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NEW APPROACH IN DEFINITION OF MULTI-STOP FLIGHT ROUTES

Summary. Optimization and profitability approaches play a crucial and central role in airline industry today. The main problem is how to overcome complexity by providing effective route schedule with minimal empty seats. So we need capable tools to re-optimize existing flight routes or to offer new one instead. This research deals about the efficient heuristic algorithm for optimal transportation of *N* different passenger contingents between ending points. We want to find out better transport plan with minimal transport cost for the route with more charging/discharging points (airports). Such optimization tool can help in sizing of appropriate airplane for definite direction, too.

NUEVA APROXIMACIÓN EN LA DEFINICIÓN DE LOS VUELOS CON LAS PARADAS MULTIPLES

Resumen. Aproximación de la optimización y rentabilidad tiene un crucial y central parte en la industria de la aviación hoy. El problema principal es como superar la complejidad al proporcionar los horarios de la ruta efectiva con un mínimo de los asientos vacíos. Por tanto necesitamos mejor instrumentos para re-optimizar rutas de vuelos actuales o en vez ofrecer nuevas rutas. Este investigación se trata sobre el algoritmo heurístico eficiente para el trasporte optimo de N diferentes contingentes de los pasajeros entre puntos finales. Queremos encontrar mejor plan de transporte con los costes mínimos para las rutas con más puntos de recargas / descargas (aeropuertos). Esta herramienta de optimización puede ayudar en dimensionamiento del avión adecuado para la dirección definida, también.

1. INTRODUCTION

Airlines companies have a big responsibility to satisfy people’s needs and in the end to gain profit. Today air transport industry is influenced indirectly by the economic recession, increase of fuel cost, stiff competition and political instability. The major change in trends of air transport development is to increase operating efficiency, productivity and profitability, so more and more routes with multiple stops (landings) are introduced. The scheduling of multi-stop flight routes is the crucial elements, especially in definition of the available airplanes, the airport slots, the airplane rental charges, airport service costs and other cost elements.

2. PREVIOUS RESEARCH AND REFERENCES

The most important issues to enhance the airline operation efficiency are flight routing and fleet scheduling. Generalized approach to multi-commodity transportation problem we can find in the early paper of Wollmer [14]. Wollmer finds out that capacity of the air corridors are virtually unlimited; however the number of flight assignments would be constrained by number of planes, pilots, same as with upper bound of seats for defined airplane type (capacity).

Many network flow techniques and models exist to solve the complex mathematical problem in flight routing. Model for fleet routes is based on the multiple commodity network flow problem (MCNFP) introduce in paper of Yan and Tseng [12].

Allocation of the expenses and revenue are the basic things that must be considered to evaluate the route profitability. Some costs can be caused directly and some indirectly. These data are very important to determine the correct calculation and profitability of the each route (Chang and Schonfeld [3]). In fleet routing and multi-stop flight scheduling the crucial elements are setting the available airplanes, the airport slots, the airplane rental charges, airport service cost (quota), fuel consumption, maintenance cost and other cost elements, which lead to the minimization of all expenses and maximization of the company's profit (Yan and Young [13]).

Short-term flight scheduling model is developed and applied to Taiwan airlines. Such model is defined as a non-linear integer program that is known as NP-hard problem. Non-linear problem is more difficult to solve than the traditional flight scheduling problem that is defined as integer linear program. The heuristic methods and algorithms can improve such approach significantly (Yan, Tang and Lee [12]).

In the research paper of Yan, Chen [9] is developed the model for Taiwan inter-city bus carriers. The model is based on integer multiple commodity network flow problem, too. In the literature many papers have been already devoted to ship routing in marine industry. Ferry fleet routing problem is solved by time space network technique that is specified to the defined time period (one day in this paper). In that technique represented by network structure, horizontal axis symbolizes airport locations and vertical axis represents the time duration (distance). Each arc between airports represents activity of ferry transport (Yan, Chen, Chen and Lou [10]).

Another group articles are concerned by vehicle routing problems. In the paper of Garaix, Artiques, Feillet and Josselin [2] the optimization of routing vehicles in freight or passenger transport is presented. During this representation for vehicle routing problem the fixed sequence arc selection problem is raised (FSASP). They proposed a dynamic programming solution method (ODT) for solving that problem. In the article written by Stojković, Soumis, Desrosiers and Solomon [7] DAYOPS model is presented. Every arc presents each flight leg which means a distance between departure and arrival. Model can be used to re-optimize the route schedule at the high level and at the lower level. The load factors, airline frequency, airplane size are necessary issues that must be taken in consideration for making the airline profitable. Choosing an appropriate airplane size for the flight route must be appropriate to the level of demand. This factor can influence a lot on the optimization of the flight route and optimum load occupancy of the airplane (Givoni and Rietveld [3]).

Maintenance costs include engine repair, consumption parts for airplane, technical support, technical documentation, maintenance staff, and other maintenance costs (Gomm [4]).

Airport service costs include landing cost and handling cost for airplane, passengers, luggage and freight transportation. Each airport determines the cost for using the airport for landing and handling their airplanes (Tatalović, Babić and Bajić [8]).

Many carriers installed the route planning software with the goal to optimize their existing routes, to increase profit and decrease expenses. With such systems we can clearly see the picture of the costs that influences on the route profitability and the way how to improve it. The software helps the pilots to find a better balance of fuel usage, flight speed and flight path. The efficiency today is the most important and the costs must be minimized wherever it is possible. Such optimization tool could be the crucial thing in any intelligent transportation and it influences on the airline profitability significantly.

3. MATHEMATICAL MODEL

Taking into account passenger demands for each airport and each destination, with sufficient amount of passengers waiting to be transported, we need optimal transportation plan to minimize shipping and loading/unloading expenses, transportation cost and cost of airport costs (connected with expenses at airport and loading process). It can help in definition of optimal airplane capacity arrangement or for evaluate the route efficiency. The problem of optimal transportation from multiple (several) airports of loading (sources) to multiple destinations (sinks) is very hard (NP-hard) optimization (combinatorial) problem.

Amounts of different passenger contingents are in firm correlation because the total capacity of airplane is limited. Passenger contingents are differentiated with i for $i = 1, 2, \dots, N$. The plane with defined capacity is shipping from the first to the last airport marked with $M+1$, with possible set of intermediate ports marked with K. The objective is to find a loading and transportation strategy that minimizes the total cost incurred over the whole voyage route consisting of M airports on the path ($M \leq K$). We need the loading plan for various passenger contingents in each airport to serve N passenger loads from loading airport to destinations (landing point).

The transportation technique explained above can be seen as the capacity expansion problem (CEP). Transmission portions of the airplane space are capable to serve N different passenger loads (multi-commodity) for $i = 1, 2, \dots, N$. For each passenger load we need a part of airplane capacity, so it looks like capacity expansion problem.

New capacity portion on the board of aircraft can be assigned to appropriate passenger load up to the given limit (maximal capacity). Used capacity can be dimensioned in two forms: by expansion or by reduction. Expansions/reductions can be done separately for each passenger contingent (load). Fig. 1 gives an example of network flow representation for multiple contingents (N) and M airports on the route. So the transportation problem can be represented by a flow diagram of oriented acyclic network.

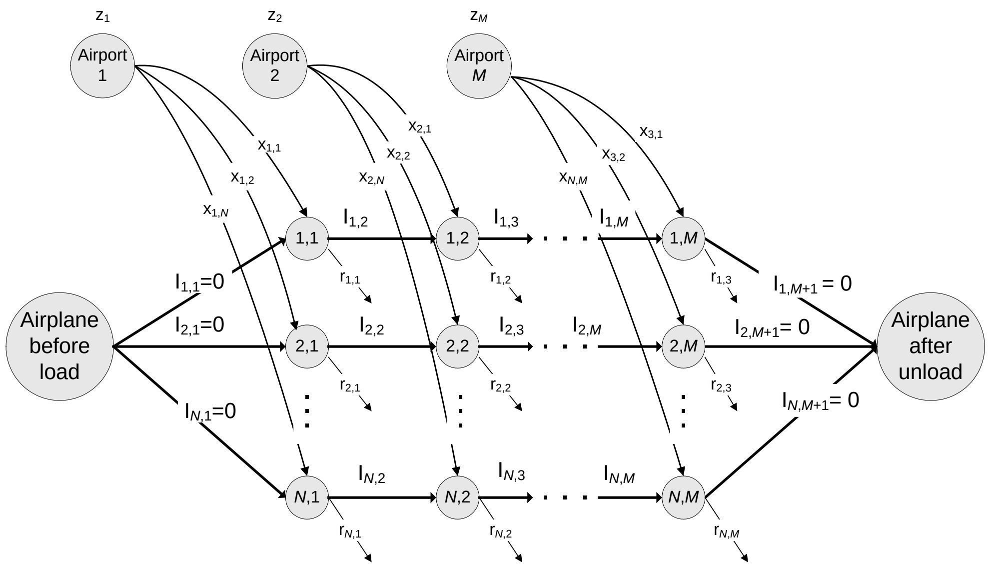

Fig. 1. Transportation problem can be represented by a flow diagram of oriented acyclic network
Fig. 1. Problema de transporte puede ser representado con un diagrama de flujo de la red orientada acíclica

Let G (V, E) denote a network topology, where V is the set of vertices/nodes, representing capacity states on the board and A, the set of arcs (links) representing traffic changes (loading/unloading, transportation, airport services etc.) between airports. Each link on the route (path) is characterized by z-dimensional link weight vector, consisting of z-nonnegative weights. In general we have multi-constrained problem (MCP) with multi-dimensional link weight vectors for M+1 links on the path $\{w_{i,m},\ m \in A,\ i = 1, \ldots, N\}$. The constraints for capacity bounds are denoted with $L_{i,m}$ ($L_{1,m}$ $L_{2,m}$, … $L_{N,m}$). For an additive measure (load of passengers) definition of the constrained problem is to find a path from the starting to the end airport with minimal weight to satisfy maximal traffic load. It is equivalent with minimal cost that is the function of all expenses and shorter distance gives lower weight. Also, the weight of each link corresponds to the amount of used capacity. As it is an additive measure more people on board cause lower transportation cost of one passenger. The objective is to find the optimal routing policy that minimizes the total cost with maximal passenger load on the path. In the context of MCP we can introduce easily the adding constraints e.g. max. length of the route.

In that CEP model the following notation is used:

i, j and k = indices for passenger load. The N facilities are not ranked, just present different types of passenger contingents from 1, 2, ... , N .

m = indices the airport of boarding and landing. The number of air of calls on the route including departure airport M (m = 1, ... , M) .

$x_{i,m}$ = quantity of i-th load of passenger amounts being loaded on board in airport m

$r_{i,m}$ = unloading of passenger i-th contingent in airport m. For convenience, the $r_{i,m}$ is assumed to be integer.

$I_{i,m}$ = the total amount of passengers transported from port m to m+1. The amount of passenger load i at departure from airport m is equivalent to arrival at airport m+1. Before the first airport of loading, $I_{i,1}$= 0 . After last airport $I_{i,M+1}$ = 0 for i =1,…, N.

$$I_m = \sum_{i=1}^{N} x_{i,m} - r_{i,m} \tag{1}$$

for $i = 1, \ldots, N$; $m = 1, \ldots, M$

Capacity values cannot be negative.

$L_{i,m}$ = maximal amount of contingent i to be boarded on airport m.

z_m = the total amount of all passengers related to airport taxes.

$$z_m \leq I_{m,} \tag{2}$$

lon_m = maximal length of the each hop, not to exceed the length of the whole route LON.

4. ALGORITHM DEVELOPMENT

Instead of a nonlinear convex optimization, that can be very complicated and time-consuming, the network optimization methodology is efficiently applied. The main reason on such approach is the possibility of discrete capacity values for limited number of contingent loads, so the optimization process can be significantly improved. The multi-constrained routing can be formulated as Minimum Cost Multi-Commodity Flow Problem (MCMCF); see [6]. Such problem (NP-complete) can be easily represented by multi-commodity the single (common) source multiple destination network.

Definition of the single-constrained problem is to find a path P from starting to end airport such that:

$$w(P) = \min \sum_{m=1}^{M+1} \sum_{i=1}^{N} w_{i,m}(I_{i,m}, x_{i,m}, r_{i,m}) \tag{3}$$

where: $I_{i,m} \leq L_{i,m}$ (4)

satisfying condition: max. distance of $P = \sum_{m_1}^{m_2} lon_i \leq LON$ (5)

for $i = 1, \ldots, N$; $m = 1, \ldots, M$

A path obeying the above conditions is said to be feasible. Note that there may be multiple feasible paths between starting and ending airport (node).

Generalizing the concept of the capacity states after loading/unloading each passenger contingent (load) m between airports on the route we define as *a capacity point* - α_m.

$$\alpha_m = (I_{1,m}, I_{2,m}, \ldots, I_{N,m}) \tag{6}$$

$$\alpha_1 = \alpha_{M+1} = (0, 0, \ldots, 0) \tag{7}$$

In formula (6) α_m denotes the vector of capacities $I_{i,m}$ for each load i and for each airport m, and we call it capacity point. On the flow diagrams (fig. 1.) each column represents a capacity point of the node, consisting of N capacity state values (for i-th passenger load).

Let C_m be the number of capacity point values at airport m (passenger load values for each contingent after departure from airport); see fig. 2. Only one capacity point is for starting and for end airport on the route: $C_1 = C_{M+1} = 1$. The total number of capacity points is:

$$C_p = \sum_{m=1}^{M+1} C_m \tag{8}$$

Horizontal links (branches) are representing capacity flows between two neighbor airports on the route. Formula (7) implies that zero values are before loading on the starting point same as after unloading on the ending point.

The objective function for CEP problem can be formulated as follows:

$$\min 1/\left(\sum_{m=1}^{M+1} f_m(I_m) - \left\{ \sum_{i=1}^{N} c_{i,m}\left(x_{i,m} - r_{i,m}\right) \right\} - h_m(I_{\max} - \sum_{I=1}^{N} I_{i,m}) - g_m(z_m) \right) \tag{9}$$

so that we have:

$$I_{i,m+1} = I_{i,m} + x_{i,m} - r_{i,m} \tag{10}$$

$$I_{i,1} = I_{i,M+1} = 0 \tag{11}$$

for $m = 1, 2, \ldots, M+1$; $i = 1, 2, \ldots, N$; $j = \mathrm{i} + 1, \ldots, N$.

In the objective function the total cost (weight) includes some different costs. As we want to incorporate minimization of expenses with profit calculation in the same optimization process than we have to introduce freight cost (passenger tickets) and all expenses have to have negative polarity; see (9). Freight cost (passenger tickets) is denoted with $f_{i,m}(I_{i,m})$. We can differentiate freight cost for each passenger load (contingent).

Transportation cost is denoted with $c_{i,m}(x_{i,m} - r_{i,m})$. The idle capacity cost $h_{i,m}$ $(I_{max}-I_{i,m})$ could be taken into account, but only as a penalty cost to force the usage of maximal capacity (prevention of unused/idle capacity). The airport taxes cost g_m (z_m) has to be introduced, too. With that cost we can include all airport expenses. Costs are often represented by the fix-charge cost or with constant value. It should be assumed that all cost of functions is concave and non-decreasing (some of them reflecting economies of scale) and they differ from one airport to another. The objective function is necessarily non-linear cost. With different cost parameters the optimization process could be strongly influenced, looking for benefits of the most appropriate transportation solution.

The profit will be reduced by transportation costs. Instead of maximization of the profit we can use minimization of the reciprocal value. Generally, the objective function is the exponential cost showing the economy of scale.

The network optimization can be divided in two steps. At first step the minimal transportation weights $d_{u,v}$ is calculated between all pairs of capacity points (neighbor airports on the route). The calculation of each weight value between any couple of capacity points has been named: capacity expansion sub-problem (CES). At second step should be looked for the shortest path in the network with former calculated weights between node pairs (capacity points); see [5]. On that network optimization level problem can be seen as a shortest path problem for an acyclic network in which the nodes represent all possible values of capacity points; see fig. 2. Then Dijkstra's algorithm or any similar algorithm can be applied.

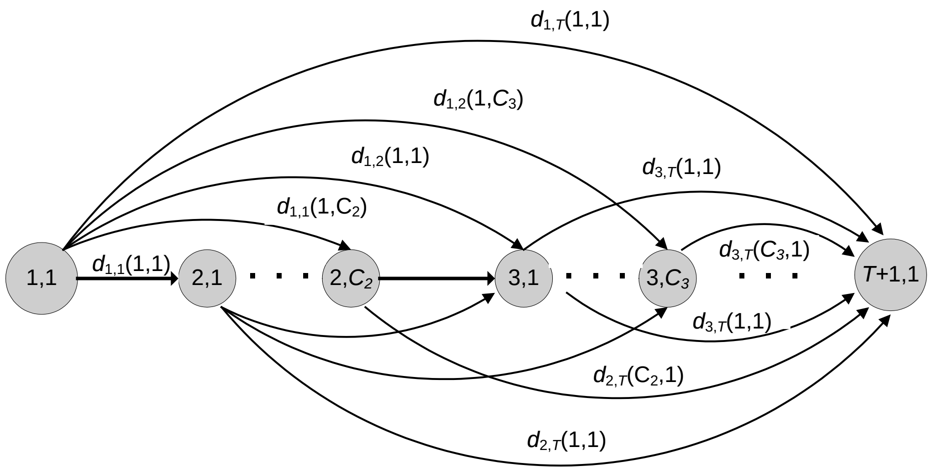

Fig. 2. The shortest path problem for an acyclic network in which the nodes represent all possible capacity points
Fig. 2. El problema de corto recorrido para la red acíclica en la que los nodos representan todos los posibles puntos de capacidad

In our optimization process number of passengers on board do not influence on voyage speed neither to oil consumption but it could be easily incorporated.

The loading strategy consists of loading/unloading plan for each airport and for each passenger contingent. The starting airport on the route can be only for loading and the last airport on the route can be only for unloading; other airports on the route may be for both. Some source airports can have limitation on passenger capacity, but most of them are main airports with capacity exceeding the plane's earning capacity (total capacity of airplane).

5. RESULTS AND DISCUSSION

In route definition we have starting airport (1) and ending airport (5), but three middle airports can also be included in the route, see fig. 3.

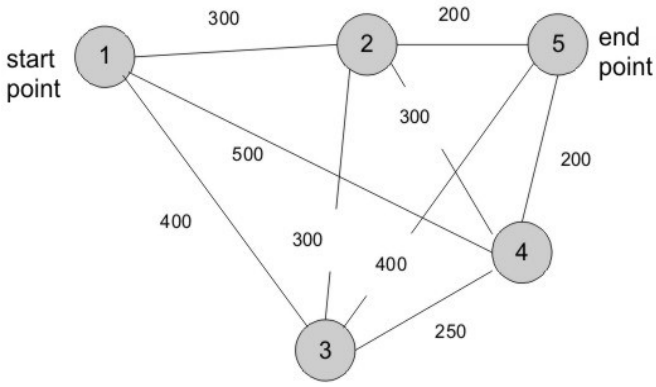

Fig. 3. Airports and distances (an example)
Fig. 3. Aeropuertos y distancias (un ejemplo)

In fig. 4 we can see traffic demands (possible transfer of passengers between airports). That information is gathered through market research or from statistics. This graph also provides the percentage of the potential passengers for particular destination in reference to total airplane capacity. In input data of seven traffic demands it is obvious that most of passengers are interested in the transfer from 1 – 4 airport and from 2 – 5 airport (40 %). According to all costs and the price determination (tickets, oil consumption, etc.) we can design the route which will be more profitable.

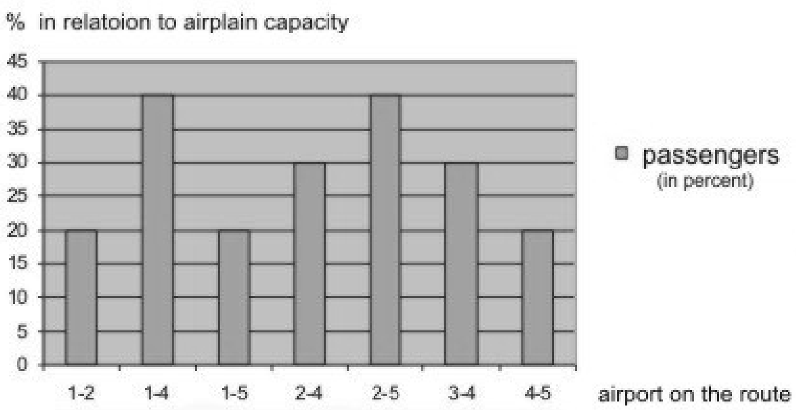

Fig. 4. Potential transfer of passengers between airports given in percentage of the airplane capacity
Fig. 4. Transferencia potencial de los pasajeros entre los aeropuertos indicados en el porcentaje de la capacidad del avión

On the fig. 5 we have an optimal route definition (loading and unloading amounts for particular airport).

On the fig. 6 the airplane occupancy on the route is presented. Amounts of passengers are given in percentage of airplane capacity. In our test-example the optimal route will be from airport 1 to airport 2, to airport 4 and finally to airport 5, excluding airport 3. The optimization solution extracts the airport 3 because it is not profitable to go away from the path (long distance).

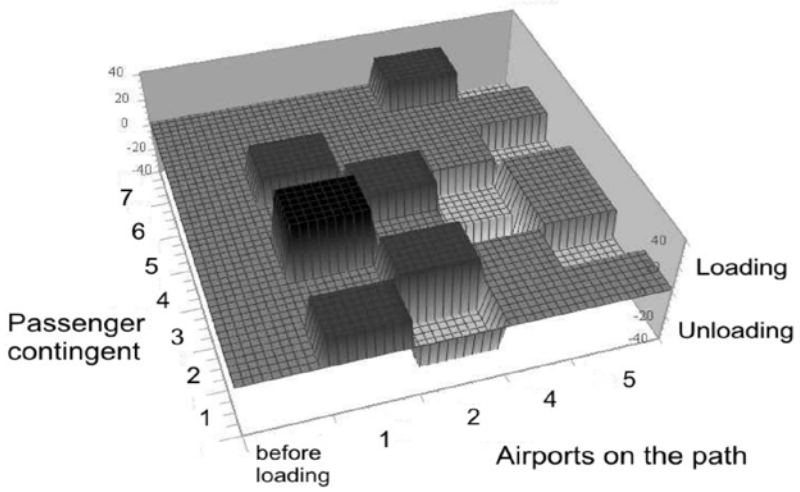

Fig. 5. Optimal solution given by loading and unloading amounts in each airport on the route
Fig. 5. Solución optima determinada por cantidades de carga y descarga en cada aeropuerto en la ruta

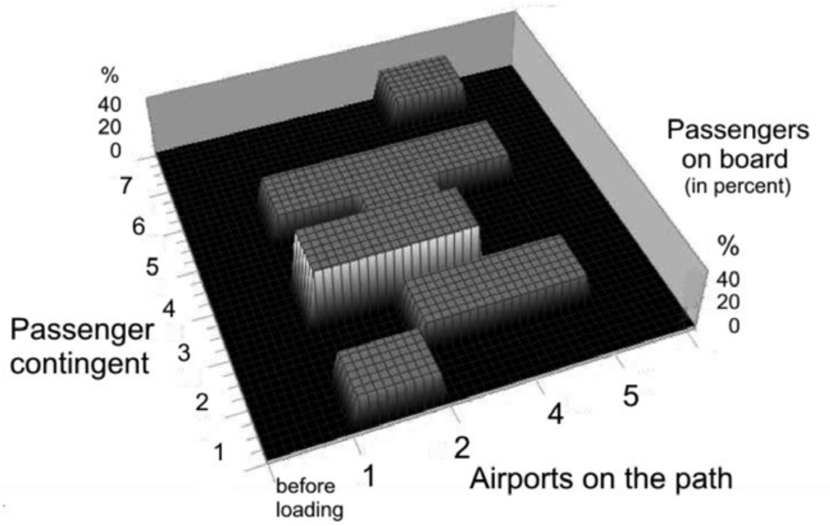

Fig. 6. Airplane's occupancy on the route with particular passenger contingent
Fig. 6. Ocupación de los vuelos en la ruta con particular contingente de los pasajeros

Fig. 7 represents the idle capacity of the airplane during the voyage.

We can see that only from airport 2 to airport 3 we have free capacity (20%) and from airport 4 to airport 5 (40%). For this example all prices for tickets/km are equal but it can be differentiated.

6. CONCLUSIONS

One of the most important problems in airline transportation is to find the sequence of passenger distribution between multiple sources and multiple destinations (stops), minimizing the transportation cost and better utilization of the airplane capacity. With optimizing their routes companies can ensure significant savings and be profitable by following the demand and easily adapt to its changes. Such optimization tool can help in sizing of appropriate airplane, too. So with the smaller planes sometimes company can transport the lower number of passenger if the demand for that particular returning flight is not so high. With comparing the data from both directions we can find the most appropriate and efficient route. Another one possibility in route definition is the change of starting or ending airport.

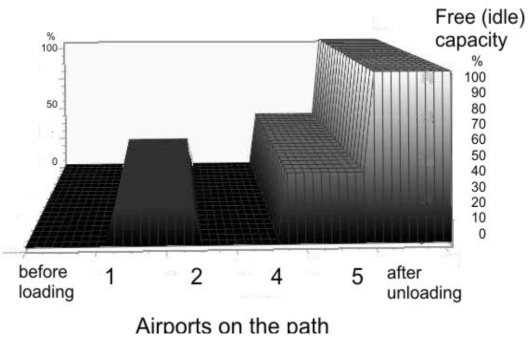

Fig. 7. Free capacity of the airplane on the route
Fig. 7. Capacidad disponible del avión en la ruta

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