]; σ^2 - variance of service time [s]; Δ – minimal time distance between vehicles [s].

2.3 Proposed formula

The presented delay model is a sum of the average waiting time from the Clayton's model and the average waiting time from the $M_{+\Delta}/G_{+\Delta}/1$ queueing model that uses the compressed queueing processes.
The proposed delay model has a form as follows (Sierpiński [14]):

$$d_M = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} + \frac{\lambda \cdot {\sigma_\mu}^2 + \lambda \cdot \left(\frac{1}{\mu} - \Delta\right)^2}{2 \cdot (1 - \rho)} \cdot (1 - \mu \cdot \Delta) \tag{8}$$

where: d_M – average delay per vehicle [s]; T_c – length of cycle [s]; G_e – effective green signal duration [s]; ρ – flow ratio [-]; λ – arrival rate [veh/s]; μ – service rate [veh/s]; ${\sigma_\mu}^2$ – variance of service time [s]; Δ – minimal time distance between vehicles [s].

This formula makes a generalization of the Webster's model (1958). Webster did use the M/D/1 queueing model and his formula has got a form as follows [15, 16]:

$$d = \frac{T_c \cdot \left(1 - \frac{G_e}{T_c}\right)^2}{2 \cdot \left(1 - \frac{G_e}{T_c} \cdot \rho\right)} + \frac{\rho^2}{2 \cdot \lambda \cdot (1 - \rho)} - 0{,}65 \cdot \left(\frac{T_c}{\lambda^2}\right)^{\frac{1}{3}} \cdot \rho^{\left(2 + 5 \cdot \frac{G_e}{T_c}\right)} \tag{9}$$

where: d – average delay per vehicle [s]; T_c – length of cycle [s]; G_e – effective green signal duration [s]; ρ – flow ratio [-]; λ – arrival rate [veh/s].

A proposed delay model (8) makes possible to generalize the service process to a common distribution. There are three curves illustrating the time delay values (expressed by this formula) as shown on the figure 3:

- $\sigma_\mu^2 = 0$ corresponds to the constant service time;
- $\sigma_\mu^2 = 4$ and $\sigma_\mu^2 = 10$ makes samples of the empirical data (the large variance is a result of the large diversification of the service time for the left-turn vehicles).

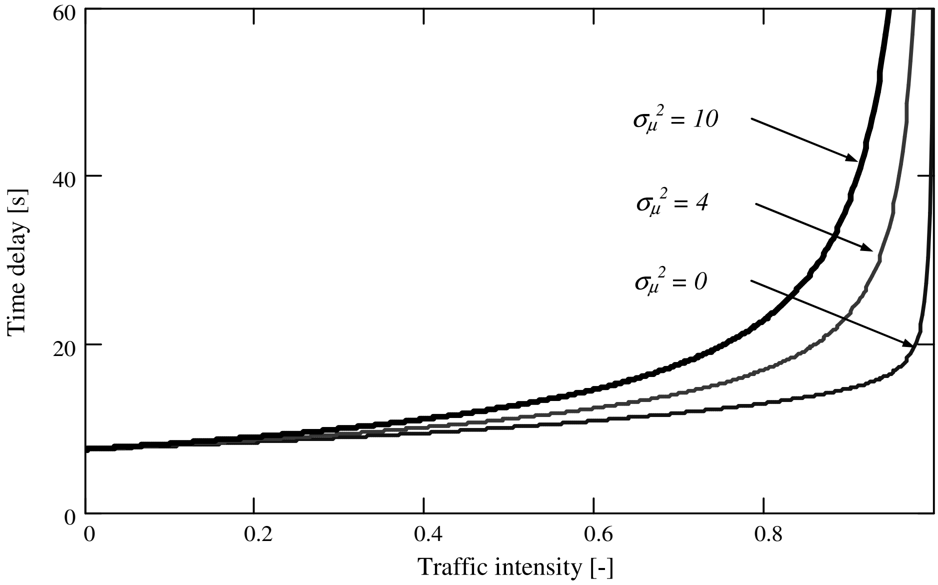

Fig. 3. The time delay for examples values of variance of service times σ_μ^2
Rys. 3. Strata czasu dla trzech przykładowych wartości wariancji czasów obsługi σ_μ^2

3. TRAFFIC MEASUREMENTS AND SIMULATION RESEARCH

Measurements on actual intersections are the basic source of data if there is a necessity of verifying traffic models. Results of observations help to create compatibility between reality and theoretical model.

Proposed model and Webster's one have been verified. There were two methods of survey applied to get these results. Measurements at actual fixed-time signalized intersections in Katowice were the first method and a video camera was used to record all stages of vehicle transition – from entering to leaving the intersection.

Each lane was analyzed separately. Traffic flow was measured for various traffic directions, traffic volume and various constructions of intersections. Traffic was observed for "straight" and "left turn" directions. "Right turn" direction was skipped by reason of "conditional drive signal"[1]. Measurement, in case of left turn, was done for independent movements and in case when vehicle transition depended on different traffic flows. Measurement was also done for mixed directions on one lane – "straight" and "left turn" with collision (with pedestrians).

Simulations in VISSIM were the second method to get values of variance of service times. The computer modelling of traffic flow allows receiving results in a very short time and they reflect the real traffic processes well enough. What is more, empiric methods are time-consuming and very often impossible to apply. A simulation program allowed to put various intensity of arrivals and make measurement for various traffic conditions. Simulation was made for 7 measurement points (in total: 630 experiments). Traffic was simulated 10 times for specific intensity of arrivals in range of traffic intensity from 0,1 to 0,9 (with skip 0,1).

[1] According to the Decree of Minister and Transport, in case of specific technical conditions of road signs and signals, and traffic safety devices, and conditions of their position on the road [6] "conditional drive signal" is allowed to be used only until 31.12.2008. Roads Administrators asked to explain this notation in August 2006. Ministry of Transport answered that this Decree will be changed in year 2007. The official opinion was: "Ministry of Transport informs that in the present state of law, impediments to using signaling device S-2 on streets (in current project) do not exist." Therefore if the notation about "conditional drive signal" will be in revision of this Decree, it will be necessary to measure traffic also in this case.

The measurements and simulations of time delays in comparison with estimated delays indicated that Webster's model still gives good estimate of time delays. Assumption about variance of service times in proposed model allows estimating time delays with smaller relative error.

Full description of these measurements and results were described on [14].

4. CONCLUSIONS

Proposed formula allows estimating time delays in case of large variations of the service time (general distribution of the service time). The service of vehicles on signalized intersections is a very complex mechanism. Usually there are many simplifications in traffic models. Many researchers take into consideration only intensity of arrival and service. Estimation of the variance of service times allows to reflect the real traffic processes well. It is possible to estimate dispersion in the process of vehicle service on signalized intersections by taking into consideration variance of service time.

The shifted exponentially distribution of the interarrivals is increasingly in use for the traffic modelling. That is because this assumption allows for the minimal time distance between the vehicles heading towards the intersection.

Proposed delay model have been verified by observations and measurements on several signalized intersections. And this model might be helpful and very useful for the urban traffic management.

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