

rotor dynamics, simulations, Simulink

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## **SIMULINK LIBRARY PROJECT FOR MODELING AND SIMULATION OF DYNAMIC PHENOMENA IN ROTATING POWER TRANSMISSION SYSTEMS**

**Summary.** This paper presents the concept and an example of usage of Simulink blocks library with which dynamic simulation of complex systems with rotating shafts, rigid rotors, bearings and couplings, general rotating power transmission systems of any configuration can be performed. The assumption is that library is modular and expandable. The main part of the library currently being developed is rigid rotor model with 6 degrees of freedom of the static and dynamic imbalance. Other components are: block modeling the bearing with mounting stiffness, damping and inertia; linear elastic-damping element and rigid beam finite element (RFEM). Also in preparation are: block modeling shaft with Timoshenko beam elements and Rayleigh damping, block modeling clutch.

## **PROJEKT BIBLIOTEKI SIMULINKA DO MODELOWANIA I SYMULACJI ZJAWISK DYNAMICZNYCH W WIRUJĄCYCH UKŁADACH PRZENOSZENIA MOCY**

**Streszczenie.** W artykule przedstawiono koncepcję oraz przykład zastosowania biblioteki bloków Simulinka, za pomocą której można przeprowadzić symulacje dynamiczne układów złożonych z wirujących wałów, sztywnych wirników, łożysk i sprzęgieł, ogólnie wirujących układów przenoszenia mocy o dowolnej konfiguracji. Biblioteka z założenia ma charakter modułowy z możliwością rozbudowy. Głównym elementem rozwijanej obecnie biblioteki jest model sztywnego wirnika o 6 stopniach swobody z niewyważeniem statycznym i dynamicznym. Pozostałe elementy to: blok modelujący łożysko ze sztywnością montażową, tłumieniem i bezwładnością; liniowy element sprężysto-tłumiący i sztywny element skończony belkowy (MSES). W przygotowaniu są także: blok modelujący wał z elementami belkowymi Timoshenki i tłumieniem Rayleigha; blok modelujący sprzęgło.

### **1. INTRODUCTION**

The questions of transmission dynamics are very important for the vehicles. This problems may be solved using classical approaches [8] or using numerical methods. Currently in simulation researches of rotors dynamics FEM is extensively applied. Many types of special finite elements modeling shaft and rotors including effects associated with rotation and unbalance are elaborated. Numerous specialist software is available (e.g. [1, 3]). An example of large commercial FEM software with

package for rotors dynamics analysis attached is ANSYS [7, 10]. Disadvantage of FEM is large number of degrees of freedom. However, it is not significant while the goal of the research is modal analysis, but may be troublesome in case of research of motion in time domain. Necessary may be applying various methods of model's degrees of freedom condensation [4], what can be a source of errors.

Below the concept of Simulink blocks library is introduced, which can be used in modeling of typical series elements of power transmission systems. Library by assumption is modular and open. Modularity allows developing models of systems of arbitrary configuration with use of available blocks. Openness means optional extending and completing the library.

Idea of decomposition of rotating system to stiff and elastic elements is applied. It partially remains the technic of continuous-discrete modeling [5]. Simulation models obtained with this method have relatively little degrees of freedom. Library is dedicated for analysis of unstable states (starts, disturbances during motion, etc.) and analysis of flexural-torsional and longitudinal vibrations.

## 2. GENERAL CONCEPT OF A ROTATING SYSTEM DECOMPOSITION

In a typical rotating system there are elements they usually have substantial mass and diameter as well as high stiffness. The elements may be treated as rigid bodies and are further called the inertial components. The example of such element is a rigid rotor. Moreover, there occur compliance elements in which elastic properties are more significant than inertial properties. Such elements are machine shafts. The figure 1 shows an example of decomposition in the typical rotating system.

It is assumed that inertial elements during motion are acting on compliance elements by kinematic input. While, compliance elements act on inertial elements by forces coming mainly from their elastic properties and internal damping. Each inertial element neighbors with one or two compliance elements (shafts).

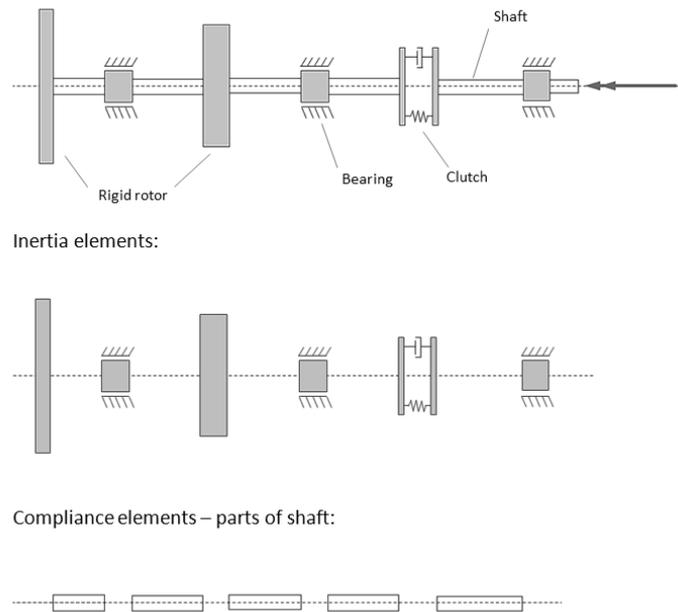


Fig. 1. Example diagram of rotating system and the proposal its decomposition  
Rys. 1. Przykładowy schemat układu wirującego i propozycja jego dekompozycji

Typically, an inertial element has six degrees of freedom (see also Fig. 3):

$$q = [x_C, y_C, z_C, \psi, \theta, \varphi]^T. \quad (1)$$

Where the first three coordinates are coordinates of a geometrical center of a component (in case of unbalanced elements it does not overlap the center of a mass), last three coordinates are the Euler's angles describing inclination of a rotor's disc and its rotation around the axis perpendicular to the disc.

In the contrary to existing in Simulink methods of modeling mechanism (Sim/Mechanics) [9] and some concepts for modeling in mechatronics [2], in proposed model two types of signals are used (Fig. 2). The kinematic signal:

$$\mathbf{q} = [q, \dot{q}, \ddot{q}], \tag{2}$$

and the force signal (generalized forces):

$$\mathbf{Q} = [Q_{x_c}, Q_{y_c}, Q_{z_c}, Q_{\psi}, Q_{\theta}, Q_{\varphi}]^T. \tag{3}$$

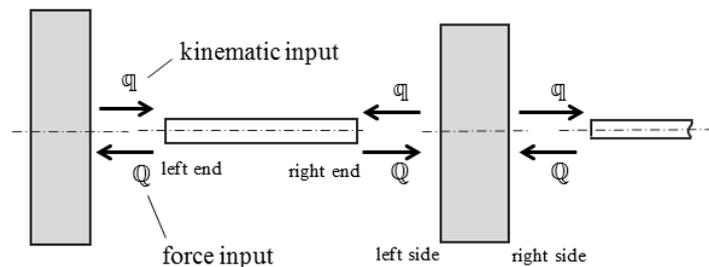


Fig. 2. Two types of signals – input kinematic from the inertial components, input force from the compliance (elastic) components

Rys. 2. Dwa rodzaje sygnałów – wymuszenie kinematyczne od elementów inercyjnych, wymuszenie siłowe od elementów podatnych (elastycznych)

### 3. MODELS OF THE MAIN ELEMENTS OF THE LIBRARY

#### 3.1. Model of a thin rigid rotor

In presented model position of a stiff rotor in space is described by three translational coordinates  $\{x_c, y_c, z_c\}$  and three rotational coordinates  $\{\psi, \theta, \varphi\}$ . Point C is a geometrical center of a rotor and it is also the beginning of a mobile system  $\{\xi, \eta, \zeta\}$  permanently connected with the rotor. Axes  $\{\xi, \eta\}$  mark a rotation plane, an axis  $\zeta$  is related to a geometrical axis of rotation (Fig. 3). Angles  $\{\psi, \theta, \varphi\}$  create a system of Euler's angles chosen in such a way to avoid overlapping of axes (blocking) during possible movements of the rotor. Angles  $\{\psi, \theta\}$  describe inclination of a rotor's plane and must be small. Rotation angle  $\varphi$  associated with the axis  $\zeta$  may have arbitrary values.

The rotor is unbalanced statically, which means that the rotor's mass center P does not overlap the geometrical center C. Unbalance can be determined with two constants: eccentricity  $e$  and angle  $\beta$ . In general case axes of a local system  $\{\xi', \eta', \zeta'\}$  connected with point P does not overlap principal axes of rotor's inertia and to describe their relative position three angles are necessary. Dynamic unbalance is usually very small and can be treated as a minor deflection of the vertical axis  $\zeta'$  [1].

Transition from the system  $\{\xi', \eta', \zeta'\}$  to the system of principal axes (1,2,3) can be performed with an assumption of minor deflection of the vertical axis, with rotation of the plane  $\{\xi', \eta'\}$  with precede angle  $\gamma$  to an axis  $\zeta'$ , and then inclining obtained system  $\{\xi'', \eta'', \zeta''\}$  with an angle  $\delta$  (rotation about an axis  $\eta''$ ). So, the dynamic unbalance is described by constants:  $\gamma$  – precede angle of principal axes,  $\delta$  – inclining angle of principal axes.

Mathematical model of dynamics of the rigid rotor is described in the work of the author [6]. Can see there how decoupling inertia coupled equations of motion.

The rigid rotor dynamic model was closed in a single Simulink block. The output of the rotor block represents the actual rotor configuration in the space. The input signal consists of resultant forces coming from the neighbouring flexible shafts, and other forces such as mass forces and damping forces. In the case of a long rotor, it is necessary to add two additional outputs representing the rigid movements of rotor ends.

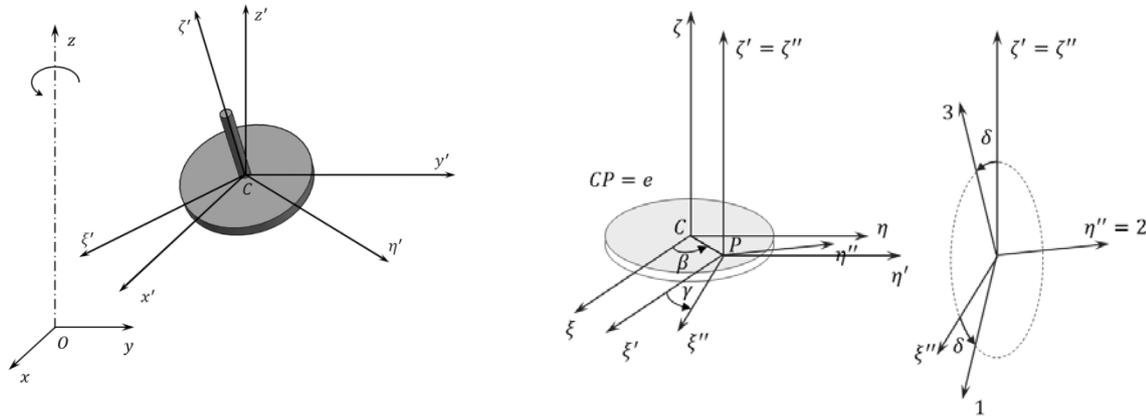


Fig. 3. Coordinates systems and definition of the static and dynamic unbalance  
 Rys. 3. Układy współrzędnych oraz definicja niewyważenia statycznego i dynamicznego

**3.2. Model of a shaft**

The following component of the library is a block modeling of a shaft which in the simplest case may be modeled by basic elastic-damping element. Shaft has twelve external degrees of freedom defining the position of its ends. Those degrees of freedom are determined by position of inertial elements neighboring with a shaft (Fig. 4). Prismatic shaft or with linearly alternating cross-section may be modeled with use of beam elements (FEM or RFEM). Additional internal degrees of freedom are calculated within a block. Method of rigid finite elements may be used also in case of shaft of complicated shape, modeling it with rigid finite elements (rigid blocks) and elastic-damping elements.

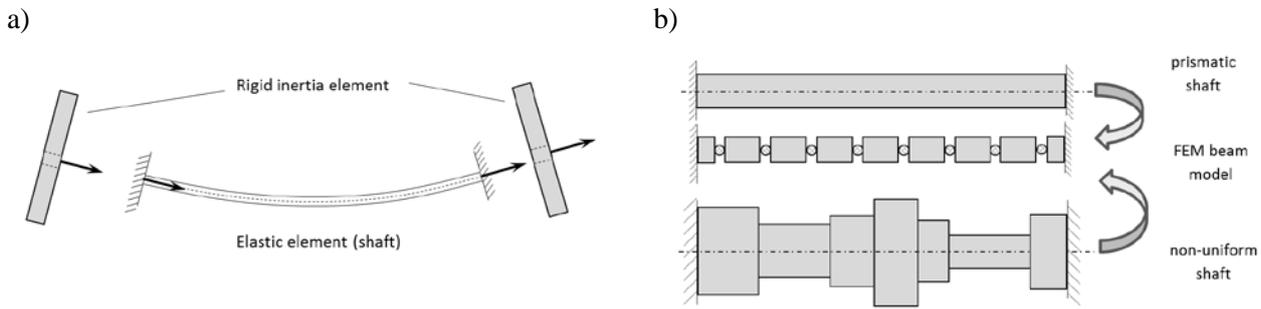


Fig. 4. Model of the shaft: (a) cooperation rigid element with a shaft, (b) division of the shaft into finite elements  
 Rys. 4. Modelowanie wału: (a) współpraca elementu sztywnego z częścią wału, (b) podział części wału na elementy skończone

RFEM method is less accurate than FEM, but its main advantage is the simple diagonal mass matrix, stiffness matrix and damping matrix structure. This simplifies the numerical procedures and strongly accelerates the calculations.

For shafts with a very complex structure it is possible to make a complete FEM model using one of the available commercial packages. Then cut down the degrees of freedom by selected method of

condensation or modal synthesis. However, keep these degrees of freedom that will allow the transfer of signals according to the idea proposed in this paper. Reduced equations of motion can be closed by a special block of Simulink.

### 3.3. Model of a bearing

Despite of a relatively small mass of a bearing, in the proposed system bearing is treated as a stiff inertial element. In this way, the problems associated with setting the boundary conditions directly in the elastic elements (shafts) can be omitted. Bearings as it is in fact, supports the shaft. The bearing should have a mass, so that it is compatible with the other flexible elements in the system.

In the simplest case of the ideal stiff bearing two degrees of freedom should be taken into account. The first rotational connected with rotation, the second translational corresponding to the longitudinal movement. In the thrust bearings translational degree of freedom is blocked. With a little effort, the friction force and motion resistance proportional to velocity can be introduced to the model. The bearing in end of the shaft may be used to startup of the system by applying torque. Can also, apply load of the system by axial force or by torque of resistance.

The more complex model which takes account of the stiffness of the bearing assembly (elasticity of the housing and the base) has to be developed with using at least two additional translational degrees of freedom. Transversal displacements of a bearing generate elastic and damping forces (so called non-rotational damping [1]). In direction of unblocked degrees of freedom additional forces stimulating vibration of base may be applied.

In even more complex models, which comply motion of internal elements of a bearing (inner ring, rolling elements - rolls or balls, basket), internal degrees of freedom may be included. They do not influence to the observed motion of the bearing – i.e. signal type of kinematic, generated by the bearing.

### 3.4. Model of a clutch

Clutch can be considered in the form of two thin rigid whirling disks interacting with each by forces, which depend on mutual position and relative motion of plates.

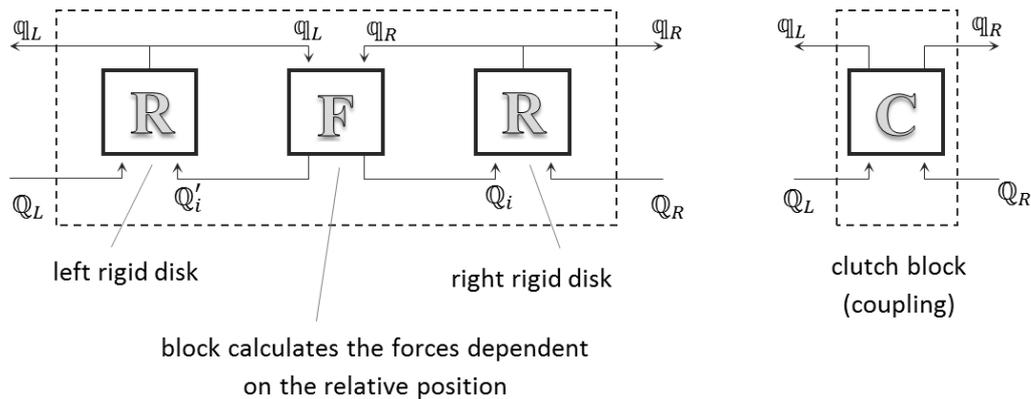


Fig. 5. General functional block diagram to clutch modeling (coupling)  
 Rys. 5. Ogólny schemat funkcjonalny bloku modelującego sprzęgło (połączenie)

General idea of the block to modeling the shafts is presented in the fig. 5. It consists of two blocks representing plates of the clutch and block calculating forces of interaction of plates by a given function. The form of a function depends on a type of clutch and may be approximately determined on a basis of static and dynamic elastic-damping characteristics. It demands experimental and simulation

studies to identify parameters of the function. Usually, unbalances of disks can be omitted due to small masses.

#### 4. NUMERICAL EXAMPLE USING THE LIBRARY

Application of the proposed library showing a simple example spinning disc supported symmetrically on two bearings (Fig. 6).

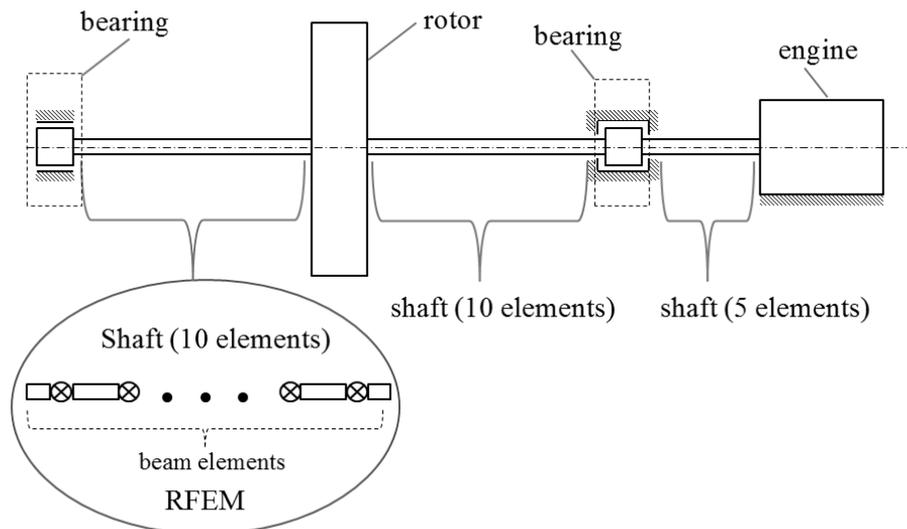


Fig. 6. Scheme of the rotating system  
Rys. 6. Schemat układu wirującego

The rotating machine consists of three inertial elements: two bearings and a rigid rotor. The polar moment of inertia of the rotor is two times larger than transversal moment of inertia. This makes that the rotor can be regarded as a thin disk. Three parts of the shaft are modeled with RFEM. Each part is divided into finite beam elements of the length 0.1m (Fig. 6).

Model of presented rotating machine made in Simulink is shown in Fig. 7. Using the constructed model is simulated very fast start-up machine. During time less than one second engine reaches 8000 rotations per minute. Engine kept constant angular acceleration and later the constant angular speed. Figure 8 shows the change over time in the rotation angle and angular velocity of the disk. Clearly visible is the impact of the torsional vibration on the speed pattern. The figure 9 shows the difference between the rotation angles of the rotor and the bearing.

The shaft has a high bending stiffness and flexural vibrations are small. Trajectories of the center of a mass and the geometric center shown in the drawings (Fig. 10, 11) are very stable.

Flexural vibrations disappear very slowly because in the system is only a weak internal damping in elastic-damping elements representing the material properties of the shaft. After stabilization of the engine speed the flexural vibration are clearly modulated by torsional vibrations (Fig. 12). Inclination of the plane of rotation represented by the angles  $\{\psi, \theta\}$  is very small, the disk rotates stable. In this case also visible is the effect of torsional vibration (Fig. 13).

#### 5. CONCLUSIONS

The library of Simulink blocks described above is during development and testing. Presented method of modeling of rotating systems is an alternative to existing systems based on FEM. Its advantage is the relatively small number of degrees of freedom which reduces simulation time and

allows the study of non-steady states with low cost of computer hardware. Author's plans, in the nearest future, to use the model to evaluate methods of active damping of torsional vibrations.

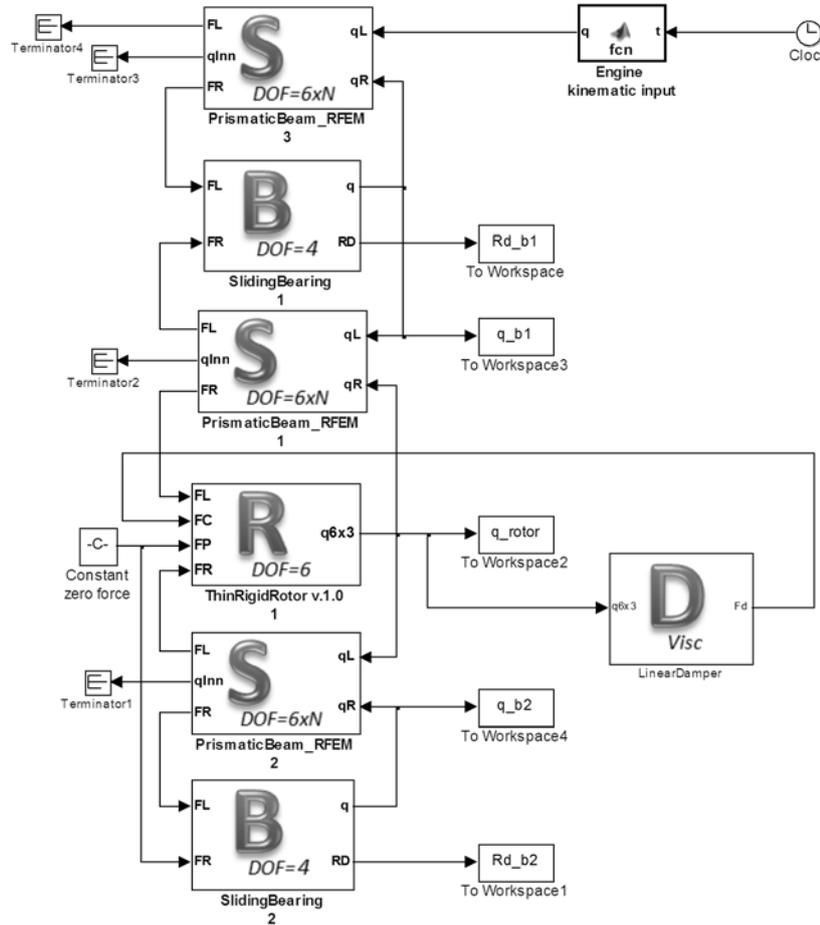


Fig. 7. Scheme in the Simulink  
Rys. 7. Schemat w Simulinku

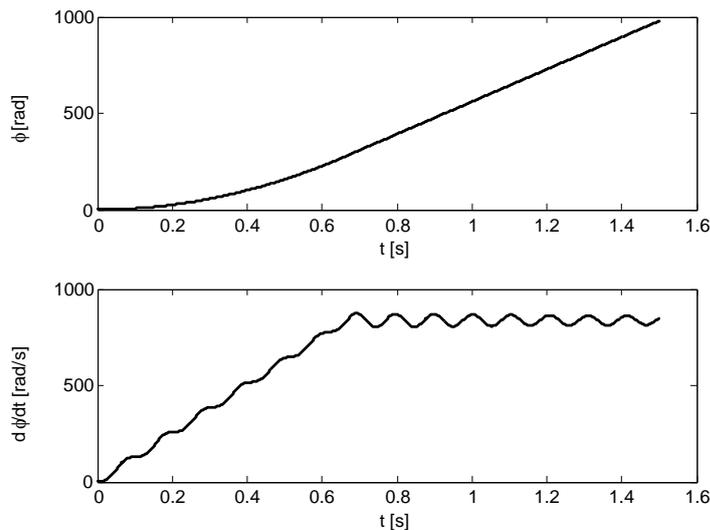


Fig. 8. The angle of rotation and the angular velocity of the rotor  
Rys. 8. Kąt obrotu i prędkość kątowna wirnika

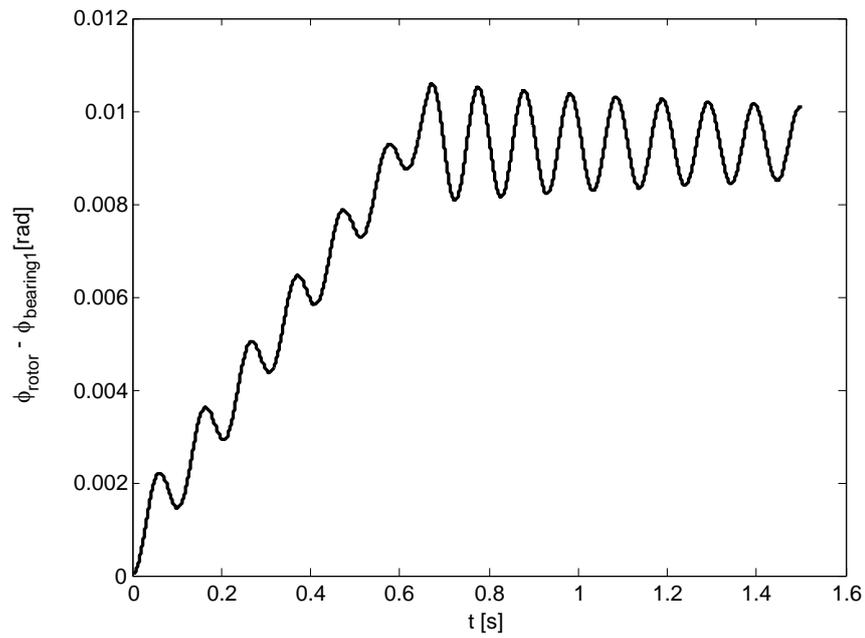


Fig. 9. Difference between the rotation angles of the rotor and the bearing  
Rys. 9. Różnica pomiędzy kątami obrotu wirnika i łożyska

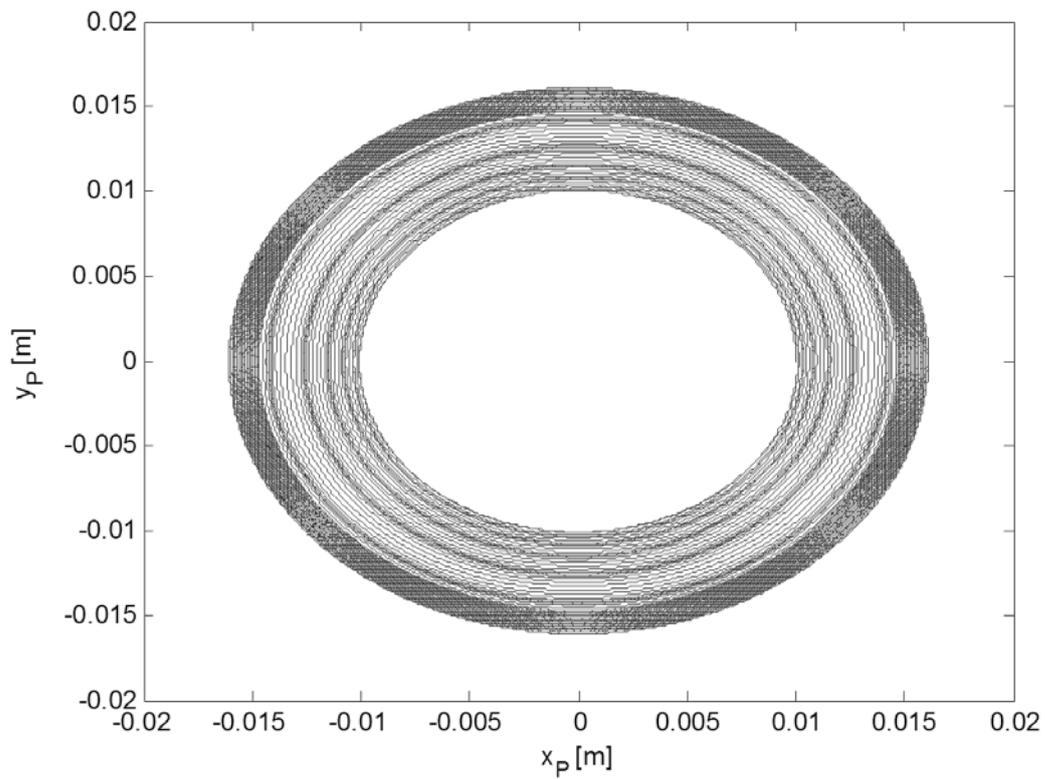


Fig. 10. Trajectory of the center of mass  
Rys. 10. Trajektoria środka masy

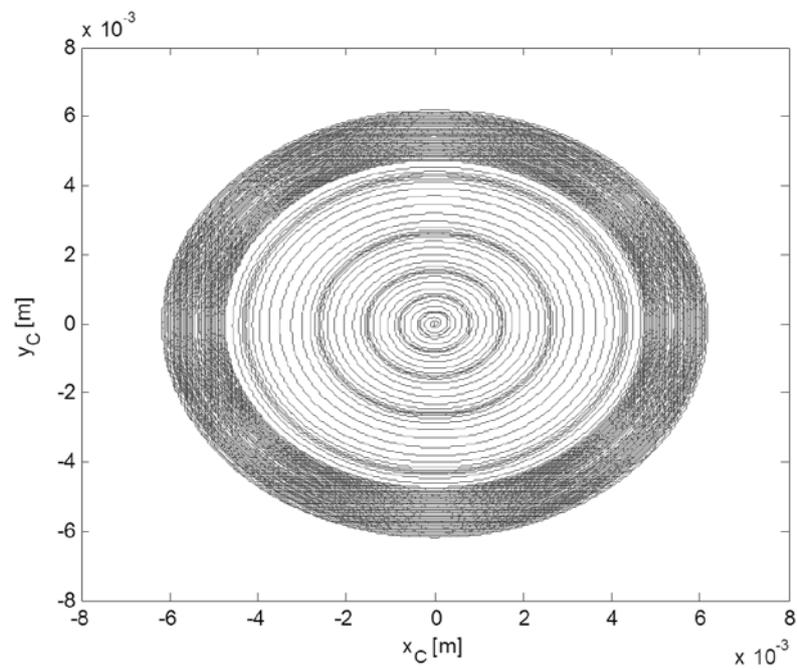


Fig. 11. Trajectory of the geometric center  
Rys. 11. Trajektoria środka geometrycznego

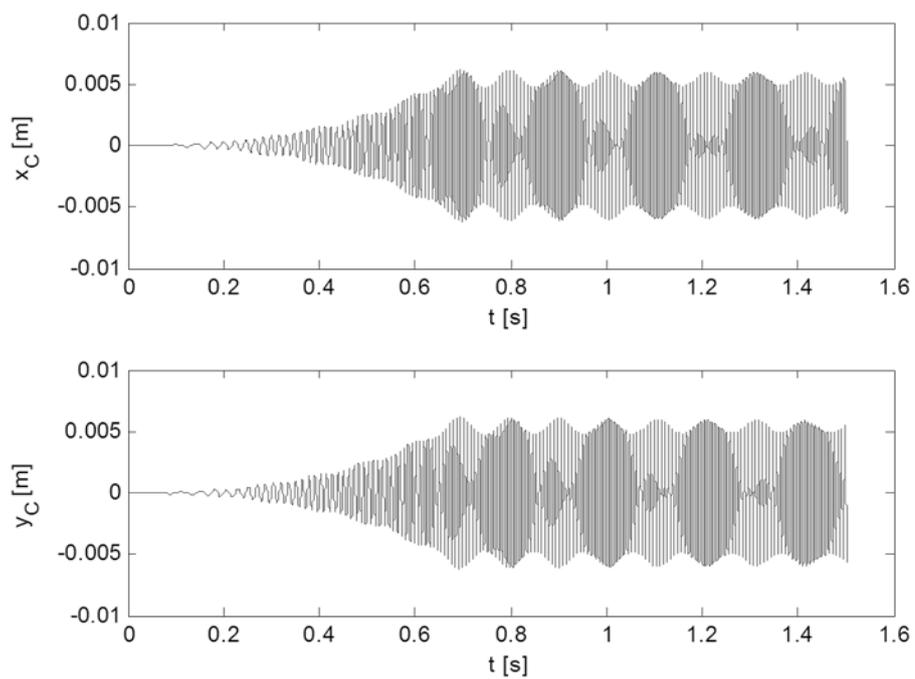


Fig. 12. Flexural vibrations of the geometric center  
Rys. 12. Drgania giętnie środka geometrycznego

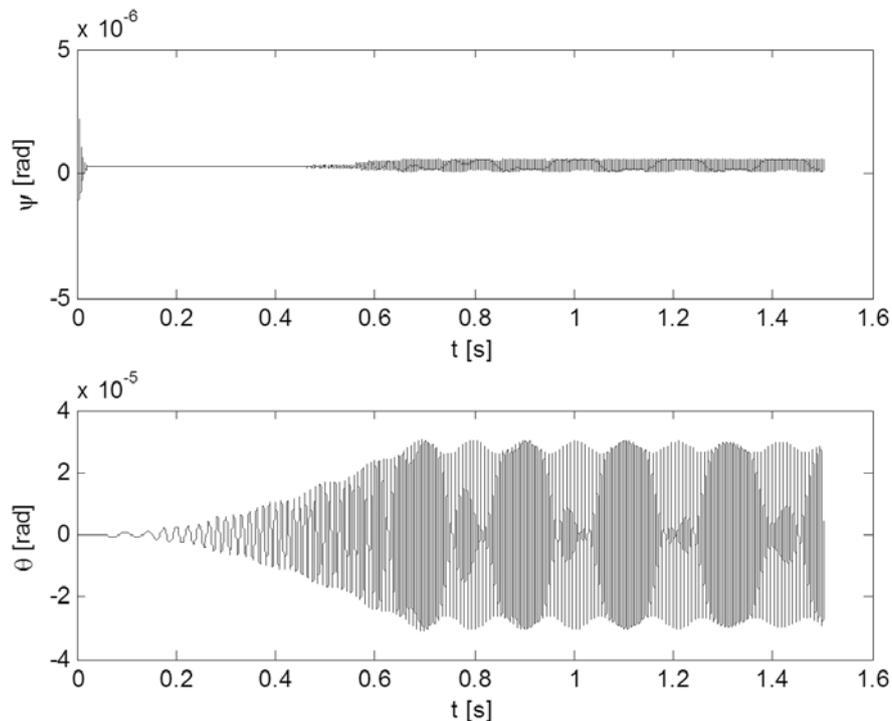


Fig. 13. Inclination of the plane of rotation  
 Rys. 13. Pochylenie płaszczyzny wirowania

## References

1. Genta, G. *Dynamics of Rotating Systems*. NY: Springer. 2005. 658 p.
2. Janschek, K. *Mechatronic System Design. Methods, Models, Concepts*. NY: Springer. 2012. 805 p.
3. Kiciński, J. *Rotor dynamics*. Radom: Wydawnictwo IMP PAN. Instytut Technologii Eksploatacji in Gdańsk. 2006. 539 p.
4. Sellgren, U. *Component mode synthesis – a method for efficient dynamic simulation of complex technical systems*. Technical Report. 2003. Sweden, Stockholm: Dep. of Mach. Design. The Royal Institute of Technology. 26 p.
5. Szolc, T. On the discrete-continuous modeling of rotor systems for the analysis of coupled lateral-torsional vibrations. *Int. J. Rotating Mach.* 2000. Vol. 6(2). P. 135-149.
6. Matyja, T. Model of stiff rotor with six degrees of freedom. *20<sup>th</sup> International Congress on Sound and Vibration*. Thailand, Bangkok, 7-11 July 2013. P. 1033-1040.
7. *Rotordynamics with ANSYS Mechanical Solutions. Tutorial*. Available at: <http://www.ansys.com>
8. Shehovtsov, V. & Sokolov-Dobrev, N. & Lyashenko, M. & etc. Influence of elements dynamic cohesiveness in power shafting on torsional vibrations spreading and dynamic equality of reducible model. *Mechanika*. 2014. Vol. 20. No. 2. P. 190-196.
9. *SimMechanics. Model and simulate multibody mechanical systems*. Available at: <http://www.mathworks.com/products/simmechanics/>
10. Thieffry, P. & Jandric, D. Turning to Rotordynamics. Tools from ANSYS work together to optimize rotating parts. *ANSYS Advantage*. 2010. Vol. IV. No. 2. P. 44-45. Available at: <http://www.ansys.com/staticassets/ANSYS/staticassets/resourcelibrary/article/AA-V4-I2-Turning-to-Rotordynamics.pdf>