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PREDICTION OF FINE PARTICULATE MATTER IN LOW-EMISSION ZONES USING A MODIFIED NUMERICAL METHOD FOR A SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

Summary. Currently, many European cities have severely exceeded the EU air quality standards and are struggling with high concentrations of fine particulate matter PM10 and PM2.5 in the air, with road transport often being one of the major polluters. One of the forms for correction of the problem that many cities in the EU are currently using is the construction of low-emission zones.

For the prediction of PM10 and PM2.5, a modified numerical method for a system of ordinary differential equations has been proposed. In the right part of this system, in addition to the main trend and the periodicity of PM10 and PM2.5, their correlation is taken into account. Against the background of the best solution obtained, a forecast is made for the emission levels in a period of one week in the town of Ruse.

1. INTRODUCTION

Modern urbanization has led to a continuous increase in the urban population. This, in its turn, has resulted in an increase of vehicles in cities, creating a number of problems related to congestion, noise, traffic accidents, frequent exceeding of permissible environmental standards and more. The current situation justifies the need to look for new policy solutions related to urban access regulations to ensure a balance between congestion, habitability, air pollution, noise levels, accessibility, damage to historic buildings and other adverse outcomes. Many cities have introduced regulations or restrictions on vehicles entering all or parts of their central areas to improve issues such as air quality, congestion or experiences in the city area. Entry may depend on vehicle emissions, payment, vehicle types, etc.

The prevention of vehicles with poor environmental performance is related to the introduction of low-emission zones (LEZs). LEZ schemes can take many forms based on the geographical area they cover, the time in which the LEZ is in force, emission standards required for vehicles to enter the area, and the types of vehicles that need to be complied with. Despite widespread public acceptance, LEZ policies may suffer from strong public opposition if they are not carefully planned with international practices in mind and if they do not take into account the economic and social characteristics of the studied area. This article presents the state of the activities for the introduction of LEZs in Bulgaria and elements of the methodology developed by the authors for the introduction of LEZs in the city of Ruse.

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According to the data included in the air quality programs adopted by the Ruse Municipal Council, the most significant source of PM10 and PM 2.5 in the city of Ruse is domestic heating, which includes emissions from heating domestic and public buildings with solid fuels and wood during the winter. Then, the share of PM10 is 61.1%, and that of PM2.5 is 79.5% of all released emissions [4]. Therefore, the city has taken measures for the gasification and use of alternative sources of energy. During the summer, the main source of these emissions is road transport.

Based on real data for the city are derived numerical models for predicting future values of PM10 and PM2.5, which are based on the numerical solution of the initial Cauchy problem for a two-dimensional system of ordinary differential equations (ODE) of the first order. The computational properties of the modified ODE system are considered.

2. LOW-EMISSION ZONES

2.1. Practice in introducing low-emission zones

Many EU countries are struggling with high concentrations of fine dust particles (PM10 and PM2.5) and nitrogen dioxide (NOx) in the air. Often, one of the reasons for this in urban areas is car traffic. Furthermore, road transport alone is responsible for 60% of NOx concentrations in cities, according to the German Environment Agency. In Paris, 65% of NOx and 36% of PM10 emissions are from transport [1].

Pollution levels in EU cities affect people’s health and can lead to premature death. According to the European Environment Agency (2017), around 430,000 people die prematurely in Europe from exposure to PM2.5, and approximately 78,000 die prematurely from exposure to Nox.

From a published interactive map [15], it can be seen that the number of premature deaths in 2019 per 100,000 inhabitants due to PM2.5 was 59 in Paris, 79 in Berlin, 45 in Madrid, 67 in Vienna, and 133 in Bucharest. In Bulgaria, these figures are even higher (169 in Plovdiv, 146 in Ruse, 142 in Stara Zagora, 127 in Varna, and 126 in Sofia).

Many cities are struggling with the balance of congestion, noise levels, air pollution, accessibility, damage to historic buildings and other consequences of modern city life. Many of them have levels of pollution that are harmful to people’s health, and much of the pollution comes from road traffic. Congested, polluted and noisy cities are not attractive to businesses and residents. Moreover, congestion has a serious impact on the economy. It costs around €100 billion, or 1% of the EU’s gross domestic product (GDP) each year.

Directive 2008/50/EC of the European Parliament and of the Council on ambient air quality and cleaner air for Europe legally requires cities that exceed the permitted limits of air pollution to develop action plans and implement the necessary measures to achieve limit values. One of the measures that many cities are implementing to reduce the number of highly polluting vehicles is the introduction of LEZs, and in recent years, their use has increased.

Low-emission zones date back to the end of the 20th century and have been developing intensively since 2000. In 1996, in Gothenburg, Malmö and Stockholm, Sweden, "ecological zones" in the city center were introduced, aimed at restricting diesel trucks with a load capacity of more than 3.5 t. In 2005, the regions of northern Italy met and prepared agreements to implement air quality measures. This led to the application of LEZs, with restrictions in the winter implemented in these areas. In July 2007, the Netherlands started implementing LEZs. The low-emission zones of Berlin and London became operational in January and February 2008, respectively. In France, Paris was the first city to introduce a LEZ in 2015.

Today, the number of planned European projects for low-emission areas is constantly increasing, from 179 in 2011 to 377 in 15 European countries in March 2022 (Tab. 1). At the same time, there are many cities that are in the process of preparing or planning for the introduction of LEZs [14]. Regardless of the country, the main goal of the introduction of LEZs is to reduce air pollution by accelerating the renewal of vehicles, thereby reducing pollutant emissions (mainly fine dust matter and nitrogen oxides).

LEZs typically meet four main criteria:

- they are implemented for a given geographical perimeter.
- they apply to all vehicles: personal, public (e.g., public transport, garbage collection vehicles), and company.
- they restrict the access of vehicles on the basis of their polluting emissions.
- they are permanent (year-round) or semi-permanent (e.g., all winter).

The addition of the fourth criterion aims to distinguish areas that work in a sustainable way from those that work only during pollution peaks, with the latter being numerous.

Table 1

European countries implement LEZs (evolution from 2011-2020)

Number of LEZ								
Country	03.2011	03.2012	03.2014	03.2015	09.2017	11. 2018	04.2020	04.2022
England	2	2	3	1	1	1	1	22
Germany	43	56	69	78	83	87 (2)	87	83
Austria	1	1	2	4	4	4	4	7
Belgium					1	2	3	4
Denmark	4	4	4	4	4	4	4	4
Spain					1	1	2	7
France					2	3	5 (3)	16
Greece				1	1	1	1	1
Italy	109	98	94	100	108	106	117	202
The Netherlands	12	12	12	13	13	13	13	16
Portugal	0	1	1	1	1	1	1	1
Czech Rep.	1	1	1	1	1	1	1	2
Sweden	6	6	7	8	8	8	8	8
Finland								1
Norway								3
Total amount	179	182	193	211	227	232	247	377

The sizing of LEZs is variable and depends on the local context. It can affect only part of the center, a whole city or groupings several parts of a city. Of course, with a larger area, there will be a stronger impact on air quality than with a smaller area. The vehicles concerned may also differ in different countries or even in different cities within the same country.

Currently, two methods are used to comply with the restrictions on access to the zones: video surveillance and immediate control (police officers with registration certificates). Video surveillance generates much higher annual revenues than direct surveillance.

In the long run, the costs of implementing, operating and adapting of the persons concerned (e.g., vehicle renewal costs, fines) are offset by health costs, which are avoided by improving air quality.

Based on the experience on an international scale, five main actions can be defined, which are related to increasing the economic and financial feasibility of LEZs:
- determining the categories of vehicles to be excluded from access to the zones,
- progressiveness of the assessment,
- possible derogations,
- financial help,
- communication.

Most LEZs operate 24 hours a day and year-round. The one exception is Italian LEZs, which are in operation only in winter.

There are no uniform LEZ regulations or standards in the Member States, although several countries (Germany, Denmark, Sweden, the Netherlands, and the Czech Republic) have adopted national LEZ regulations. Even within Member States with national LEZ regulations, each city may be free to apply LEZs with local conditions and administration, creating a mix of restrictions and procedures.

As of 30.05.2022, no low-emission zone has been introduced in Bulgaria. The two cities listed in the National Program for Improving Atmospheric Air Quality (2018-2024) as examples have started work on planning low-emission zones. These are the city of Sofia and the city of Plovdiv.

In December 2021, a study was published to assess the impact of the introduction of a LEZ in Sofia [2].

The study proposes two different schedules for the introduction of LEZs: one in which a new stage is introduced every two years from 2022 to 2032 and one on an accelerated schedule in which each new stage is introduced annually from 2022 to 2027. The scheme of LEZs, which is designed for Sofia, is presented in Tab. 2.

Table 2

Design of a low-emission zone in Sofia

Stage	Minimal standard		Input schedule	
	Diesel	Gasoline	Two-year interval	Accelerated
1	Without restrictions	Without restrictions	2021	2021
2	Euro 3	Euro 2	2022	2022
3	Euro 4	Euro 3	2024	2023
4	Euro 5	Euro 3	2026	2024
5	Euro 6	Euro 4	2028	2025
6	Euro 6d	Euro 5	2030	2026
7	Euro 7	Euro 6d	2032	2027

Currently, in Sofia, there is a technological possibility for the introduction of two low-emission zones (two rings): small and large [13].

The introduction of automatic control by technical means is envisaged, namely by cameras with automatic recognition of registration numbers, as well as by readers for the environmental group, which the Ministry of Transport and Communications has developed in [3]. Cars entering and leaving the designated areas will be detected in real time. When detecting a car that does not meet the minimum environmental group set in the defined zones or zone, the system will automatically compile the necessary documents for sanctioning.

It is planned that the system will be connected to the registers of the Ministry of Interior and the Ministry of Transport and Communications to the extent necessary for the exchange of data on the establishment of the ecological group and for the future service of the sanction.

In October 2021, the Municipality of Plovdiv published a public procurement with the subject "Development of a methodology for the introduction of low-emission zones" for a period of six months. A contractor company has been appointed. No public information on the progress of public procurement has been published yet.

Under a contract with the Municipality of Ruse, a team of the University of Ruse, with the participation and coordination of the authors of this article, developed a methodology for the introduction of the LEZ "Ruse C," the geographical scope of which is shown in Fig. 1.

The methodology offers two alternative charts:

Scheme 1: with a schedule for the introduction of the respective restrictions of the cars in accordance with the European norms (Tables 3 and 4).

Scheme 2: with a schedule for the introduction of the respective restrictions on the cars in accordance with the Bulgarian euro groups, introduced by Ordinance H32 (Tables 5 and 6).

The choice of scheme depends on administrative decisions and access to relevant databases.

It is envisaged that LEZs will apply to all types of vehicles, with the exception of some, due to their special purpose (e.g., vehicles with special traffic regimes; emergency aid; fire safety; utilities for repatriation of vehicles; trucks and specialized vehicles of emergency services, specialized construction equipment, and vehicles for people with permanent disabilities).

Fig. 1. Map of the geographical realm of LEZ in Ruse

Table 3

Schedule for the introduction of LEZ restrictions for diesel cars under Scheme 1 (according to Euronorms)

Date of introduction for the respective year 01 November

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Euro 6	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Euro 5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
Euro 4	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
Euro 3	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No
Euro 2	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No
Euro 1	Yes	Yes	Yes	No	No	No	No	No	No	No	No
Before Euro	Yes	Yes	No	No	No	No	No	No	No	No	No

Table 4

Schedule for the introduction of LEZ restrictions for petrol cars under Scheme 1 (according to European standards)

Date of introduction for the respective year 01 November

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Euro 6	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Euro 5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Euro 4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No
Euro 3	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
Euro 2	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No
Euro 1	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No
Before Euro	Yes	Yes	No	No	No	No	No	No	No	No	No

In both schemes, a gratis period of two years is provided for preparation. The gradual introduction of the restrictions is expected to take place from 1 November, given the onset of the winter season and the increase in the values of fine dust particles. In 2032, both schemes show a total ban on entering the area for cars refueled with conventional fuels.

Table 5

Schedule for the introduction of LEZ restrictions for diesel cars under Scheme 2 (according to the national Bulgarian Eurogroups)

Date of introduction for the respective year 01 November

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Group 5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Group 4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Group 3	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No
Group 2	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No
Group 1	Yes	Yes	No	No	No	No	No	No	No	No	No

Table 6

Schedule for the introduction of LEZ restrictions for petrol cars under Scheme 2 (according to the national Bulgarian Eurogroups)

Date of introduction for the respective year 01 November

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Group 5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Group 4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Group 3	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
Group 2	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
Group 1	Yes	Yes	No	No	No	No	No	No	No	No	No

The presence of LEZs implies low values of harmful substances, which, however, are variable, depending on a number of factors. This calls for solutions to predict the presence of harmful substances, including PM10 and PM2.5.

In case of increased values, the population can take appropriate measures to ensure their personal safety. At the same time, this is valuable information for the municipal administration when making decisions for holding mass events in the LEZ, as well as the implementation of other activities (e.g., construction and repair of the LEZ).

2.2. Method for predicting fine dust particles

The main goal is to make the most accurate forecasts possible for the future values of fine dust matter. Analysts use a large amount of data to predict the trends and dynamics of a given set of data over a period of time. This data can be used to predict the movement of PM10 and PM2.5 values for the time after the given period.

The statistical methods most commonly used by analysts are cluster analysis, regression analysis, time series, and neural networks. Time series analysis uses statistics that are dynamic over time [16]. The independent variable in this method is time, and the goal is to predict future values based on past and present data.

One of the first classical tools for time series analysis was based on the theory of approximation. The time series is presented as a sum (sometimes a product) of its constituent components (e.g., trend, periodicity). An approximation function is constructed with the help of which a component of the time series is filtered. An approximate function is built to the calculated row again, which filters the next component in the row. This is done several times consecutively. In the end, the stochastic (accidental) part remains [11]. It is represented as a random variable that can be simulated using a random number generator [6]. Once the functions that approximate the individual components of the timeline are found, future values are predicted.

Another commonly applied approach for time series forecasting is the so-called autoregressive integrated moving average approach. It was developed in 1970 by George Boxing and William Jenkins [9]. The time-varying variance poses a problem. Robert Engle took this variance variability into account and proposed a model for autoregressive conditional heteroskedasticity (ARCH). At a later stage, the generalized ARCH model, exponential ARCH model, and threshold ARCH model were also developed.

Neural networks are used to predict future values and quantities based on historical data, quantities and other information [10]. One major advantage of using neural networks for forecasting is that in addition to quantitative data, history and other inferred factors can be used (very often, these factors are not quantitative in nature). Neural networks are widely used, including in areas such as currency markets [8], credit card approval and fraud detection, and image recognition. Their drawback is that large data sets are used. This requires more complex computational procedures that require significant computational time and RAM resources.

In this paper, a modified ordinary differential equations (ODE) approach was considered to model PM10 and PM2.5 values over a period of time. The advantages of the modification are demonstrated using numerical examples. For this purpose, real data from the city of Ruse, Bulgaria, were used. The proposed models for predicting future PM10 and PM2.5 values are based on numerical methods for solving the initial Cauchy problem for a two-dimensional first-order ODE system. A number of models were fitted to historical data to determine the best predicted values. The final forecast was given as a weighted average of all forecasts for the relevant fine particulate matter. The weights were the inverse of proportional to the "marginal error" (final error).

The approach is based on that proposed in [5] and is a summary of a two-dimensional ODE system of the first line.

Let the moments (chronologically arranged in ascending order) and the observed values of PM10 and PM2.5 be time series for the same period:

$$t_0, t_2, \dots, t_n \; and \; x_0, x_1, \dots, x_n; \; y_0, y_1, \dots, y_n, \; i = \overline{1, .. n}, \tag{1}$$

where t_i is the moment at which the i^{-th} observation is done, x_i, y_i are the currently observed values of PM10 and PM2.5, respectively, at the moment t_i.

Similar to [5, 7, 12], this time series with PM10 and PM2.5 as a time-dependent can be compared with a system of ordinary differential equations of the first order with initial conditions (Cauchy problem),

$$\begin{aligned} x'(t) &= g_1(t, x, y) \\ y'(t) &= g_2(t, x, y), \\ x(t_0) &= x_0, \; y(t_0) = y_0, \end{aligned} \tag{2}$$

which describe the values of the time series in the given discrete time moments. The functions $\mathrm{g}\,(\mathrm{t}, \mathrm{x}, \mathrm{y})$ can be largely arbitrary.

The main purpose of considering the time series (1) as two-dimensional and not as two separate one-dimensional lines was that it takes into account the interaction between PM10 and PM2.5. This means that the influence of both the time and the previous values of the respective type of fine dust particles must be considered in order to predict the value of one of them.

In the one-dimensional case (when predicting an instrument $\mathrm{y} = \mathrm{y}\,(\mathrm{t})$) for $\mathrm{g}\,(\mathrm{t}, \mathrm{y})$, the authors in [5, 7, 12] offer the following form:

$$g(t, y) = a(t)y + s(y), \tag{3}$$

where $a(t) \; and \; s(y)$ can be primitive, power, exponential, or logarithmic functions, among others. They can be extended to a certain degree under certain conditions. In addition, some periodicity can be observed in the data. Therefore, the function $\mathrm{g}\,(\mathrm{t}, \mathrm{y})$ of the form (3) can consist of a polynomial and a trigonometric part.

Suppose that the first derivative has the form:

$$g(t, y) = \left(\textstyle\sum_{i=0}^{M} a_i t^i\right) y + b_0 + \textstyle\sum_{j=1}^{N} b_j \sin\left(\frac{2\pi j}{\theta} y + c_j\right), \tag{4}$$

i.e.,

$$y'(t) = \left(\sum_{i=0}^{M} a_i t^i\right) y + b_0 + \sum_{j=1}^{N} b_j \sin\left(\frac{2\pi j}{\theta} y + c_j\right). \quad (5)$$

The unknowns here are the coefficients:

$$a_0, a_1, \dots a_M, b_0, b_1, \dots, b_N, c_1, c_2, \dots, c_N, \theta. \quad (6)$$

Forecasting PM10 and PM2.5 in this article was proposed in the right part of system (2), in which the functions $g_1(t,x,y)$ and $g_2(t,x,y)$ take the following form:

$$g_1(t,x,y) = \left(\sum_{i=0}^{M_1} a_i^{11} t^i\right) x + \left(\sum_{i=0}^{M_2} a_i^{12} t^i\right) y + b_0^1 + \sum_{j=1}^{N_1} b_j^{11} \sin\left(\frac{2\pi j}{\theta^{11}} x + c_j^{11}\right) + \sum_{j=1}^{N_2} b_j^{12} \sin\left(\frac{2\pi j}{\theta^{12}} y + c_j^{12}\right);$$

$$g_2(t,x,y) = \left(\sum_{i=0}^{M_1} a_i^{21} t^i\right) x + \left(\sum_{i=0}^{M_2} a_i^{22} t^i\right) y + b_0^2 + \sum_{j=1}^{N_1} b_j^{21} \sin\left(\frac{2\pi j}{\theta^{21}} x + c_j^{21}\right) + \sum_{j=1}^{N_2} b_j^{22} \sin\left(\frac{2\pi j}{\theta^{22}} y + c_j^{22}\right).$$

The coefficients to be determined are:

$$a_0^{11}, a_1^{11}, \dots a_{M_1}^{11}, a_0^{12}, a_1^{12}, \dots a_{M_2}^{12}, b_0^1, b_1^{11}, \dots, b_{N_1}^{11}, b_1^{12}, \dots, b_{N_2}^{12}, c_1^{11}, \dots, c_{N_1}^{11}, c_1^{12}, \dots, c_{N_2}^{12} \theta^{11}, \theta^{12}$$
$$a_0^{21}, a_1^{21}, \dots a_{M_1}^{21}, a_0^{22}, a_1^{22}, \dots a_{M_2}^{22}, b_0^2, b_1^{21}, \dots, b_{N_1}^{21}, b_1^{22}, \dots, b_{N_2}^{22}, c_1^{21}, \dots, c_{N_1}^{21}, c_1^{22}, \dots, c_{N_2}^{22} \theta^{21}, \theta^{22}. \quad (7)$$

These coefficients are $4N_1 + 4N_2 + 2M_1 + 2M_2 + 6$ in count. If $n \geq 4N_1 + 4N_2 + 2M_1 + 2M_2 + 7$ (something that will always be assumed), they can be found by solving an inverse problem using a numerical one-step explicit or implicit (or a combination of both) method:

$$\frac{(x_{k+1} - x_k)}{h} = Z(g_1) \text{ or } x_{k+1} = x_k + hZ(g_1),$$
$$\frac{(y_{k+1} - y_k)}{h} = Z(g_2) \text{ or } y_{k+1} = y_k + hZ(g_2), \quad (8)$$

where $h = t_{k+1} - t_k$, g represents the right part of (2), and $Z(g)$ is a specific one-step numerical method (e.g., explicit, implicit method of Euler, Runge-Kuta). Similar to [12], the latter can be used $4N_1 + 4N_2 + 2M_1 + 2M_2 + 7$ values of the time series (1), and, by means of (8), a system of nonlinear equations with respect to the coefficients (7) is reached. After solving the system, the required coefficients were obtained (7). They were then used to find the next y_{n+1} currently unknown value of the moment t_{n+1} (i.e., in (2), all coefficients were known, and we could calculate the next y_{n+1} value by the numerical method Z). The disadvantage of this approach is that not all the information from time series (1) is used, as many values from line (1) are taken as necessities to close the system of nonlinear equations, with a specific choice of M_1, M_2, N_1 and N_2. As shown in [5], this shortcoming was easily overcome by taking any number of values k ($n \geq 4N_1 + 4N_2 + 2M_1 + 2M_2 + 7$) from series (1), as the predefined system of nonlinear algebraic equations was solved by the least squares method (LSM). The LSM can also be used in its weight variant (i.e., to give the weight function). It is logical for the newer values in the time series (1) for the weight function to set a higher weight (i.e., the weight function is increasing or, in extreme cases, constant). In this article, the growing function [5] setting the weight of LSM errors was as follows:

$$w(t) = t^{\alpha}, \ t > 0, \alpha > 0. \quad (9)$$

Different experiments were performed at different weight values α (when $\alpha > 1$ function (9) is convex and when $\alpha < 1$ – concave). More precisely, instead of the weight function (7), the weights were used:

$$w_i = w(t_i) = \left(\frac{t_i - t_0}{t_k}\right)^{\alpha}, \ t_i > 0, \alpha > 0, i = \overline{0,..k}. \quad (10)$$

One advantage of using (10) as a weighting function is that each error is weighted in the interval [0,1]. The error furthest in time is given a weight of 0, and the error closest (last) is given a weight of 1. The choice of weighting function is subjective, and the analyst may choose another, largely arbitrary weighting function. It is recommended that the function increases (errors closer to the prediction time are more valuable). To find the coefficients (7) by LSM, a nonlinear system of equations must be solved with respect to these coefficients. For systems of nonlinear equations, it is not possible to determine in advance the number of different solutions (it is also possible that the system has no solution). Most numerical methods (based on the iterative principle) for solving the nonlinear system of coefficients (7) need an initial approximation. With different choices of initial approximations, different solutions for the coefficients of (7) are usually also obtained. Different solutions of (7) would also lead to different predictions. In this paper, the option chosen is to generate a pseudo-random set of L number of different initial approximations. For each value, the interval in which it lies was preselected. Different solutions were found. One of these solutions was selected from the time series by choosing l values last and setting them aside to construct an error and validate the selected solution of (7). We will call these values "validation values." The next value was predicted and compared with the actual value. The real value was included in the time series. The value of the error between the actual and predicted value was reported sequentially for the selected l time series values. The final error was calculated as a weighted average of the obtained l errors. An increasing discrete function was chosen for the weight function of these l errors. After a particular choice of M_1, M_2, N_1, N_2 and α, the solution of (7) with the smallest final error was taken to construct (8). The future values of x_{n+1} and y_{n+1} in the moment t_{n+1}. were then predicted. Given the known moments and time series values at those moments (1), the following approach was applied to predict the next time series value at a future moment:

1) The time series compared a system of two ordinary differential equations with initial conditions (2) (Cauchy problem).
2) For functions $g_1(t,y)$ and $g_2(t,y)$, type (4) was selected, fixing the parameters M_1, M_2, N_1 and N_2.
3) For the numerical solution of (2), a specific numerical explicit-implicit method was chosen from the one-step method of Euler, Milne, Runge-Kuta and others. In the present work, Euler's explicit-implicit method was chosen (11):

$$\begin{cases} x_{i+1} = x_i + \left(\frac{h_k}{2}\right)\left(g_1(t_i, x_i, y_i) + g_2(t_{i+1}, x_{i+1}, y_{i+1})\right) \\ y_{i+1} = y_i + \left(\frac{h_k}{2}\right)\left(g_1(t_i, x_i, y_i) + g_2(t_{i+1}, x_{i+1}, y_{i+1})\right) \\ \qquad x(t_0) = x_0, \ y(t_0) = y_0 \end{cases} \tag{11}$$

4) The last k values of the time series (1) ($n \geq k > 2N + M + 4$) were selected.
5) The inverse problem with the unknown coefficients was solved (6). The solution was reduced to solving an overdetermined system of nonlinear equations.

5.1) A method was selected to solve the overdetermined system of nonlinear equations. This could be, for example, the LSQ method or the minimax method.

5.2) A weight function is chosen to apply this method (the weight function is nondecreasing. In this particular paper, the form is (10)).

5.3) After applying the selected method (in this paper, it was the LSQ method) with the selected weight function, the redefined system reduced to a nonlinear system of equations with unknown coefficients (7).

5.4) An iterative numerical method was applied to solve the system. A set of L different pseudo-random initial approximations necessary to find the coefficients (7) was generated. Different solutions for (7) were obtained.

6) T the last l values of (1) were subtracted to compute errors obtained as differences between the predicted values and the actual values. In this way, the chosen solution of (7) was validated. The final error was computed as a weighted average of the l errors obtained. Here, again, the weights increased.
7) The solution of (7) that has the smallest final error was selected.
8) The chosen solution (7) was replaced in (8) at k=n, and the future values of x_(n+1) and the values of y_(n+1) at time t_(n+1) were predicted.
9) Steps (1)-(8) were repeated with different choices: the numerical method implementing the solution of the initial Cauchy problem, the parameters M_1,M_2,N_1,N_2,k,l,L, the numerical method for solving the overdetermined system of nonlinear equations, the weight function for solving the preset system of nonlinear equations and the weight function for estimating the final error.

10) The various estimates were obtained, along with their final errors. A weighted average, whose weights are inversely proportional to their terminal errors, was taken as the final prediction.

3. EMPIRICAL ANALYSIS

Based on the average weekly values of the levels of PM10 and PM2.5 particles from the period 01.04.2017 to 17.04.2022, various forecasts have been made for their average value in the next week (i.e., from 18.04.2022 to 24.04.2022). The explicit-implicit Euler method for the numerical solution of the Cauchy problem (2) was applied. For some days or weeks, there was no information. This is not a serious problem, as in the numerical method used, the step h_k was variable. The redefined system of nonlinear equations was solved by weight LSM, with weight function (10). For each specific choice of parameters M_1, M_2, N_1 *and* N_2 and certain choices of α in (10) $L = 40$ different pseudo-random initial approximations were made in the range $[-50;\ 50]$ to determine the coefficients (7). The last $l = 6$ data were left for validation, and the weights given when calculating the final error were $\frac{2}{13.5}; \frac{2.1}{13.5}; \frac{2.2}{13.5}; \frac{2.3}{13.5}; \frac{2.4}{13.5}; and\ \frac{2.5}{13.5}$, respectively. The tables below show the values of the predictions and the corresponding final errors for specific values of the parameters $M_1, M_2, N_1, N_2\ and\ \alpha$. For brevity, the values of the parameters M_1, M_2, N_1, N_2 were presented as a quadruple in the same order.

From Tables 7 and 8, after averaging with weights inversely proportional to the final errors, for the predicted value of the particles PM10 and PM2.5 from 18.04.2022 to 24.04.2022, the obtained values were $x = 36.9294 \frac{\mu g}{m^3}\ and\ y = 10.1304\ \mu g/m^3$, respectively.

Table 7

Predicted values by parameters $M_1, M_2, N_1, N_2\ and\ \alpha = 1.25$

Value $[M_1, M_2, N_1, N_2]$	Estimated value for PM10 [μg/m³]	Estimated value for PM2.5 [μg/m³]
[1,1,4,4]	38.4792	8.9977
[2,2,6,6]	38.5048	9.7828
[1,3,5,5]	35.3482	10.1468
[3,1,5,5]	36.8538	10.5304
[2,2,6,8]	34.7874	10.8409
[2,2,8,6]	34.9484	9.8202

Table 8

Final errors for the predicted values, for parameters $M_1, M_2, N_1, N_2\ and\ \alpha = 1.25$

Value $[M_1, M_2, N_1, N_2]$	"Final errors" for PM10	"Final errors" for PM2.5
[1,1,4,4]	2.7459	1.0024
[2,2,6,6]	0.3637	2.1298
[1,3,5,5]	2.4617	0.0361
[3,1,5,5]	0.4448	0.6379
[2,2,6,8]	1.1037	1.0287
[2,2,8,6]	0.9245	0.6466

From Tables 9 and 10, after averaging with weights inversely proportional to the final errors, for the predicted values of PM10 and PM2.5 from 18.04.2022 to 24.04.2022, the obtained values were $x = 37.4500 \frac{\mu g}{m^3}$ and $y = 9.2838 \frac{\mu g}{m^3}$, respectively.

Table 9

Predicted values for M_1, M_2, N_1, N_2 and $\alpha = 1$

Value $[M_1, M_2, N_1, N_2]$	Estimated value for PM10 [μg/m³]	Estimated value for PM2.5 [μg/m³]
[1,1,4,4]	41.6736	11.1295
[2,2,6,6]	38.1329	9.6047
[1,3,5,5]	39.0743	8.0493
[3,1,5,5]	34.9311	9.0883
[2,2,6,8]	35.9334	10.8437
[2,2,8,6]	36.2668	8.9023

Table 10

"Final errors" for the predicted values, for parameters M_1, M_2, N_1, N_2 and $\alpha = 1$

Value $[M_1, M_2, N_1, N_2]$	"Final errors" for PM10	"Final errors" for PM2.5
[1,1,4,4]	2.3649	1.5716
[2,2,6,6]	2.0091	0.1599
[1,3,5,5]	0.8763	0.5370
[3,1,5,5]	1.1554	0.1409
[2,2,6,8]	1.2683	1.6189
[2,2,8,6]	1.9607	0.8330

From Tables 11 and 12, after averaging with weights inversely proportional to the final errors for the predicted value of PM10 and PM2.5 from 18.04.2022 to 24.04.2022, the obtained values were $x = 36.5479 \frac{\mu g}{m^3}$ and $y = 9.7958$ μg/m^3, respectively.

Table 11

Predicted values, by parameters M_1, M_2, N_1, N_2 and $\alpha = 0.75$

Value $[M_1, M_2, N_1, N_2]$	Estimated value for PM10 [μg/m³]	Estimated value for PM2.5 [μg/m³]
[1,1,4,4]	38.5973	10.4807
[2,2,6,6]	36.2576	9.8530
[1,3,5,5]	37.9226	10.8375
[3,1,5,5]	33.2419	8.2893
[2,2,6,8]	36.1285	9.6117
[2,2,8,6]	35.3299	8.8541

For the final forecast, the three obtained values were averaged, resulting in the final forecast of $36.9758\ \mu g/m^3$ for PM10 and $9.7367\ \mu g/m^3$ for PM2.5. These values were projected as weekly averages for the week of 18.04.2022 to 24.04.2022. The actual values of the particulate matters for the period from 18.04.2022 to 24.04.2022 are given in Table 13.

By averaging the data from Table 7, the actual values of PM10 and PM2.5 for the period under consideration were obtained. Their real average values for the week under review were $36.7286\ \mu g/m^3$ for PM10 and $9.7571\ \mu g/m^3$ for PM2.5. It can be seen that these values are very close to the predicted weekly averages. The relative error was less than 1%. Experiments were performed with other values of the parameters M_1, M_2, N_1, N_2 and α. The results obtained were similar to the results presented here.

Table 12

Final errors for the predicted values for parameters M_1, M_2, N_1, N_2 and $\alpha = 0.75$

Value $[M_1, M_2, N_1, N_2]$	Final errors for PM10	Final errors for PM2.5
[1,1,4,4]	2.0267	1.8719
[2,2,6,6]	0.7741	0.9536
[1,3,5,5]	0.6656	0.4703
[3,1,5,5]	3.0551	0.9093
[2,2,6,8]	0.3467	1.2720
[2,2,8,6]	2.9736	1.1220

Table 13

Real values of PM10 and PM2.5 for the period from 18.04.2022 to 24.04.2022

Particle\ date	18.04.22	19.04.22	20.04.22	21.04.22	22.04.22	23.04.22	24.04.2022
PM10 [µg/m³]	28.6	28.10	32.2	36.8	42.3	48.2	40.90
PM2,5 [µg/m³]	9.04	7.22	8.78	14.28	13.55	7.98	7.45

4. CONCLUSION

One of the applied solutions in modern cities to deal with the problems caused by road transport is the introduction of LEZs. There is still no city in Bulgaria with LEZs. The authors are actively working on the introduction of LEZs in Sofia, Plovdiv and Ruse. A methodology has been developed for the introduction of an LEZ in Ruse, which envisages its establishment in the central part of the city. There is a gratis period for the introduction of the first restriction of two years. In 2032, the process of building the low-emission zones is expected to be completed.

The proposed LEZ in the city of Ruse will limit the year-round entry of cars with internal combustion engines using liquid gasoline and diesel fuel starting in 2032. Thus, only electric vehicles will be allowed to enter the zone to seriously reduce car emissions in the central part of the city.

With the applied modified numerical method for a system of ordinary differential equations, a forecast for one week ahead for the period 04/18/2022 to 04/24/2022 was obtained with the following average values: 36.9758 for PM10 and 9.7367 for PM2.5. For this period, the real average values of PM10 and PM2.5 particles were 36.7286 for PM10 and 9.7571 for PM2.5. These values are very close to the predicted weekly averages, as the relative error is less than 1%.

The validation of the method used for the numerical solution of the initial Cauchy problem for a two-dimensional ordinary differential equations system of the first order confirms the good applicability of the method for forecasting PM10 and PM2.5.

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