travel mode choice; public transport; game theory

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GAME THEORY APPROACH TO OPTIMIZING OF PUBLIC TRANSPORT TRAFFIC UNDER CONDITIONS OF TRAVEL MODE CHOICE BY PASSENGERS

Summary. Consideration was given to the problem of making decisions on public transport management. Three parties making decisions were marked out: passenger flow, transport operator and municipal authorities. Passengers choose between public and private transport evaluating the value of time. The value of time is modeled by uniform distribution. The transport operator seeks to maximize the profit by varying the public transport frequency. The decrease in the frequency decreases the passenger flow of the public transport and cost of transportation. It's important for the municipal authorities to determine the frequency minimizing the time loss of the population and cost of transportation. The existence of the decision concerning the problems of management was proven. The numerical example characterizing the influence of the parameters on the problem solution was given.

ТЕОРЕТИКО-ИГРОВОЙ ПОДХОД ПРИ ОПТИМИЗАЦИИ ИНТЕНСИВНОСТИ ДВИЖЕНИЯ ГОРОДСКОГО ПАССАЖИРСКОГО ТРАНСПОРТА В УСЛОВИЯХ ВЫБОРА ПАССАЖИРАМИ СПОСОБА ПЕРЕДВИЖЕНИЯ

Аннотация. Рассмотрена проблема принятия решений по управлению городским пассажирским транспортом. Выделены три объекта принимающих решения: пассажиропоток, транспортный оператор и муниципальные органы власти. Пассажиры выбирают между общественным и личным транспортом на основе оценки стоимости своего времени. Стоимость времени пассажиропотока считается распределенным равномерно. Транспортный оператор стремится максимизировать прибыль, изменяя частоту движения. Снижение частоты движения снижает пассажиропоток на общественном транспорте и транспортные расходы. Для муниципалитета важно определить частоту движения, минимизирующую потери времени населения и транспортные расходы. Доказано существование решения в задачах управления. Приведен численный пример описывающий влияние параметров на решение задачи.

1. INTRODUCTION

Public transport reforming is connected with introduction of market mechanisms in management [2, 8, 9]. The implementation of incentives and restrictions for passengers and transport operators will allow efficient developing of public transport system. Therefore it's necessary to set the most efficient way of regulating the passenger transportation system, including the determination of the municipal authorities’ policy with account of transport operators’ and passengers’ interests.

Public transport is a social-economic system with a passenger being a basic element of it. Due to the increasing level of automobilization a passenger activity is rising. A passenger activity is the existence of a great number of strategies (capability to mode choice and the route), and objective function as well (minimizing of losses linked with transportation).

Thus, the considerable changes under the increasing level of automobilization in the developing nations led to the necessity of constructing new mathematical models of urban passenger transportation management systems. Management systems should take into account the existence of active agents (transport operators and passengers) in public transport [7].

The difference of interests leads to the necessity of applying the game theory for making decisions by the participants of urban passenger transportation system. However, at the moment the game theory is hardly applied for characterizing public transport system [4]. Optimization of public transport management should be based on seeking the equilibrium between the interests of passengers, transport operators and municipal authorities.

2. PARTICIPANTS OF PUBLIC TRANSPORT MANAGEMENT SYSTEM

It’s offered to consider two statements of the problem of public transport management. They are connected with different approaches to public transport management: whether public transport is managed by private operator or by municipal authorities.

The first statement implies that the transport operator is aware of how passengers will respond to the change in public transport operation. It means that, reducing the interval of travel; the transport operator incurs additional costs and gains additional revenue from the passengers who have chosen public transport. As a result we have the most simple one-criterion problem with one variable.

The second statement of the problem is connected with the necessity of taking into account additional impact of public transport on the urban environment. Passenger transportation by private vehicles requires much more resources than by public transport [9]. Therefore, optimizing the passenger transportation, it’s necessary to take into account the total loss of people and transportation costs.

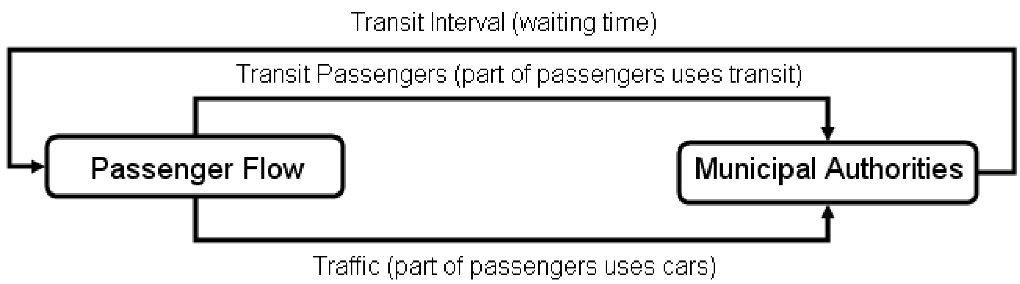

Fig. 1. Public transport management system
Рис. 1. Система управления городским пассажирским транспортом

Figure 1 shows that controlling the interval of travel, the municipal authorities vary the passenger flow in public transport and private vehicles flow.

2.1. Passenger flow

Nowadays the travel mode choice is studied much [5]. There are two main modes of traveling for long distances: a private car and public transport.

The most important characteristic of efficiency of cars is their higher speed of movement. It deserves noting that the time of transportation by public transport includes [9]:

– travel time from origin to transit station,

– travel time from entering station to vehicle platform,
– vehicle waiting time,
– vehicle boarding time,
– vehicle travel time,
– vehicle alighting time,
– travel time from vehicle platform to station exit,
– travel time from station exit to final destination.

The time of car transportation [9]:
– reaching the parking from the place where the need for transportation occurred;
– leaving the parking;
– transportation by car;
– parking;
– reaching the destination from the parking place.

Choice is also influenced by the cost of transportation, speed of the public transport and cars on the given part of the road network, cost of car transportation, as well the importance of travel. “Importance” is expressed in terms of the person’s value of time used for travel [5, 6].

Passengers can be divided into 3 categories:
1) those who use public transport in any case;
2) those who can change their travel mode choice depending on the conditions;
3) those who use a car in any case.

A person chooses mode in advance (car or public transport) from the knowledge of transport frequency. Let’s suppose in the model that the risk of travel by car or bus is disregarded (no sensitivity to risk) and the mode chosen in terms of average characteristics, that is, the mode is chosen uniquely for the given value of time.

In contrast to [6] where the value of time was characterized with exponential distribution, let’s assume that the value of time is described by uniform distribution [1], then public transport is used by the part of the population with lower income (as a car allows reducing the total travel time while increasing the financial spending).

The main parameters of the model of passenger flow:

p_0 - the part of the passengers who always use public transport;

p_1 - the part of the passengers who always use cars;

a , b - parameters of uniform distribution of the value of time for the passengers who are ready to change travel mode ($a < b$);

$\gamma_0 \le a$ average value of time of the passengers always using public transport;

$\gamma_1 \ge b$ average value of time of the passengers always using cars;

p - the probability of using public transport for the passengers who are able to change mode;

t_0 – average time of travelling by public transport (with the exception of time of waiting);

Δt – average time of waiting for public transport;

t_1 – average time of travelling by car;

c - costs of using car;

β - fare at the public transport.

The part of the passengers who are able to change travel mode $(1 - p_0 - p_1)$.

Let’s calculate the average losses of passengers per unit of travel.

The time losses of the passengers who are not ready to change the mode in terms of money $\gamma_0 p_0$ and $\gamma_1 p_1$.

Let's consider in detail the passengers having the probability of changing the mode. The value of time of changing the mode

$$a+(b-a)p \; . \tag{1}$$

Then the average value of time of the passengers who have chosen public transport

$$\frac{a+a+(b-a)p}{2}=a+\frac{(b-a)p}{2} \; . \tag{2}$$

The average value of time of the passengers who have chosen a car

$$\frac{b+a+(b-a)p}{2}=\frac{b+a}{2}+\frac{(b-a)p}{2} \; . \tag{3}$$

To calculate the value of time it's necessary to multiply the average value of time (2, 3) by the probability of choice of the given mode and by the time of travelling.

$$\left(a+\frac{(b-a)p}{2}\right)p(t_0+\Delta t)+\left(\frac{b+a}{2}+\frac{(b-a)p}{2}\right)(1-p)t_1=$$
$$=\frac{b-a}{2}(t_0-t_1)p^2+a(t_0+\Delta t-t_1)p+\frac{b+a}{2}t_1. \tag{4}$$

The costs of travelling will be as follows

$$\beta p+c(1-p) \; . \tag{5}$$

The total costs of flow per one travel are the sum of the costs of travelling by car and public transport (with account of the passengers not willing to choose mode):

$$G=\left(\frac{b-a}{2}(t_0+\Delta t-t_1)p^2-(c-\beta-a(t_0+\Delta t-t_1))p+c+\frac{b+a}{2}t_1\right)(1-p_0-p_1)+$$
$$+\gamma_0 p_0(t_0+\Delta t)+\gamma_1 p_1 t_1+\beta p_0+cp_1 \tag{6}$$

The aim of the flow of population is to minimize the total costs of travelling by varying the parameter p . To find the solution let's take the first derivative of (6) with respect to the parameter p.

$$G'=[(b-a)(t_0+\Delta t-t_1)p-(c-\beta-a(t_0+\Delta t-t_1))](1-p_0-p_1) \; . \tag{7}$$

The second derivative is more than zero, which shows the downward convexity of the function of costs and ensures the existence of the unique decision in the admitted set $p\in[0,1]$:

$$G''=(b-a)(t_0+\Delta t-t_1)(1-p_0-p_1)>0 \; . \tag{8}$$

It should be noted that the solution (9) is based on (7).

$$p=\begin{cases}1, & (t_0+\Delta t-t_1)\le\dfrac{c-\beta}{b};\\[2ex] \dfrac{(c-\beta-a(t_0+\Delta t-t_1))}{(b-a)(t_0+\Delta t-t_1)}, & \dfrac{c-\beta}{b}<(t_0+\Delta t-t_1)\le\dfrac{c-\beta}{a};\\[2ex] 0, & (t_0+\Delta t-t_1)>\dfrac{c-\beta}{a}.\end{cases} \tag{9}$$

2.2. Transport operator

Transport operator's profit directly depends on the probability of the travel mode choice by passengers. The probability of using public transport

$$\overline{p}_0=p_0+(1-p_0-p_1)p \; . \tag{10}$$

The probability of using cars

$$\overline{p}_1=p_1+(1-p_0-p_1)(1-p) \; . \tag{11}$$

The interval of travel of public transport under the determined flow of transport is two times more than the time of waiting, therefore the frequency of public transport

$$\mu = \frac{1}{2\Delta t} \ . \tag{12}$$

Transport operator's profit is the difference between the revenue from selling tickets to passengers and costs of transportation.

$$H = \overline{p}_0 \lambda\beta - \alpha\mu \ , \tag{13}$$

where α is cost of one round-trip of a vehicle.

If to substitute the passenger decision into the formula of the revenue (9), then we'll have

$$H = \left(p_0 + (1 - p_0 - p_1) \frac{\left(c - \beta - a\left(t_0 + \frac{1}{2\mu} - t_1 \right) \right)}{(b-a)\left(t_0 + \frac{1}{2\mu} - t_1 \right)} \right) \lambda\beta - \alpha\mu \ . \tag{14}$$

2.3. Municipal authorities

For optimal city development it's necessary to save two components: time of population and costs of transportation. Herewith it's already determined that time of population can be presented in terms of money [1, 5]. Costs of transportation are also a broad notion. Except direct expenditures they may include road maintenance expenditures, ecological damage from transport operation.

In the given case passengers make a decision on mode choice, and the municipal authorities managing public transport seek to minimize costs of transportation and time losses of passengers. In this case a conflict arouses between passenger flows and the municipal authorities.

The function of losses of the system "city" consists of several components [6, 7]:

1) time losses of passengers connected with travelling by public transport;
2) damage to the urban environment from public transport operation;
3) time losses of population while travelling by cars;
4) damage to the urban environment from cars.

The average value of time of bus passengers

$$\overline{\gamma}_0 = \left(a + \frac{(b-a)p}{2} \right)(1 - p_0 - p_1)p + \gamma_0 p_0 \ . \tag{15}$$

The average value of time of those traveling by car

$$\overline{\gamma}_1 = \frac{b + a + (b-a)p}{2}(1 - p_0 - p_1)(1 - p) + \gamma_1 p_1 \ . \tag{16}$$

Herewith the average value of time of all the passengers $\overline{\gamma}_0 + \overline{\gamma}_1$. Let's introduce additional parameters c_0 and c_1, which characterize the damage to the urban environment per travel of public transport and cars respectively.

Then the losses of the city will be as follows

$$F = \left(t_0 + \frac{1}{2\mu} \right)\left[\left(a + \frac{(b-a)p}{2} \right)(1 - p_0 - p_1)p + \gamma_0 p_0 \right]\lambda + c_0\mu +$$
$$+ t_1\left[\frac{b + a + (b-a)p}{2}(1 - p_0 - p_1)(1 - p) + \gamma_1 p_1 \right]\lambda + c_1\lambda\left(p_1 + (1 - p_0 - p_1)(1 - p) \right) \to \min \ . \tag{17}$$

Herewith it's easy to prove the downward convexity of the given formula with respect to the parameter μ .

3. STATEMENTS OF PROBLEMS OF PUBLIC TRANSPORT MANAGEMENT

Let's consider two statements of the problem. In the first one transport operator maximizes its profit under varying the interval of travel (or the public transport frequency). Such a management system can be called management by private transport operator.

The second statement of the problem implies minimizing the total expenditures of the transport and passengers. Thus, management is done by the municipal authorities.

3.1. Private transit

The problem of optimization the traffic intensity of public transport by a private operator consists in maximizing the profit (14).

$$H=\left(p_0+(1-p_0-p_1)\frac{\left(c-\beta-a\left(t_0+\frac{1}{2\mu}-t_1\right)\right)}{(b-a)\left(t_0+\frac{1}{2\mu}-t_1\right)}\right)\lambda\beta-\alpha\mu\to\max\ . \tag{18}$$

To prove the existence of the unique solution let's check the upward convexity in terms of the variable (μ). For this we'll take the second derivative from (14):

$$-\frac{(1-p_0-p_1)\lambda\beta(t_0-t_1)[a(t_0-t_1)+(c-\beta-a(t_0-t_1))]}{(b-a)\left((t_0-t_1)\mu+\frac{1}{2}\right)^3}<0\ . \tag{19}$$

3.2. City-owned transit

In the given case it's necessary to solve a more complicated problem of management. Passengers minimize their losses (3), and the municipal authorities - theirs (17).

$$\begin{cases} G\to\min\limits_{p} \\ F\to\min\limits_{\mu} \end{cases}. \tag{20}$$

Thus, we get the statement of the problem in the form of game $\langle\{PF,MA\},p,\mu,-G,-F\rangle$. Two participants (Passenger Flow and Municipal Authorities), which have a continuous set of strategies. To prove the existence of the solution (Nash equilibrium) under the given conditions it's enough that the statement of the problem satisfies a number of conditions [3].

Firstly, a set of strategies is compact, convex and not empty. It's evident that the probability of choice of the mode p satisfies this condition. The frequency of public transport μ also satisfies this requirement under the evident condition that it is restricted from above (the frequency of public transport is limited by traffic capacity of roads).

Secondly, payoff functions of players (in the given case players' expenditures) are continuous in all the strategies of all the players and quasiconcave in their own strategies. Continuity is evident, and quasiconcavity is replaced by a stronger characteristic - convexity upward.

Therefore there's Nash equilibrium in the problem.

4. THE NUMERICAL EXAMPLE

The problem solution in any of the two statements is accomplished by numerical methods. Let’s see how the frequency of public transport varies depending on the parameters of the model. Herewith private transport operator and municipal authorities come to different solutions.

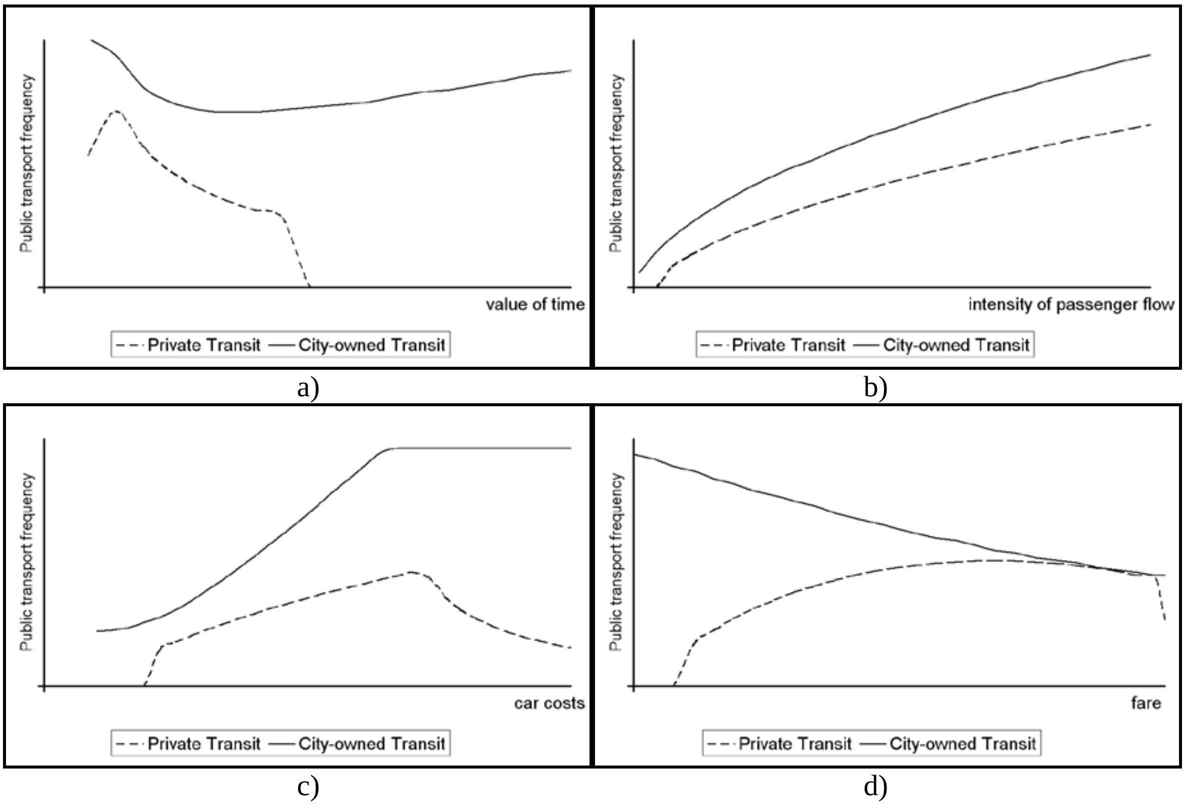

Fig. 2. Influence of the parameters of the models on the equilibrium frequency of public transport: a) cost of a passenger-hour; b) passenger flow intensity; c) cost of owning a car; d) fare at public transport

Рис. 2. Влияние параметров моделей на равновесную интенсивность движения общественного транспорта: a) стоимость пассажиро-часа; b) интенсивность пассажиропотока; c) стоимость владения автомобилем; d) тариф на общественном транспорте

In the developing nations the level of income is changing greatly, which is reflected in the cost of time. Fig. 2a shows that under a very low level of life it makes no sense for a transport operator to increase frequency – passengers have practically no choice. On the other hand, under a high income level of population it’s more difficult to attract passengers to public transport. Thus, transport operator has incentives to increasing the frequency just in a narrow range of value of time. But it’s for the benefit of rational city development to have higher frequency under a low value of time (large passenger flow in public transport). Under a high value of time it’s necessary to improve the quality of passenger service to attract more hard-to-please passengers to public transport (to reduce the flow of private vehicles).

Fig. 2b shows that the growth of passenger flow intensity leads to the growth of frequency of public transport. Fig. 2c shows what impact a cost of using a car will have. For transport operator the diagram is similar to fig. 2a, and by the way for the same reasons. For the municipal authorities the more the probability of using public transport is, the more its frequency is. And lastly, fig. 2d shows that the growth of fare for passenger transportation leads to reducing the frequency of public transport. That is, it’s important for the municipal authorities to minimize the fare. For transport operator there is an optimal fare at which the profit and frequency are maximum.

5. CONCLUSIONS

In conclusion it will be noted that public transport should be regulated by the municipal authorities, as private operator will reduce frequency of public transport (fig. 2), thereby having increased the flow of private vehicles which will lead to the growth of finance for construction and maintenance of transport infrastructure. Further research is connected with the generalization of the obtained results on a great number of passenger flows and transport operators. Also the improvement of the models is connected with taking into account the traffic capacity of roads and congestion.

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