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**OPTIMIZATION OF PERIODIC UNITARY ONLINE SCHEDULE OF  
TRANSPORT TASKS OF HIGHWAY ROAD TRAINS**

**Summary.** The paper is devoted to the problem of optimization of transport tasks' execution schedule. The exploitation of a group of interdependent highway road trains is considered. The task differs from the known ones by limitations of the duration of a cycle within which a maximum number of goods transportation orders must be fulfilled. There are also limited intervals of time when loading and unloading of vehicles is permitted at appropriate points of a transport network. The solution of the problem consists in distributing a given number of road trains between orders, finding the shortest path for each of them, and determining the time of their departure and arrival provided the absence of idle runs and with minimal time delays of the process. It is shown that a maximum number of completed tasks are proportional to the duration of the cycle up to a certain limit only. The optimization is carried out for two options: in the absence and presence of restrictions on repeatability of freight flows from one point. The used algorithm guarantees an exact solution for an acceptable period of time for a small-dimensional task.

**1. INTRODUCTION**

Freight capacity of vehicle fleets significantly exceeds a quantity of orders to perform long-distance road transportation in Ukraine. In addition, in spite of the management efforts, the transport companies' operating costs are increasing annually. For example, Ukrainian carriers are trying to reduce idle mileage. However, the majority of administrative decisions are not systemic. They totally rely on various information systems such as demand forecasting and individual crew scheduling. These systems allow reducing idle runs and downtime for some road trains. Instead, they get underutilization, downtime, and even breakdowns of orders. Managers do not take into account basic principles of medium and large transport systems, in which vehicles can be used effectively. That is the reason why the problem of efficiently coordinated operations should be resolved under the following conditions. First, separate vehicles of a fleet ought to be coordinated properly. Second, a demand for cargo transportation must be systemic by nature, and, preferably, regular. Third, the information on the origin and properties of orders should be used as efficiently as possible.

Carriers who use automated information systems forecast oncoming orders and distribute them by means of available vehicles. Thus, they are able to partially avoid wasteful expenditure on transportation. However, the requirements for orders dynamics are increasing. They are often interdependent. This imposes additional restrictions on road trains scheduling. The content of a scheduling problem is that two contradictory requirements apply to it. On the one hand, the schedule must match the information received on potential freight flows and completely satisfy the known demand. At the same time, idle mileage is excluded. On the other hand, a complex structure of transport processes in the fleet of vehicles that interact as well as casual appearance and cyclical nature of orders

leads to forced downtime of trucks due to inconsistency of their schedule. In this regard, criteria and limitations are important for the schedule. The aim of this research is to optimize scheduling of vehicles providing freight services in the network and investigate its optimal conditions.

## 2. LITERATURE SOURCES REVIEW

### 2.1. Scheduling problems

All existing software programs that help to solve scheduling problems do not provide the schedule which is efficient enough for a group of vehicles, taking into account a cyclical nature of their exploitation. On the other hand, such tools exist in a modern theory of operation research [1]. Transportation scheduling, cyclic scheduling, and online scheduling are particularly noteworthy sections of this theory [2].

When speaking about cyclic scheduling, classical objective functions (such as the maximum of tasks completion moments, the sum of tasks completion moments, or the minimum of late tasks, etc.) do not work, and that the quality of the schedules obtained for such tasks is characterized by other parameters (such as density of the schedule, duration of the cycle of each operation, etc.). Cyclic schedules play an important role in organizing goods production and delivery. They provide a rhythmic loading of equipment and executants, and allow us to effectively plan both resources supply and products sale. The complexity of the tasks of cyclic schedules with various technological limitations developing was investigated in [3, 5, 6, 8, 12]. Most of the problems considered are *NP*-difficult in hard sense. Some researchers consider it appropriate to use approximated and heuristic algorithms for their solving [4]. On the other hand, there are precise algorithms for solving problems based on branch and bound methods. In most cases, the main attention is paid to the problem of cyclic time minimizing in schedules with restrictions of a minimum number of completed orders [5].

There is a different approach to the cyclic scheduling problem, one of which is a conceptually simple extension of its constraint-based formulation, which has been praised recently. The main advantages of using such a framework are availability of existing techniques and extensibility of constraint-based representations. This allows us to transparently exploit the power available in any of modern constraint satisfaction search engines, utilize other constraint reasoning techniques such as constraint propagation and consistency checks, and use the problem formulation within larger constraint decision problems. In addition, it has to be noted that the formulation lends itself to the use of methods for schedules reordering after a failure, and that it can be extended to handle more complex problems such as resource-constrained project scheduling [6].

The main feature of the problem, formulated in some works, deals with an infinite set of tasks whose precedence graph has a simple periodic structure. For example, the latest schedule notion with deadlines in a scheduling theory can be extended to a basic cyclic scheduling problem. In the general case, the latest schedule exists, but it is not always computable. In particular cases of cyclic production scheduling, more specific results allow calculating the latest starting times and showing that the associated time sequences have the same periodic structure as the earliest ones [8].

### 2.2. Online scheduling problems

Online scheduling is characterized by the need to decide on a schedule construction if there is some incomplete information about an input example. However, a degree of information completeness can vary [2]. Dynamic scheduling problems involve real-time constraints such as task arrival time, route changing, etc. But most of existing problems, whose data are supposed to be known a priori, are static.

In some recent research, a multiple shift scheduling problem has been considered and modelled as a covering problem. For a cyclical multi-routing problem, in which a right-hand side varies, a corresponding polyhedron has at most one fractional extreme point [3]. This means that a cyclical schedule for multiple transport crews can make the best overall schedule rather than a single schedule for each particular, or even all available vehicles. However, a solution of the cyclic multi-routing

problem schedule described in [3] cannot be used to achieve the goal, because it provides required periods of idle vehicles (aircrafts) connected with the rest of the crew.

The problem of online freight flow management can be considered as an optimal number of order cards in a production and transportation integrated system, which provides reduction in logistics costs. All known methods of the efficient management in conditions of continuously changing internal and external environments are directed to increase the system adaptation to a dynamic condition for executing a production plan and minimizing logistics costs. Such approach will allow us to manage all types of decision making under uncertainty and provide real-time freight flow management for costs reduction. However, a significant disadvantage of methodology of unclear sets and logic lies in need for large volumes of original teaching materials under operational control transport undertaking, which are not always available [7].

POP-Star is the algorithm of scheduling that produces multiple object types in low volumes and is based on a partial order planner which is guided by best-first search algorithms and landmarks. The best-first search uses heuristics to help the planner to create complete plans while minimizing a makespan. The algorithm takes landmarks which are extracted from user's instructions given in structured English as input. Using different topologies for landmark graphs, it becomes possible to create schedules for changing object types which will be processed in different stages. Results show that POP-Star can create and adapt schedules with changing product types in low-volume production [11].

The time flow algorithm describes the time which is needed to produce one occurrence of one task that consists of several operations. A corresponding mixed integer linear programming has been derived as a way to solve these problems. Unfortunately, this approach can solve problems with only a little number of operations and machines [12].

The problem of a periodic schedule minimizes the time difference between two succeeding occurrences of one operation for a given set of constraints [13]. This time difference is called cycle time. The other objective, which is not taken into account, is to minimize the time flow of a procedure.

The issue of establishing the complexity of the problem is a global matter of scheduling theory. The complexity of the problem is a function of many parameters on which this problem depends mainly. Mixing two examples of different types can fundamentally affect the resulting complexity of the task.

Some researchers have presented highly efficient schedules to meet demands while taking into account an entire integrated logistics chain. In particular, they present simulation models of railway networks with trains circulating in closed loops of a railroad with double-track and single-track stretches. High sensitivity of the system requires modelling algorithms, which can properly represent such cases. Authors insist that the model will not be validated if all created railway system complexity does not appear during modelling and system representation phases. The presented algorithm is highly adequate but, on the other hand, is too complicated to be successfully computed by users [14].

Scheduling might become a huge problem, the complexity of which depends on system parameters, restrictions on targets and a number of operations. It has been demonstrated how changing an insignificant condition, at first glance, can turn the task of schedule building by classical polynomial algorithms to the one which is in *NP*-hard problem [10, 13]. One can schedule different ways in task planning for small systems, including a manual one or using any heuristic methods. The main thing is that it should be proved to be correct ultimately. However, this process should be automated in practice for medium, large, or dynamic systems.

### 2.3. Routing problems

Due to the fact that modern transport technologies have reached a high level, time delays have become a dominant factor determining the productivity of vehicle fleets. Traditional methods of their routing, which give good results in terms of length of connections and minimization of density, cannot guarantee the fulfilment of requirements for the speed of transport schemes. In this respect, special attention has been paid to the ways of keeping time restrictions at the stage of global routing in recent years [2, 12, 15]. Despite the urgency and importance of the task, due to transportation costs, there are no methods which perform routing that satisfies both restrictions on time and carrying capacities of way criteria as well as total length of the route.

Thus, the problem of routing and scheduling of road trains forms a complex task. Its solution is complicated over the need to consider additional obstacles that arise while implementing a transport task. In that case the dynamics of the schedule changes.

A typical routing task is TSP (travelling salesman's problem). It is *NP*-hard in strong sense, for both minimum and maximum of a Hamiltonian cycle length. Possible variations of TSP were researched in the paper [15]. They are single vs. multiple depots, a number of salesmen, fixed charges, and time windows. In the last variation, certain nodes of a network which are necessary to be visited in specific periods of time are called time windows. This is an important extension of mTSP (multiple traveling salesman's problem) with time windows (mTSPTW – multiple traveling salesman's problem with time windows). The solution of the problem is obtained through an iterative method using time window discretization. The advantage of the proposed algorithm is that it provides a guaranteed optimal solution with a small number of variables. However, this method ignores the benefits of cyclic schedules.

The most common practical interpretation of TSP is that a salesman seeks for the shortest tour through  $n$  clients or cities. This basic problem underlies several vehicle routing applications, but in this case a number of side constraints usually come into play [16].

The result of a constant search for the best practices for solving TSP is the use of bionic algorithms, including genetic and evolutionary ones [17]. The results of some experimental studies have shown high efficiency of these algorithms, and some test cases have shown their undeniable advantage over existing methods.

Transport accessibility can be analysed by using a number of different methods. The problem with each of them lies in the difficulty when obtaining data to measure this phenomenon.

A few articles deal with possibilities of optimizing a transport network. Some of them use allocation models within a graph theory to obtain results for an addressed optimization problem. The latter is subsequently applied in a postal network to determine an optimum location of postal facilities while minimizing costs. Moreover, some researchers describe possibilities of identifying and calculating input variables of the used model, creating an underlying network as well as possible further improvements of the obtained solution. The results can serve as a basis for modification of the used model for simulating similar networks [18].

The values of additional time reserves, which should be included in both a train schedule and a schedule for a freight turnover to provide conditional interaction of railroads and industrial enterprises in Ukraine, were proposed [19].

### 3. FORMULATION OF THE PROBLEM

A set of orders  $P = \{1, 2, \dots, p\}$  is known over period  $T$ , which may be performed without interruption by one or more vehicles of a given set  $M_k$ ,  $k = 1, \dots, m$ , where  $p$  is order index. The content of each order is to deliver cargo group  $Q_{i,j}$  from some point  $g_i$  to another  $g_j$ ,  $i, j = 1, \dots, n$ . Thus, the group

can be delivered in  $Z$  parts  $Q_{i,j} = \sum_{z=1}^Z q_{i,j,z}$ , that each of them  $q_{i,j,z} \leq q_k$ , where  $q_k$  is nominal carrying capacity of a  $k$ -th vehicle. We deal with  $Q_{i,j} \geq q_k$  in this task for any  $k = 1, \dots, m$ . This means that each

of available vehicles can serve no more than one customer simultaneously. This corresponds to the condition of transportation unitary. The time service of a  $p$ -th customer is a random variable with known distribution of assessment and therefore expectation of time  $t_p$ , which consists of values:

$$\bar{t}_p = t_{ij} + t_l + t_u + t_{org}, \quad (1)$$

where  $t_{ij}$  is the movement between  $i$ -th and  $j$ -th transport items,  $t_{l,k}$  is the duration of loading operations of  $k$ -th vehicle, and  $t_{org}$  is the duration of organizational downtime. Due to accepted assumption of unitary orders, components  $t_u$  and  $t_{org}$  can be seen as a permanent component in the performance of total

flows which does not depend on its schedules, but on technology of transportation. Therefore, these elements can be combined with  $t_{i,j}$  and considered as  $\bar{t}_p = t_{i,j}$ .

Each order has the following characteristics: time  $eb_i$ , no sooner than it starts functioning in  $g_i$ , allowed interval  $eb_i + \Delta_i$ , during which the implementation has to be initiated. Then, moment  $ef_i$  comes, after which the order cannot be performed by unavailability of a  $g_i$  point of departure. The same characteristics are a collection of cargo points: the earliest point in time when the goods can be delivered to point  $j - eb_i$ , allowed arrival interval with the load at point  $j - eb_i + \Delta_i$ , and the most recent time of order  $-ef_i$ .

Transport items, which are the shippers, are characterized not only by the points of origin and cancellations but cyclical nature of such moments. Orders  $p$ , which arise periodically at point  $g_i$ , are the meaning of freight flows to  $g_j$  sized by  $q_{i,j}$ , are called homogeneous and denoted by index  $p_l$ . Tact  $\tau_{i,l}$  exists at  $0 \leq \tau_{i,l} < \infty$ , between the moment of occurrence of similar regular orders, which, in general, is a random variable. If  $\tau_{i,l} = 0$ , then the shipment with cargo from a  $g_i$  item is not limited by a deadline. If  $\tau_{i,l} = \infty$ , then the order at this point is disposable.

The entire set of orders  $P$  displays an oriented graph  $A(G,U)$ , where  $G$  is a set of nodes,  $\{g_0, g_1, \dots, g_{p+1}\}$ , symbolizing transport points. Node  $g_0$  is fictitious, and symbolizes a formal start of freight flows. Node  $g_{p+1}$  is also fictitious, and symbolizes the end of a planned cycle with duration of  $T$ . A set of arcs  $U$  reflects planned freight flows from  $g_i$  to  $g_j$  points. The arcs of graph  $A$  are weighed. The weight of each of them is given  $q_{i,j}$ , which is the volume of the freight that must be arranged. If there is a demand, it is shown in graph  $A$  by arc weight  $q_{i,j} > 0$ , but all other links appear by arcs with weight of  $-\infty$ .

Graf  $A$  has no loops, so that, if there are two nodes,  $g_h$  and  $g_x$ , among which there are arcs such as  $q_{h,x} > 0$  and  $q_{x,h} > 0$ , then they add the third one, which is  $g_{h+1}$ , that  $q_{h,x} > 0$ ,  $q_{x,h} = 0$ , and  $q_{x,h+1} > 0$ . Thus, the loop is eliminated due to duplication of nodes.

Between any of two orders ( $p_x$  and  $p_y$ ), a relationship of advantages might appear, which can be formalized in a set of additional arcs with zero weight. The same arc with zero weight is an associated node  $g_0$  with those nodes that arc starts with weight  $q_{x,h} > 0$ , i.e.  $q_{0,x} = 0$  for all  $x$ , that  $q_{x,h} > 0$ ,  $h=1 \dots p$ , and although  $q_{y,p+1} = 0$  for all  $y$ , that  $q_{x,y} > 0$ , for all  $x=1 \dots p$ . Due to these circumstances, no general formulation of the problem exists.

All  $m$  vehicles may be involved in the process of transportation. They must run simultaneously, performing several orders sequentially. This means that one must find  $k$  chains in graph  $A$ , which starts at node  $g_0$ , passing through a maximum number of points relating to existing orders, and ends at node  $g_{p+1}$ . The idle run is not allowed. Therefore, the desired chains have to pass only those points for which  $q_{x,y} > 0$ . If the chain reaches point  $g_y$ , and then there are no integral or non-zero weighted ways in graph  $A$ , it goes to point  $g_{p+1}$  at the same time. The transport cycle will be considered complete for this vehicle, despite the fact that the reserve of time is not performed to take more orders. On the other hand, if an unsuccessful run allowed prolonging a chain of orders, the loss of time there would be disproportionate to the loss of funds on idle mileage. In this regard, the criterion of maximum transportation, which means maximum quantity of completed orders, is used. The problem is similar to typical tasks of mTSP in this formulation with a few differences.

1. Period  $T$  is not a predetermined value (its shortest numeric value is not searched).
2. Repeated and even cyclical passing of truck transport at any point is allowed, as well as partial order performing.
3. The length of any link of any chain value is variable. It depends on sequence of orders. The orders can be carried out over a limited period of time, so, ordering that does not start in time can be executed with a delay.
4. The number of specified vehicles may not be equal to the number of vehicles actually engaged in transportation.

#### 4. METHODS OF SOLUTION

The known algorithm for solving TSP is not quite suitable to find a guaranteed approximate solution. Therefore, we used a combination of these algorithms [1, 8, 10, 12].

Variable  $x_{i,j}$  of this task is the number of trips that are led from  $g_i$  to  $g_j$  for period  $T$ ,  $x \in \{0, 1, \dots, Z_j\}$ . However, if  $Q_{i,j} = 0$ , then  $x_{i,j} = 0$ . Restrictions of these variables are as follows:

$$\sum_{j=1}^n x_{i,j} - \sum_{i=1}^n x_{i,j} = 0, \quad (2)$$

which means that the number of flows that come to any node equals to the number of flows that come out of this node except start  $g_0$  and final  $g_{n+1}$ ;

$$\sum_{j=0}^{n+1} x_{0,j} - \sum_{i=0}^{n+1} x_{i,0} = \sum_{j=0}^{n+1} x_{0,j} - 0 = R, \quad (3)$$

which means that the number of flows that come from  $g_0$  is equal to given maximum number of vehicles given:

$$\sum_{j=0}^{n+1} x_{n+1,j} - \sum_{i=0}^{n+1} x_{i,n+1} = 0 - \sum_{i=0}^{n+1} x_{i,n+1} = -R, \quad (4)$$

which means that the number of flows that come to the end node equals to the given maximum number of vehicles.

Restrictions of total duration of all operations cycle are as follows:

$$\sum_{i=1}^n \sum_{j=1}^n (t_{i,j} + h_{i,j}) \cdot x_{i,j} \leq t_{\Sigma}, \quad (5)$$

which means that regardless of the amount involved in both, the process of vehicle operations and total duration should not exceed a certain value which is set in a range of some importance to advance to the minimum at which the solution to the problem exists. Value  $h_{ij}$  is delay / advancing of the order, which is calculated by the following expression:

$$h_{i,j} = \begin{cases} \theta_{0,i}^{b,l+1} - eb_{i,j} & \text{if } eb_{ij} < \theta_{0,i}^{b,l} \\ eb_{i,j} - \theta_{0,i}^{e,l} & \text{if } \theta_{0,i}^{b,l+1} - eb_{i,j} \end{cases}, \quad (6)$$

where  $\theta_{0,i}^{b,k} = (k-1) \cdot \tau_{0,i}$ ,  $\theta_{0,i}^{e,k} = (k-1) \cdot \tau_{0,i} + \Delta_{0,i}$  are respectively the beginning and the end of the order.

The criterion is the maximum number of orders that can be performed during period  $t_{max}$ :

$$\sum_{i=1}^n \sum_{j=1}^n x_{i,j} \rightarrow \max. \quad (7)$$

First, find in advance a large numerical value of  $t_{\Sigma}$ . In this sense, function (5) takes the maximum possible number of orders that can be performed during  $T$ :

$$Z = \sum_{i=1}^n \sum_{j=1}^n \left\lfloor \frac{Q_{i,j}}{q_k} \right\rfloor, \quad (8)$$

where square brackets mean dividing without the rest.

Expressions (2) – (7) indicate that this problem is linear programming. For its solution, gradient method has been used [5]. The relative error of solving is less than 5%. The maximum number of iterations is less than 100 with small data matrix dimension ( $7 \times 7$ ). So, maximum time solving is less than 0.01 sec. per iteration. The solution of the problem with  $t_{\Sigma} = t_{max}$  is guaranteed and initiating at any given  $R$ . It is provided that constraints (4) are practically absent. In fact, graph  $A(G, U)$  exists, in which all arcs  $U$  are saved. Since this graph can contain loops, an unambiguous optimal performance schedule for this graph cannot be built, according to the theorem [1]. If there is a time limit for implementing a transport cycle, like maximum permissible duration  $T \leq t_{max}$ , then, such a clear schedule does not make

sense, because the request fulfils all tasks performed optimal conditions in any order. Then, period  $T$  in this graph becomes an arbitrary value.

If duration of  $t_{max}$  is reduced, the solution of the problem will change. To do that, subgraph  $A'$  without any loops will be found in graph  $A$ , which will lead to an unambiguous schedule execution of certain orders. Choosing these arcs of graph  $A$ , matching (9), let us perform its ordering searching for a critical path and parallel chains [1]. Another subgraph  $A'' = A / A'$  remains unchanged, i.e. it does not require a clear schedule for orders execution, since the conditions of tasks are fulfilled. Graph  $A''$  is the subject of ordering next steps of the algorithm. The scheduling construction algorithm is interactive. The goal of the iteration is to arrange a disordered graph of freight flows if this could find an expression in accordance with (6). If any arc, which meets the conditions (6), cannot be reduced more than to  $t_{\Sigma}$ , the schedule for which a maximum number of orders executed in compliance extremely minimal period of execution of all orders in view of their authorized frequency is found. Example of such a schedule is shown in Fig. 1.

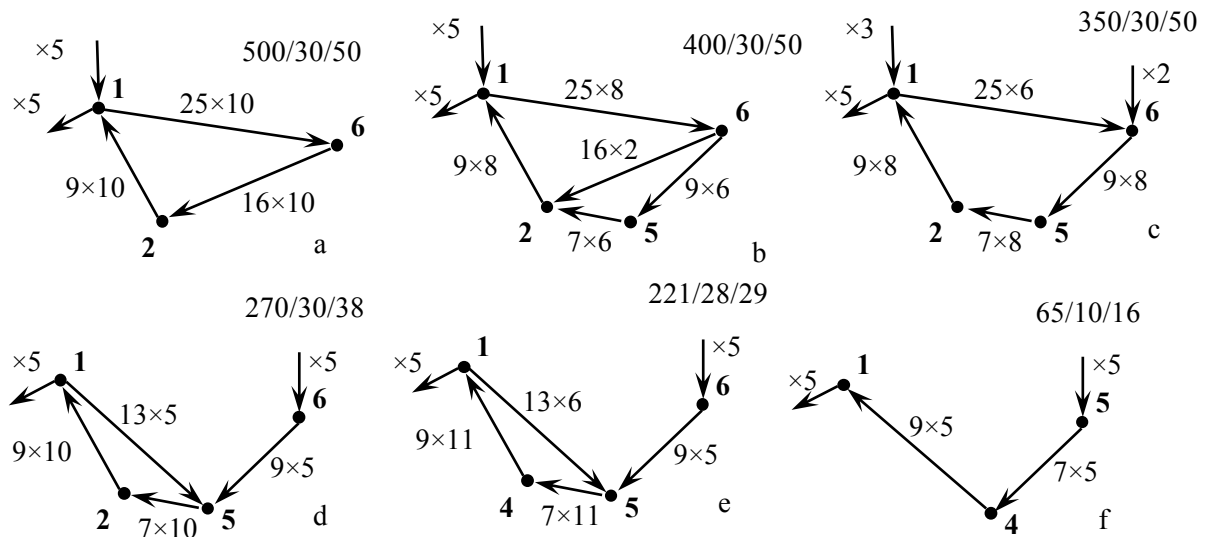


Fig. 1. Modeling of transport cycle restructuring under the influence of limiting total duration of orders

### 5. SOLUTION OF THE PROBLEM USING A TEST MODEL. THE RESULTS ANALYSIS

Here is the algorithm used in a test model. The graph contains 6 nodes and arcs between each pair of nodes. There are no loops in the graph. Nodes  $g_0$  and  $g_7$  are fictitious. Matrix of values  $t_{i,j}, Q_{i,j}, eb_{i,j}, ef_i, \Delta_i$  must be used but is not presented here. Loading capacity of vehicles is signed by  $q_k$ . Maximum number of vehicles involved in transportation is  $R = 10$ . Maximum total duration of transportation is  $t_{\Sigma} = 500$  hours. Examples of changes in the structure of an overall transport process in the case, where  $R=5$  involved vehicles, are shown in Fig. 1. Each scheme in a common subgraph is a graph of disordered relationships. The nodes are its transportation items  $g_1...g_6$ , and arcs are its cyclical flows of vehicles with loads. Its weight, indicated above each arch, is  $\bar{t}_{i,j} \times z_{i,j}$ . For example, the weight of arc  $q_{1.6}$  in Fig. 1a is  $25 \times 10$ . This means that the average duration of goods delivery from item  $g_1$  to  $g_6$  is 25 hours, and is repeated 10 times. Since the entrance to this scheme is completed by 5 vehicles, and the same is on the way out, it becomes obvious that each one makes two cycles. The maximum number of cycles that are achieved as a criterion is 30. As it can be seen from the scheme, all its operations are synchronous, which ensures minimum downtime of vehicles and readiness for anticipation of sending. The maximum total duration of transportation is  $t_{\Sigma} = 500$  hours. Characteristics of this scheme are given in the upper

right corner of Fig. 1a:  $t_{\Sigma}/Z/T$ . The duration of a planned period  $T$  is found as the length of a critical path in the graph: 1–6–2–1. A separated subgraph definitely outlines a vehicle schedule. In Fig. 1b, the total duration limit of operations results in reducing the number of cycles and the addition of 1-6-2-1, 1-6-5-2-1 cycles are seen. The schedule of operations becomes more stressful because one needs to coordinate the flow of cars that blend in node  $g_2$ . This time delay will eventually occur. The duration of the planned period remains the same. Further reduction sets a limit problem that leads to fewer cycles, and more intense schedule of car and cycle times. In each of the following schemes, the schedule is unambiguous. Finally, in Fig. 1f, a linear transport scheme without closed cycles is seen. In this scheme, the desired schedule without downtime of vehicles and cargo is very difficult to build because of the length and size of transactions related  $eb_{i,j}$ .

Fig. 2 shows the dependence of maximum number of orders, which cyclic vehicles can perform in a transport system, on maximum total duration of the process.

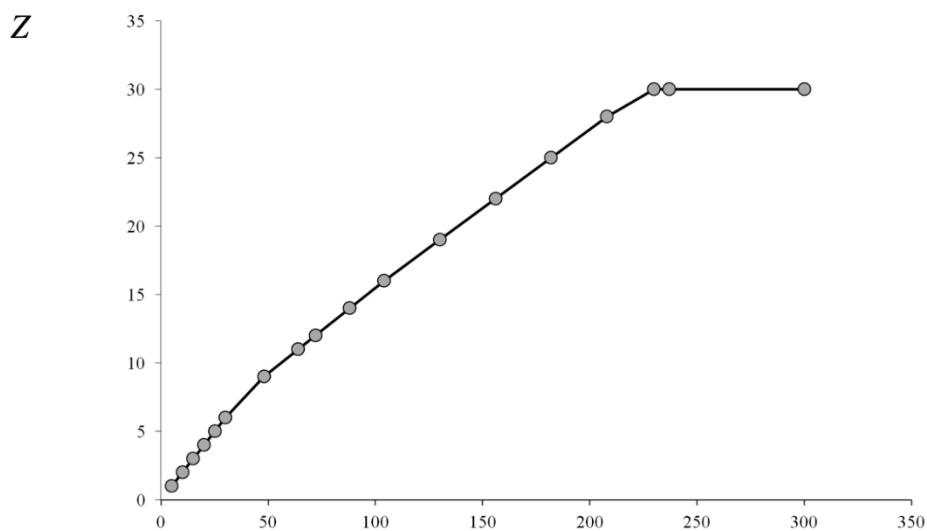


Fig. 2. Dependence of the total number of orders executed on permitted duration of a periodic transport cycle

As it can be seen from the dependence, the maximum number of orders that a road vehicle interaction in the transport system can perform approaches the maximum possible value for the system asymptotically when expanding restrictions on total duration of transport. This is due to the fact that small transport systems (2 – 3 circular routes), the items of which on the graph reflect the value 5...50, execute orders short in duration and schedule, which makes their execution tight and allows a lot of downtime. With an increase of more than 50 and up to 210 hours, the number of orders that are executed in one cycle increases gradually. More complex execution order, which ignores smaller systems, has been taken into account. It has to be noted that this kind of dependence is not influenced by changing a number of vehicles involved.

Similar studies have been conducted with additional restrictions on the number of orders that may apply to only one transport point during period  $T$ . In particular, if a number of arrivals at each item of a network is more than one, then the problem becomes of TSP with many routes content. Here, this problem is solved by the method set forth above. As a result, the dependence of the maximum number of cycles permits total duration of orders by analogy.

As it is shown in Fig. 3, this dependence also approaches a sustainable value  $Z$  asymptotically. The function also does not depend on the number of cars available  $R$ . Moreover, the actual quantity of vehicles, involved in the process, is usually less from the set (Fig. 4).

Fig. 4a shows that the transportation involves 10 vehicles. However, the number of arcs that go from the initial node is actually 4. The number of orders executed is 9.



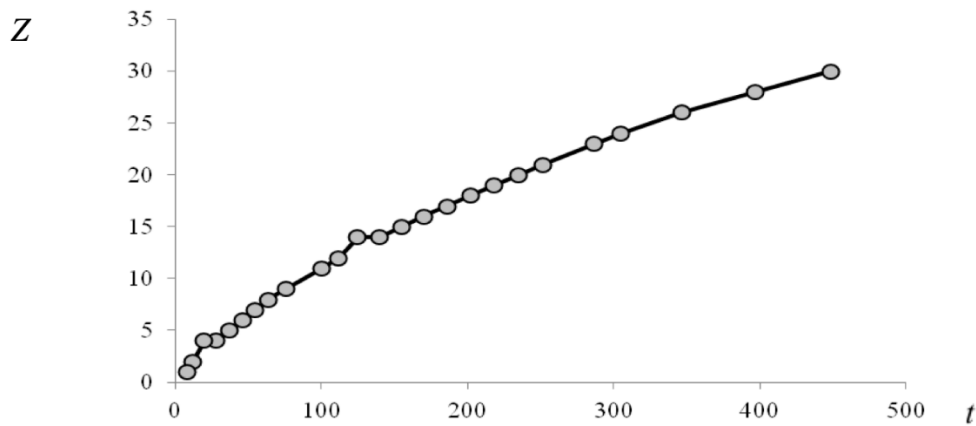


Fig. 3. Dependence of the total number of orders executed on total permitted length of a single transport cycle

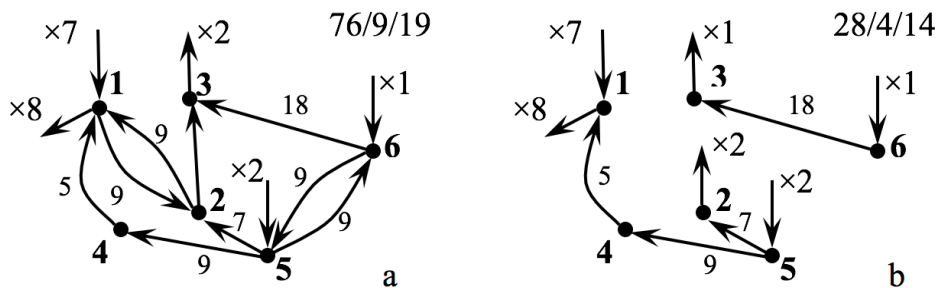


Fig. 4. Modelling of transport cycle restructuring under the influence of limiting total duration in a single route of each item

The transport scheme in Fig. 4.b has been achieved by restriction of its total duration. Here, ten vehicles are available. It is clear that the number of vehicles used is 3, which corresponds to the number of arcs, which go from source nodes. Four orders are fulfilled. If we could reduce the total cycle time to more than 2.7 times, the maximum cycle time would decrease by only 35%.

The number of cars involved affects only the maximum value for which we can get a solution of the problem (Fig. 5).

The duration of a transport cycle, which is called tact, is important in organizing schedules of vehicles in medium and large vehicle systems. This value is a measure of the information collected as the forecast freight depends on the duration of the forecast period. In addition, the forecast period can be distinguished in medium and large transport systems. The value of information supply particularly increases in large systems because cars interact in complex traffic cycles, and a great deal of transport points is serviced. Therefore, the amount of information for such systems is a crucial criterion. This can be seen on the basis of dependence in Fig. 6.

### 6. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

The construction of the schedule of vehicles that interact during a planned cycle should be carried out by the criterion of maximum transportation or a maximum number of orders if flows are unitary. Availability of time limits for orders execution and absence of idle runs make this task complex. None of the known algorithms which could guarantee an optimal schedule construction are unsuitable for such applications. However, if an iterative change in the duration of the cycle is applied, then the problem

can be solved by using linear programming methods. Thus, it will be possible to get a directed graph of material flows, which can definitely build an optimal schedule.

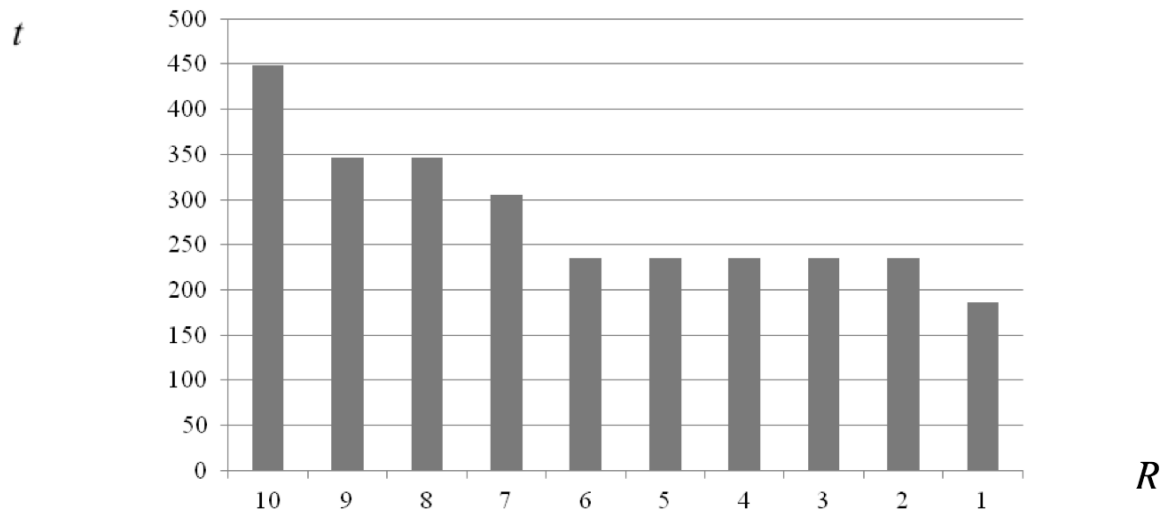


Fig. 5. Dependence of maximum total duration of cycle operations on given vehicles

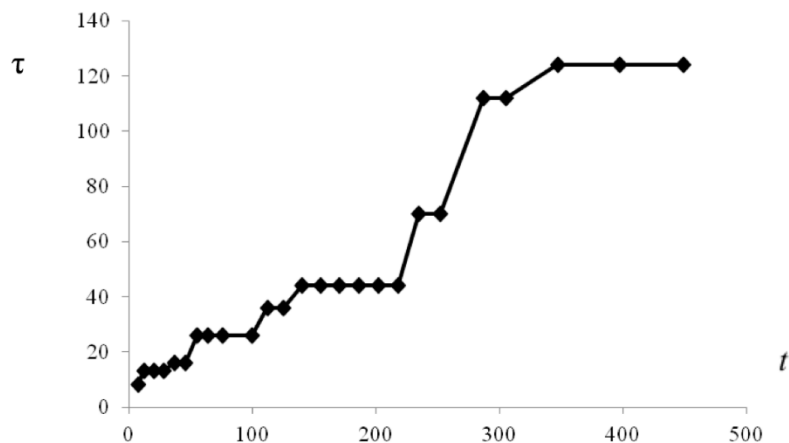


Fig. 6. Dependence of duration of the planned period forecast on maximum total duration of operations

Reduction of the time limit for the cycle naturally leads to reduction of the number of executed orders. However, this decreases as the required number of trucks comes to a certain limit. The execution of individual orders loses contents of a single production problem in its achievement. The case in question is small transport systems for which the integrative effect is absent.

The problem, which has been formulated and solved, is one of the variants of real production situations. In order to build the best schedule for any other concerning small and medium transport systems, the algorithm described here with corresponding change restrictions can be used.

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