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THE INFLUENCE OF THE «TRAIN-TRACK» SYSTEM PARAMETERS ON THE MAXIMUM LONGITUDINAL FORCES' LEVEL

Summary. The main objective of the simulation is to study the effect caused by the parameters of the longitudinal profile on the maxima of longitudinal forces in freight trains of increased length during adjustment braking and running-out. To decrease the number of numerical experiments, some empirical formulae for estimating the maximum longitudinal forces during the motion of freight trains along the track with various configurations of its longitudinal profile's gradient changes have been obtained for the first time. Comparison of those forces with the permitted values, from the point of view of the railway stock strength and eventual vehicle derailment, has been performed. As a result of numerical integration of the system of non-linear differential equations of train motion for the considered driving modes, the values of the greatest longitudinal shock and quasi-static forces, as well as the dependence of the latter on the train length, initial braking velocity, on the algebraic difference of gradients and the length of the horizontal area that separates two gradients with opposite signs are estimated. The proposed mathematical model and methodology can be applied during standardization of the longitudinal profile's parameters from the point of view of the freight traffic safety for the trains of various length.

1. INTRODUCTION

To secure vehicles' safety, it is necessary to estimate maximum values of longitudinal forces within a train at various transient modes of motion on difficult sections of the longitudinal profile, as well as to compare those values with those allowed from the point of view of rolling stock elements' strength and vehicles' derailment [1].

At the same time, to increase the railway network capacity and to decrease the maintenance costs, the weight norms for freight trains are being revised in many countries. Longitudinal forces, dangerous from the running safety point of view, arise in the trains of increased mass and, especially, of increased length [2 - 4]. Thus, to provide the permissible level of longitudinal forces during braking (the most dangerous running mode), the coordinated control of locomotives is necessary. Research on the long trains' longitudinal dynamics during braking using mathematical modelling was performed in some articles [5 - 7]. The obtained results are used both in design of new absorbing devices and air distributors [8], as well as in the development of the new ways of traffic control for conventional and coupled trains [5] and energy-consuming modes of train running control [6, 7].

In real conditions, perturbations of the train caused by track gradient changes often are superimposed by perturbations arising from train driver' actions, such as traction climb or drop, or brakes release or actuation [9 - 12].

Analysis of the experimental and theoretical data shows that the greatest compressive longitudinal shock forces arise when a stretched train is entering a section of the concave profile, and the greatest tensile shock forces arise when a compressed train is entering a section of a convex profile. The specific initial states of clearances in the inter-vehicle connections can be predetermined by previous actions of the train driver aimed at controlling train movement, or (more likely) by the previous part of the track profile [13].

As shown in a paper [13], among the two cases of a train moving along the sections of a track with a concave or convex profile outline, only one is enough to be considered, as another one's solution differs only with sign. In the same paper, it was noted that, in a statistically non-uniform (heterogenous) train, the highest expected force values, estimated by the rule 3σ, exceed the corresponding force values in a uniform (homogenous) train of the same mass by no more than twenty per cent. Therefore, the highest force values in a uniform train as it moves along a section of a concave profile were evaluated.

Of the possible train management issues, the most dangerous one, from the point of view of the occurrence of the compressive longitudinal forces' maxima, is the case of adjustment braking, when a stretched train enters a section of concave profile. The occurrence of the tensile longitudinal forces' maxima is possible when releasing brakes after a compressed train enters a section of convex profile. As shown in Blochinas et al. [13], the compressive and tensile forces maxima, caused by shock loading during braking and releasing brakes are approximately the same. Therefore, hereon we will consider only adjustment braking.

As it is known, longitudinal shock forces arise in those transient processes, the flow of which is influenced by clearances in the inter-vehicle joints. If the clearances do not affect these processes, then the resulting longitudinal forces are close to quasi-static. The magnitude of these forces determines the wheel sets' stability against derailment [14 - 16].

Thus, in our case, to determine the maxima of compressive longitudinal shock forces, we investigate the mode of adjustment braking when a stretched train is entering a section of a concave track, and to determine the maxima of quasi-static compressive forces, we deal with the same mode but when a compressed train is entering a section of a concave profile.

Calculations for trains were made taking into account all their real-world characteristics (mass; vehicle parameters, i.e., bearings type, axles number, pneumatic systems operation mode, length, and pressing type (one- or two-way); power characteristics of inter-vehicle connections; braking systems characteristics; etc.) for the given initial velocities and control modes (adjustment braking with release of the brake line by 0.08 MPa as well as running-out). The integral values of the braking system and inter-vehicle connections parameters were taken based on the results of many years of numerous experimental experiments conducted in the Dnipro National University of Railway Transport with trains of different lengths in real-life conditions on a number of railways in Ukraine and Russian Federation.

In order to decrease the number of numerical experiments, it makes sense to develop an empirical formula for evaluation of the maximal compressing forces depending on the longitudinal track profile parameters for the trains of various length during adjustment braking.

2. METHODOLOGY

In the railroad design practice, an adjustment curve is usually replaced by a respective circumscribed polygon, the sides' length of which equals to that of the profile's straight section length L with the gradient difference between adjacent elements of Δi (the sides of the polygon). While conducting numerical experiments, the largest gradient differences on a local profile and the smallest length values of straight profile section, at which longitudinal forces' maxima do not exceed the values allowed, were determined.

We will assume that the gradient change of the concave profile is formed by two gradients of different signs, separated by a horizontal zone (i.e. with zero gradient). Thus, the algebraic difference

of gradients ranged from 0.01 to 0.02 with increment of 0.005. The length of the horizontal zone separating two gradients is to be ranged from 200 m to 350 m, with 50-m increment.

In the study of adjustment braking, it was assumed that freight cars are equipped with air distributors with conditional No.483 switched to medium operational mode, composite shoes and spring-frictional shock-absorbing devices. We consider the trains of normal length of 850 m, 1050 m, and 1250 m, consisting respectively of 60, 75, and 90 four-axle open-goods wagons of a 14-meter length, and mass of 100 tonnes, with a locomotive placed at the head of the train. The rotating masses' inertia coefficients have been taken into account during estimation of the longitudinal forces. The paper deals with the homogenous train with fully loaded wagons, so coupler slacks have not been taken into account. The initial velocities were assumed to be $V = 60, 80, 100$ km/h.

As a computational scheme for mathematical modelling of train motion, a one-dimensional chain of solid bodies (vehicles) connected by essentially non-linear deformable elements, which allow to take into account clearances in inter-vehicle connections, was developed [17, 18]. Braking forces, the forces of main resistance to the train's forward motion, as well as the resistance forces from the track profile affecting each vehicle, were considered as the external forces.

The magnitude of the braking force affecting each vehicle depends on the braking mode (adjustment braking level, full adjustment braking or emergency braking), location of the vehicle in the train, velocity and the brake shoe type, and is therefore determined as follows:

$$B_j = -m_j \cdot c_j \cdot k_j(t) \cdot \varphi_j \cdot f_{\mathrm{c}} \tag{1}$$

where m_j – is the number of brake shoes in the j-th vehicle (either a locomotive or a wagon) driven by one brake cylinder; c_j – is the number of brake cylinders in that j-th vehicle; φ_j – is the friction coefficient between the brake shoe and the wheel which depends on the brake shoes type, brake shoes pressure force applied to the wheel, and velocity [19, 20]; $k_j(t)$ – is the pressing force applied to each brake shoe ; f_{c} – is a coefficient depending on weather conditions and presence of sand in the locomotive and taking into account changes in the wheel - rail cohesion; and j – is the vehicle number.

Coefficient of the brake shoes friction against wheel is determined by the formula [19]:

$$\varphi_j(t) = d_1 \frac{k_j(t) + d_2}{k_j(t) + d_3} \cdot \frac{V_j(t) + d_4}{V_j(t) + d_5} \tag{2}$$

where the coefficients d_1, d_2, d_3, d_4, d_5 – are depending on brake shoes type [19].

The number of brake shoes for each vehicle is determined upon its main characteristics – number of axles and pressing type (one- or two-way).

The pressing force applied to the brake shoe depends on the mode of operation of the air diffuser (empty, medium, or loaded) and takes into account that the brake cylinders' filling rate is slowing down the further away they are from the brake line release source.

The pressing force applied to each brake shoe is calculated for each vehicle using the following formula:

$$k_j(t) = \frac{1}{m_j} A_{cj} \cdot P_j(t) \cdot n_j \cdot \eta_{nj} \tag{3}$$

where m_j – is the number of brake pads in a j-th vehicle (either a locomotive or a wagon), which are driven by one brake cylinder; A_{cj} – is the brake cylinder's piston surface area; P_j – is the compressed air pressure in the cylinder; n_j – is the gear ratio of the lever transmission; and η_{nj} – is the lever transmission efficiency factor. The function P_j depends on the air distributor's type and braking mode. The dependencies P_j correspond to air distributors with conditional No.483, which were obtained for trains of various lengths using pressure sensors installed in different train sections, during

numerous experiments conducted with trains of different lengths in real conditions on a number of railways in the Ukraine and Russian Federation.

The brake cylinders filling characteristics depend on the brake line release amount. The more the brake line release, the higher is the pressure in a brake cylinder. In order to decrease the running velocity, one usually uses the release by 0.07 - 0.08 MPa. The brake cylinders filling characteristics obtained from experimental data, taking into account all the stochastic factors namely for such braking modes, were used for the modelling of those processes.

In modelling the braking process, it was taken into account that the air distributor on each vehicle is activated when the braking wave from the source of release of the brake line (locomotive) reaches it. The change in pressing force applied to the brake shoe during filling in the brake cylinder is described by a piecewise-linear law. While determining the pressing force, the brake shoes approaching stage is taken into account, and the time for filling in the brake cylinders corresponds to the adjustment braking [20].

As an example, the dependencies of the braking force in the leading wagon on the running velocity and the running time, obtained as a result of mathematical modelling of the 60-wagon train during its full service braking from the 40 km/h on a horizontal track section, have been shown in the Figures 2 and 3 respectively.
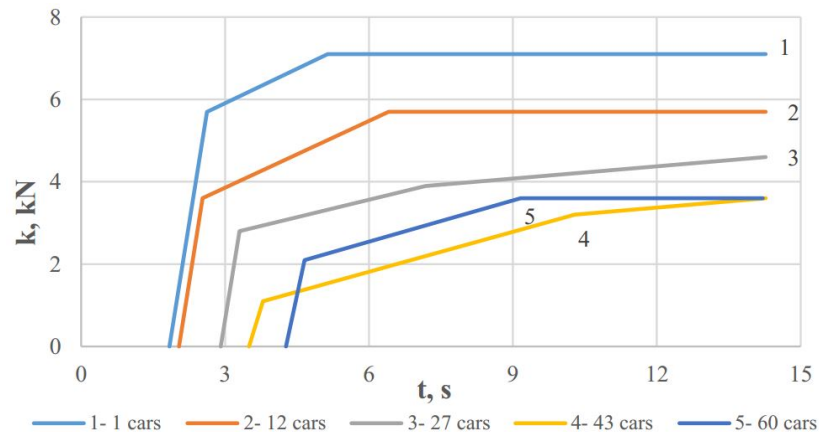

Fig. 1. Dependence of the pressing forces on one brake shoe upon time for the different wagons in a 60–wagon train during the adjustment braking with release of the brake line by 0.7 MPa

Here the lines 1, 2, 3, 4, and 5 correspond to the wagons with sequence numbers 1, 12, 27, 43, and 60 in the train. As one can see, braking in the different wagons begins at the moment of reaching the wagon by the brake line's release wave and after approaching the wheel by the brake shoe. The braking wave velocity was set to 280 m/s. The time of approaching the wheel by the brake shoe varied between 1.4 s. and 1.8 s.

As an example, the dependencies of the braking force in the leading wagon on the running velocity and the running time during the full service braking from the 40 km/h have been shown in the Figures 2 and 3, respectively.
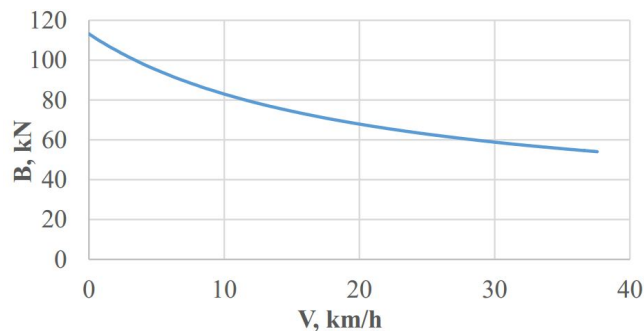

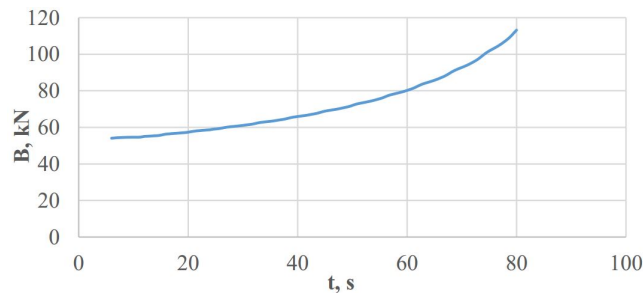Fig. 2. Dependence of the braking force in the leading wagon upon the running velocity

Fig. 3. Dependence of the braking force in the leading wagon upon the running time

The main resistance force to the train's forward motion and the resistance forces from the track profile affecting each vehicle were determined using the known formulas [19], which take into account the axle load, the type of vehicles and bearings, gradient value, plane curve value, and velocity.

3. SOME RESULTS

To obtain the highest longitudinal forces' values, the search for the place on the concave section of the longitudinal profile, which is most unfavourable for switching on the automatic brakes, was carried out first.

The dependence between the greatest longitudinal shock forces' values and the point of switching on the automatic brakes at a velocity of 100 km/h on a concave section of a longitudinal profile with a gradient of 0.015 and a horizontal separation area with length of 250 metres (see Fig. 5), for a train consisting of 60 wagons, is shown in Fig. 4. Here $x0$ is the distance between the start of the transitional (horizontal) area and the place where braking begins.
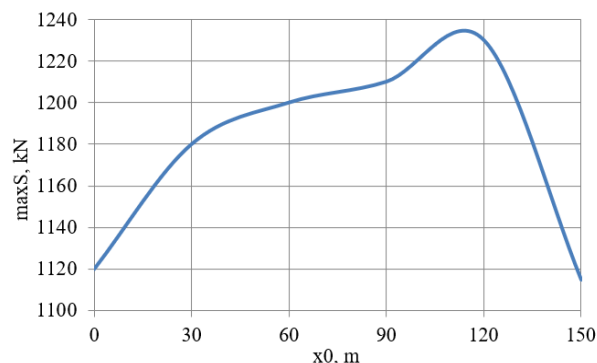

Fig. 4. Dependence of the greatest longitudinal shock forces' values from the point of switching on the automatic brakes
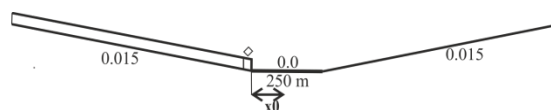

Fig. 5. A concave track section scheme

It was necessary to perform up to five iterations to find the magnitude of the highest compressive shock force. To reduce the number of numerical experiments, the empirical dependencies between forces' maxima and the point of switching on the automatic brakes on a concave profile were obtained.

To achieve this, the compressive shock and quasi-static forces' maxima were obtained for stretched and compressed freight trains respectively, of the aforementioned lengths, running out along concave sections of the profile with the parameters indicated before.

Some modelling results can be seen in Table 1.

Adjustment braking from velocities of $V = 60$ km/h, 80 km/h and 100 km/h, of pre-stretched and pre-compressed freight trains of different length on a horizontal track section are also considered. Fig. 6 shows the dependencies of the highest longitudinal forces in trains of various length during adjustment braking at the horizontal area on the initial train's velocity. Here, the dotted lines correspond to the values of the longitudinal shock forces' maxima, whereas the solid lines correspond to the values of the longitudinal quasi-static forces' maxima. Lines 1, 2, and 3 correspond to trains consisting of 60, 75, and 90 cars, respectively.

Table 1

Longitudinal compressive forces' maxima (kN) in stretched (numerator) and compressed (denominator) freight trains of different length during running-out, for various algebraic difference of gradients and various initial velocities

| Δi	V, km/h	60 Cars				75 Cars				90 Cars			
		$L=200$m	$L=250$m	$L=300$m	$L=350$m	$L=200$m	$L=250$m	$L=300$m	$L=350$m	$L=200$m	$L=250$m	$L=300$m	$L=350$m
0.01	60	$\frac{540}{250}$	$\frac{525}{240}$	$\frac{520}{230}$	$\frac{520}{205}$	$\frac{625}{380}$	$\frac{585}{364}$	$\frac{560}{340}$	$\frac{560}{320}$	$\frac{740}{830}$	$\frac{740}{810}$	$\frac{660}{780}$	$\frac{620}{735}$
	80	$\frac{656}{255}$	$\frac{615}{245}$	$\frac{580}{230}$	$\frac{575}{210}$	$\frac{725}{390}$	$\frac{680}{375}$	$\frac{640}{345}$	$\frac{610}{325}$	$\frac{830}{850}$	$\frac{805}{835}$	$\frac{760}{820}$	$\frac{735}{790}$
	100	$\frac{735}{260}$	$\frac{690}{250}$	$\frac{660}{240}$	$\frac{630}{220}$	$\frac{805}{390}$	$\frac{775}{380}$	$\frac{730}{348}$	$\frac{700}{325}$	$\frac{900}{835}$	$\frac{865}{835}$	$\frac{830}{830}$	$\frac{790}{805}$
0.015	60	$\frac{630}{380}$	$\frac{615}{350}$	$\frac{600}{340}$	$\frac{600}{310}$	$\frac{705}{530}$	$\frac{660}{505}$	$\frac{660}{480}$	$\frac{660}{475}$	$\frac{830}{1050}$	$\frac{780}{1005}$	$\frac{750}{950}$	$\frac{720}{890}$
	80	$\frac{750}{375}$	$\frac{705}{365}$	$\frac{680}{340}$	$\frac{675}{320}$	$\frac{815}{540}$	$\frac{775}{515}$	$\frac{730}{480}$	$\frac{730}{465}$	$\frac{965}{1130}$	$\frac{930}{1120}$	$\frac{900}{1090}$	$\frac{840}{1025}$
	100	$\frac{840}{390}$	$\frac{800}{355}$	$\frac{745}{350}$	$\frac{745}{330}$	$\frac{930}{540}$	$\frac{880}{510}$	$\frac{830}{475}$	$\frac{790}{460}$	$\frac{1065}{1190}$	$\frac{1030}{1175}$	$\frac{980}{1090}$	$\frac{920}{1025}$
0.02	60	$\frac{690}{500}$	$\frac{682}{460}$	$\frac{685}{440}$	$\frac{685}{410}$	$\frac{775}{685}$	$\frac{750}{640}$	$\frac{750}{625}$	$\frac{750}{615}$	$\frac{930}{1255}$	$\frac{870}{1185}$	$\frac{830}{1105}$	$\frac{830}{1060}$
	80	$\frac{823}{495}$	$\frac{766}{470}$	$\frac{770}{450}$	$\frac{770}{415}$	$\frac{906}{685}$	$\frac{850}{645}$	$\frac{830}{600}$	$\frac{830}{585}$	$\frac{1075}{1370}$	$\frac{1000}{1315}$	$\frac{960}{1245}$	$\frac{930}{1180}$
	100	$\frac{932}{510}$	$\frac{880}{475}$	$\frac{840}{455}$	$\frac{840}{430}$	$\frac{1035}{710}$	$\frac{980}{660}$	$\frac{915}{620}$	$\frac{905}{615}$	$\frac{1250}{1370}$	$\frac{1200}{1330}$	$\frac{1110}{1275}$	$\frac{1050}{1215}$
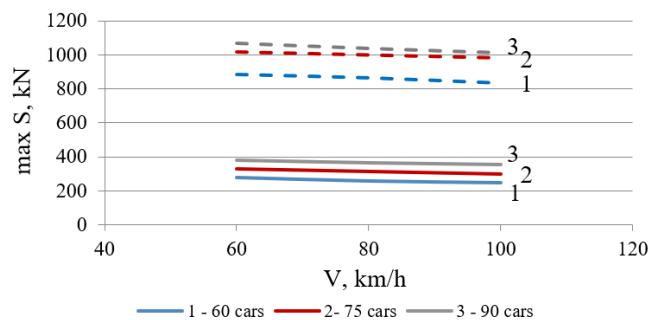
Fig. 6. Dependence between the initial train's velocity and longitudinal shock (dotted lines) and quasi-static (solid lines) forces' maxima during adjustment braking on a horizontal area

Using the data provided in Table 1 as well as the dependencies shown in Fig. 6, the following empirical formulae for determining the maxima of compressive longitudinal shock (4) and quasi-static (5) forces during adjustment braking on the concave sections of the profile were obtained:

$$\max S_{brake\,shock}^{prof} = 0.5\max S_{runout\,shock}^{prof} + \max S_{brake\,shock}^{flat} \tag{4}$$

$$\max S_{brake\,quasistatic}^{prof} = \max S_{runout\,quasistatic}^{prof} + \max S_{brake\,quasistatic}^{flat} \qquad (5)$$

where $\max S_{runout\,shock}^{prof}$ and $\max S_{runout\,quasistatic}^{prof}$ stand for the maxima of compressive forces of a shock and quasi-static nature, respectively, caused in a running-out train by the profile change, and $\max S_{brake\,shock}^{flat}$ and $\max S_{brake\,quasistatic}^{flat}$ stand for similar forces caused by adjustment braking at a horizontal area.

As an example, the dependencies of the maxima of longitudinal compressive shock forces $\max S_{Shock}$ during adjustment braking of a 60-car train from the initial speed over the profile gradient change of 0.015 are shown in Fig. 7. Line 1 corresponds to the case of the horizontal separating area length being taken at 200 m, whereas line 2 corresponds to the case of the separating area of 250 m length. Every $\max S_{Shock}$ value in this figure corresponds to the most unfavourable case of superimposing the disturbance caused by the train run control over disturbance caused by the profile gradient change.

In this figure, solid lines are obtained as a result of numerical integration of the system of non-linear differential equations of train motion, and dashed lines are obtained using the empirical formula (4). As one can see from Fig. 7, the corresponding values of the forces (the solid and dashed lines) differ by no more than 14%. This allows us to use the aforementioned empirical formula for the approximate estimation of the greatest longitudinal shock forces in trains during adjustment braking on track sections of a broken profile.
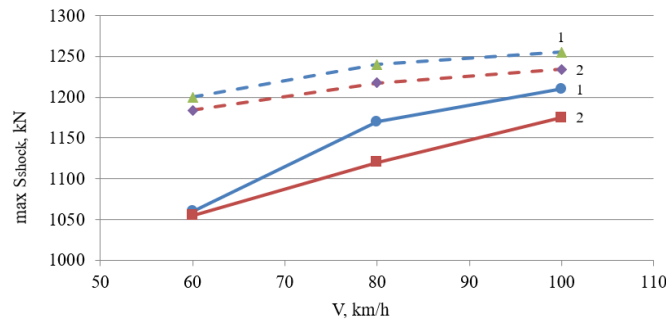

Fig. 7. Dependencies between the maximal longitudinal shock forces and the velocity of a 60-car train moving along the track with 0.015 gradients' difference

The dependencies between the compressive quasi-static forces' $\max S_{quasistatic}$ and velocity during adjustment braking on a concave track section with parameters $\Delta i = 15\,‰$ and $L = 250$ metres are shown in Fig. 8. Here, each $\max S_{quasistatic}$ value corresponds to the most unfavourable case of superimposing the disturbance caused by the train run control over disturbance caused by the profile gradient change.
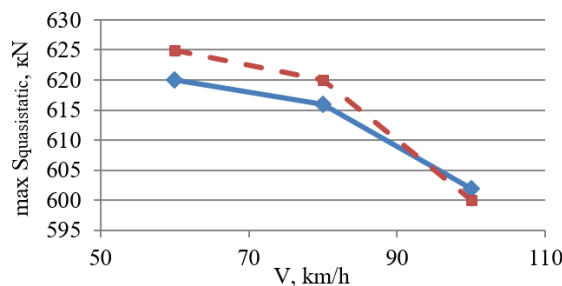

Fig. 8. Dependencies between the maximal compressive quasi-static forces and the velocity during adjustment braking on a concave track section

In this figure, solid lines represent results obtained after numerical integration of the system of non-linear differential equations of train motion, and dashed lines represent those obtained using the empirical formula (5). As one can see, the corresponding values of the forces represented by solid and dashed lines differ by no more than 5%. This allows us to use the obtained empirical formula for an approximate estimation of the maximal quasi-static longitudinal forces in trains during adjustment braking on track sections of a broken profile.

The maxima of longitudinal compressive shock and quasi-static forces during adjustment braking for the trains of different length on a concave track section with various parameters, obtained by formulae (4) and (5) are provided in Table 2.

Table 2

Longitudinal compressive shock (numerator) and quasi-static (denominator) forces' maxima (kN) in freight trains of different length during adjustment braking, for various algebraic difference of gradients and various initial velocities

Δi	V, km/h	60 Cars				75 Cars				90 Cars			
		$L=200$m	$L=250$m	$L=300$m	$L=350$m	$L=200$m	$L=250$m	$L=300$m	$L=350$m	$L=200$m	$L=250$m	$L=300$m	$L=350$m
0.01	60	540/540	520/515	520/510	520/485	625/710	585/694	560/670	560/650	740/1210	700/1190	660/1160	620/1115
	80	656/515	615/510	580/490	575/470	725/705	680/690	640/660	610/640	830/1215	800/1200	765/1185	735/1155
	100	735/510	690/490	660/490	630/470	805/690	775/680	730/648	700/625	900/1190	865/1190	830/1185	790/1160
0.015	60	630/660	598/630	600/620	600/590	705/865	660/835	660/810	660/805	830/1430	780/1385	750/1330	720/1270
	80	750/635	705/625	680/600	675/565	815/655	775/830	730/795	730/780	965/1495	930/1485	900/1455	840/1390
	100	842/640	800/605	745/600	745/580	930/835	880/810	830/775	790/760	1095/1445	1030/1430	980/1405	920/1365
0.02	60	690/780	682/740	685/720	685/690	775/1015	750/970	750/955	750/945	930/1635	870/1565	830/1485	830/1440
	80	823/755	766/735	770/700	770/665	906/1000	850/960	830/915	830/900	1075/1735	1000/1680	960/1610	930/1545
	100	932/760	880/715	840/705	840/685	1035/1010	980/955	915/920	905/915	1250/1720	1200/1685	1110/1630	1050/1570

As it is known, the permissible value of the compressive shock longitudinal force equals 2500 kN, and the value of the compressive quasi-static forces for loaded wagons should not exceed 1000 kN [21]. While analysing the data provided in Table 2, one can say that adjusting braking of compressed 90-cars trains on a concave section of the track with a gradients' difference $\Delta i = 0.010$ or more can lead to derailment of wagons, as the greatest compressive forces exceed the permissible value of 1000 kN.

Below, as an example, the dependencies of the longitudinal shock forces' maxima for various parameters of the track with longitudinal concave profile during adjustment braking of the trains of different length are shown in Figs. 9–11.

Fig. 9 shows the dependencies between the maxima of compressive shock forces $\max S_{shock}$ and the separation area length for a 90-car train during its adjustment braking from 80 km/h velocity. In this figure, lines 1, 2 and 3 correspond to the gradients of $\Delta i = 0.010$, 0.015 and 0.020, respectively.

The calculations show that the longitudinal shock forces' maxima decrease with an increase of the length of the straight area separating gradients of opposite signs and with decrease of the gradients' algebraic difference.
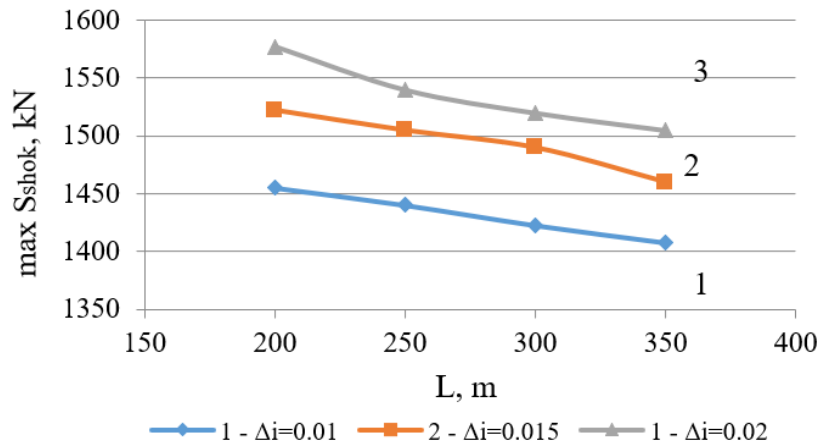
Fig. 9. Dependence between the maxima of compressive shock forces and the separation platform length for a 90-car trains during adjustment braking from 80 km/h velocity

Fig. 10 shows the dependencies between the maxima of longitudinal shock forces $\max S_{shock}$ and the algebraic difference of the gradients with a separation area length of 250 metres for trains of different length during adjustment braking from an initial velocity of 80 km/h. Here, lines 1, 2, and 3 correspond to trains of 840 metres (60 cars), 1050 metres (75 cars) and 1250 metres (90 cars) length, respectively.
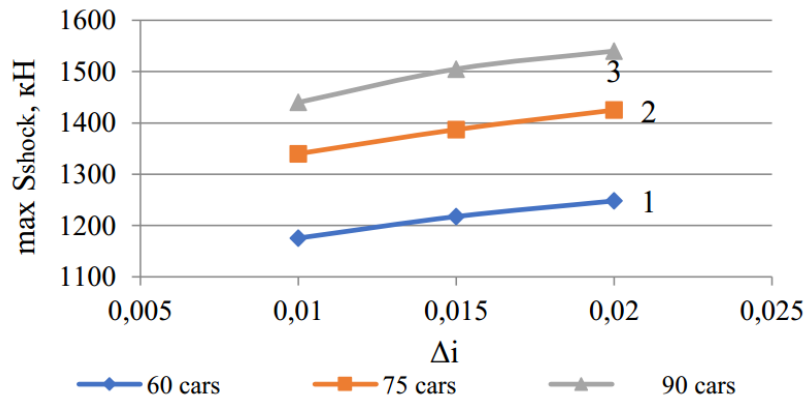

Fig. 10. Dependence between the maxima of shock forces and the gradients' algebraic difference during adjustment braking with a 250-m separation area, for trains of different lengths.

Fig. 11 represents the longitudinal shock forces' maxima $\max S_{shock}$ depending on the gradients' algebraic difference and initial velocity during adjustment braking of a train 1050-metres long (75 cars). In this figure, lines 1, 2 and 3 correspond to initial velocities of 60 km/h, 80 km/h, and 100 km/h, respectively.
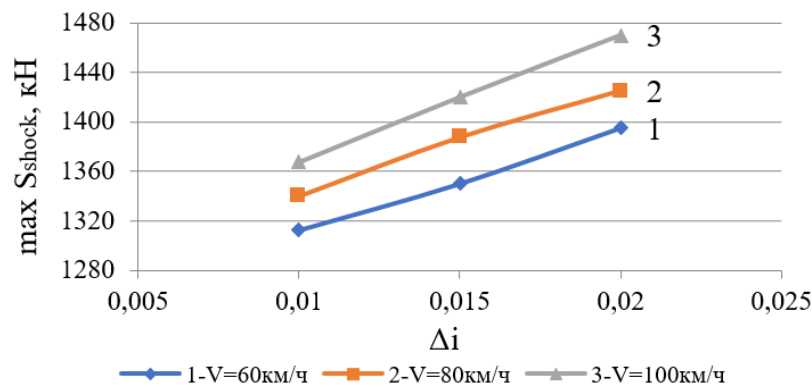
Fig. 11. Dependence between the maxima of longitudinal forces and the gradients' algebraic difference during adjustment braking of a 75-car train for various initial velocities

The results shown in Figures 10 and 11 can be used for the determination of the longitudinal track profile's permissible parameters (maximal difference of gradients, maximal separation area length) from the running safety point of view for the trains of various length.

4. CONCLUSIONS

With the help of mathematical modelling, it has been shown that the longitudinal forces' maxima level depends on the place on a concave section of the track, where the automatic brakes were turned on.

Empirical formulae have been obtained for the first time, which would allow to reduce significantly the number of numerical experiments necessary for determining the maximum longitudinal forces of a shock and quasi-static nature during adjustment braking on a concave track section. The performed research shows that the error in determination of the longitudinal shock forces maxima using the proposed empirical formulae does not exceed 14%, and the error for the maximal forces of quasi-static nature does not exceed 5%. This allows to use the obtained empirical formulae for the approximate estimation of the longitudinal compressive force's maxima in trains of various length during adjustment braking on the track section of a broken profile.

As a result of using the proposed methodology and empirical formulae for the given running modes (adjustment braking with release of the brake line by 0.08 MPa and running-out), the longitudinal shock and quasi-static forces maxima as well as the dependence of the latter's on the train length, initial braking velocity and the algebraic difference of gradients and the length of the horizontal area that separates two gradients with opposite signs were obtained. The calculations show that the longitudinal shock forces' maxima decrease with an increase of the length of the straight area separating gradients of opposite signs and with decrease of the gradients' algebraic difference.

The maximum longitudinal forces' level increases with the increase of initial velocity and train's length. The longitudinal forces' maxima obtained for various longitudinal track profiles, different train lengths and running velocities have been analysed. It has been shown that, from the point of view of rolling stock strength and eventual derailment prevention, for a 90-wagon (1250 metres length) train and the given length of a horizontal separation area, the maximum difference in gradients should not exceed 0.01, or the separation area's length should be increased significantly.

The comparison of the longitudinal forces obtained as a result of train motion mathematical modelling and those obtained with the empirical formulae, shows that the formulae (4) and (5) can be used for the preliminary approximate estimation of the maximal longitudinal forces of shock and quasi-static nature in the trains during adjustment braking in the track sections of o broken profile.

The proposed mathematical model can be used for determining the parameters of the longitudinal track profile from the point of view of freight traffic safety for the trains of various length. The methods developed as well as the dependencies between the train forces' maximal values and the

longitudinal track profile's parameters, obtained through mathematical modelling, can be further used in the development and correction of the Norms and Regulations for Construction, governing the design of the track longitudinal profile. These results will allow to reduce the amount of excavation work on the newly built tracks, as well as to increase the railway traffic safety.

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