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KINEMATICS OF POINT-TO-POINT CONTACT OF WHEELS WITH A RAILS

Summary. The railway transport in a maximum degree is conservative. For example, the design of wheelsets, practically, has not changed for the last 150 years. Such a long life was given to this tendency by its high reliability and simplicity of the design. It is considered, that the rigid connection of wheels by means of an axis provides a self-centering of wheelset within the limits of a cross backlash of a railway. The tread contact surfaces have a difficult profile. Researches of many scientists, dealing with the questions of interaction of wheels with rails, show that the classical approach to the wheelset design leads to many problems of the railway transport. Therefore, the authors of this article strived for deeper consideration of kinematics of a movement of vehicle by a railway.

КИНЕМАТИКА ДВУХТОЧЕЧНОГО КОНТАКТИРОВАНИЯ КОЛЕСА С РЕЛЬСОМ

Аннотация. При упоминании о консервативности железнодорожного транспорта имеют в виду, прежде всего, конструкцию колесных пар, практически не изменившуюся за последние 150 лет. Столь долгую жизнь этому движителю обеспечили его высокая надежность и простота конструкции. Считается, что жесткое соединение через ось колес с профилированными поверхностями катания обеспечивает самоцентрирование колесных пар в пределах поперечного зазора в рельсовой колее. Однако, как показывают исследования многих ученых, занимающихся вопросами горизонтального взаимодействия колес с рельсами, именно этот классический подход к конструированию колесных пар является причиной многих проблем эксплуатации подвижного состава. Этим объясняется стремление авторов данной работы к более глубокому рассмотрению кинематики направления экипажей рельсовой колеей. В статье рассматривается кинематика двухточечного контактирования колес с рельсами с учетом перераспределения кинематических параметров контактирования между контактами.

At theoretical researches of horizontal dynamics of rail vehicle the driving of a single wheelset with small speed, as a rule, is considered without taking into account a creep in contacts. The trajectory of a wobble, thus, depends on a profile of driving surfaces of wheels and initial deviation from the average position in a track. In the majority of researches of horizontal dynamics the radiuses of the tread contact surfaces are described by linear functions without taking into account a point-to-point contact of wheels with rails [1]:

$$R_1 = R_0 + \Delta_1(y)$$
$$R_2 = R_0 + \Delta_2(y) \tag{1}$$

where: R_1, R_2 – radiuses of taping lines in points of contact with rails; R_0 – average radius of taping lines relevant to the central position of a wheelset; y – a cross displacement of the wheelset in relation to the central position; $\Delta_1(y)$, $\Delta_2(y)$ – increment function of radiuses of wheels depending on the cross displacement of the wheelset.

The instant meaning of a curvature of a wheelset trajectory moving thus can be determined from a ratio:

$$\Re = \frac{\Delta_1(y) - \Delta_2(y)}{2R_0 + \Delta_1(y) + \Delta_2(y)} \cdot \frac{1}{s} \tag{2}$$

where: 2s – a distance between taping lines. If y(x) – function describing a moving trajectory of the geometrical centre of the wheelset:

$$\Re = -\frac{\sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3}}{\frac{d^2y}{dx^2}} \tag{3}$$

However, the majority of variants of combinations of wheels and rails profiles can form, so called, point-to-point contact. Thus under "points" we mean the centers of contact spots. Practically, while moving in direct sites of the way, the point-to-point contact can exist for a very short time. It is basic in curve sites of the way. The spatial distribution of contact forces caused by the form of tread surfaces of a wheel and a rail is the reason of a differential creep in contacts and occurrence of moving resistance forces. Schematically, the point-to-point contact is a statically indefinable system with a parameter of nondefinability, equal to a unit. Therefore, in mathematical modeling of distribution of loadings in contacts it is necessary to take deformations of contact areas into account [2]. It is necessary to consider a combination of two groups of parameters: for the first contact (I), more removed from a crest, and for the second (II) – crest contact.

The geometrical parameters of the point-to-point contact in the numerical integration of the equations of wheelset movement are defined as a result of coordinates analysis of contact points of wheels with rails. The radiuses of tread contact surfaces (R_I, R_{II}) and profiles grade (g_I, g_{II}) in points of contacts are related to such parameters.

The contact spots centers on the tread surface, generally, do not lay in an axial plane of the wheelset, as it is accepted in the simplified consideration of the task in flat statement.

A wheel profile in three-dimensional system of coordinates can be described, as a surface of rotation with a forming line, relevant to a working surface profile:

$$x_w^2 + z_w^2 = [r(y_w)]^2 \tag{4}$$

where: $r(y_w)$ – function describing a wheel structure.

The rail profile is described as a cylindrical function: $z_r = z_r(y_r)$.

The displacement of contact points in the direction Z (increment of the tread surface radius – ΔR_Ψ) can be described by a formula:

$$\Delta R_\psi = 1{,}235 \frac{\psi^2}{g^{1{,}82}} \tag{5}$$

and displacement on an axis X (forestalling of a contact b) by the formula:

$$b = \frac{\psi}{0{,}15\psi^2 + 3{,}6 \cdot 10^{-4} g} \tag{6}$$

where: *Ψ* – wheel-rail climbing corner; *g* – grade of the tread surface in a contact point.

The picture of sliding speeds in contacts is shown on a fig. 1. A point C – instant centre of a wheel rotation; K_I – point of the first contact; K_{II} – point of the second contact at Ψ = 0; K_{II}* – a point of the second contact at Ψ ≠ 0; V_{sxI} , V_{sxII} – absolute speeds of sliding in I and II contacts. From a proportion:

$$\frac{CK_{II}}{OC} = \frac{V_{sxII}}{V_w} \tag{7}$$

where: V_w – forward speed of the wheel centre. Taking into account, that

$$V_w = \dot{x} \cdot \cos\psi + A\dot{\psi} \tag{8}$$

where: *2A* – distance between planes of taping lines, and OC = R_{II} – CK_{II}, we receive:

$$CK_{II} = R_{II} \frac{V_{sxII}}{V_w + V_{sxII}} \quad \text{or} \quad CK_{II} = R_{II}\varepsilon_{xII} \tag{9}$$

where: ε_{xII} – relative longitudinal sliding in crest contact.

From a Fig. 1:

$$CK_{II}^* = \sqrt{(CK_{II} + \Delta R_\psi)^2 + b^2} \tag{10}$$

$$\mathrm{tg}\,\varsigma = \frac{b}{CK_{II} + \Delta R_\psi} \quad \text{or} \quad \mathrm{tg}\,\varsigma = \frac{b}{R_{II}\varepsilon_{xII} + \Delta R_\psi} \tag{11}$$

where: ς – a grade corner of a contact plane Ω to a vertical. The plane Ω passes through I and II contacts and is parallel to the wheelset axis (Fig. 1).

Projections of sliding speeds in system of coordinates of the wheelset:

$$V_{sII}^* = \dot{\varphi} \cdot CK_{II}^* . \tag{12}$$

Here: $\dot{\varphi}$ – angular velocity of the wheel rotation.

$$V_{sxII} = V_{sII}^* \cdot \cos\varsigma \quad V_{syII} = V_{sII}^* \cdot \frac{\sin\varsigma}{g_{II}} \quad V_{szII} = V_{sII}^* \cdot \sin\varsigma \tag{13}$$

Total speed of sliding:

$$V_{sII} = V_{sII}^* \sqrt{1 + \frac{\sin^2\varsigma}{g_{II}^2}} \quad \sin\chi = \frac{g_{II}\sin\varsigma}{\sqrt{g_{II}^2 + \sin^2\varsigma}} \tag{14}$$

where: χ – corner between a vector of total sliding speed in crest contact and horizontal plane.

The relative longitudinal sliding in contacts is defined by the following ratio:

$$\varepsilon_{xI} = \frac{\dot{\varphi}R_I - V_w}{\dot{\varphi}R_I} \qquad \varepsilon_{xII} = \frac{\dot{\varphi}R_{II} - V_w}{\dot{\varphi}R_{II}} \tag{15}$$

"Pure" rolling is not possible at the point-to-point contact. Let's introduce the tread surface equivalent radius R_e, with which we shall define theoretical meaning of radius appropriating wheel rolling with speeds V_w and $\dot{\varphi}$ without sliding:

$$V_{sx} = \dot{\varphi} R_e - V_w = 0 \qquad \text{whence: } R_e = \frac{V_w}{\dot{\varphi}} \tag{16}$$

From the scheme of sliding speed distribution at point-to-point contact, shown on the Fig. 1:

$$\frac{V_{sxI}}{V_{sxII}} = \frac{R_e - R_I}{R_e - R_{II}} \tag{17}$$

From ratio (15) and (17):

$$R_e = R_I R_{II} \frac{\varepsilon_{xII} - \varepsilon_{xI}}{\varepsilon_{xII} R_{II} - \varepsilon_{xI} R_I} \tag{18}$$

The meanings of sliding ε_{xI} and ε_{xII} indirectly depend on an adhesion force $\vec{F}_{sx} = \vec{F}_{sxI} + \vec{F}_{sxII}$, and the distribution of contact adhesion forces F_{sxI} and F_{sxII} between contacts, in turn, depends from normal (N_I and N_{II}) and vertical (P_I and P_{II}) contact loads.

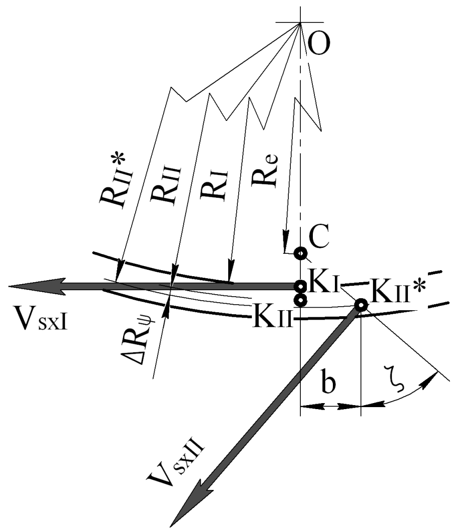

Fig. 1. The scheme of distribution of contact spots centers and sliding speeds at point-to-point contact
Рис. 1. Схема расположения центров контактных пятен и распределения скоростей скольжений при двухточечном контактировании

The dependences description $F_{sxI}(P_I, \varepsilon_{xI}), F_{sxII}(P_{II}, \varepsilon_{xII})$ is given in a monograph [2].

Two contacts exist simultaneously only on a transitive site dy_I - dy_{II} [3]. And, at $dy \le dy_I$ $P_I = P_0$, $P_{II} = 0$; at $dy \ge dy_{II}$ – $P_I = 0$, $P_{II} = P_0$. On transitive site, at $dy_I \le dy \le dy_{II}$: $P_I + P_{II} = P_O$.

Having accepted an assumption of a linear loadings change from P_I to P_{II}, it is possible to receive the laws of their change on the overlapping site $dy_I \le dy \le dy_{II}$:

$$P_I = P_0 \frac{dy - dy_I}{dy_{II} - dy_I}; \quad P_{II} = P_0 \frac{dy - dy_{II}}{dy_I - dy_{II}}, \tag{19}$$

From expressions (2), (3), (5) and (15) equations of wheelset wobbling will look like this:

$$\frac{d^2y}{dx^2} = -s\frac{R_{e1} + R_{e2}}{R_{e1} - R_{e2}}\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)^3}, \tag{20}$$

where: R_{e1}, R_{e2} – equivalent radiuses of the first and second wheels of the wheelset.

The method given in this article allows describing the mechanism of wheelset movement by a railway more precisely. It has a large importance for theoretical researches of horizontal dynamics of rail vehicles.

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