

take-off operation, modeling

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## **METHODS OF MATHEMATICAL MODELING OF THE TAKE-OFF OPERATION OF A COMMERCIAL AIRCRAFT**

**Summary.** While observing the dynamic increase in the intensity of air traffic, the issue of constant controlling and monitoring every individual operation carried out during the flight process becomes a vital matter. One of the most significant operations in this process is the aircraft's take-off procedure. On the airports such procedures are observed quite frequently, with a rate of several dozen seconds up to few minutes. The correctness of carrying out the mentioned guidelines has a crucial impact on the traffic capacity of the runway, number of operations completed within the area of the airport and, above all, safety of passengers. The research and analysis of these processes cannot, for obvious reasons, be done on objects in real conditions. Therefore, there is a tendency to use IT tools and methods for the purpose of the analysis of the operations which occur in the area of the airport. In order to make use of the computer simulation it is essential to have mathematical models of these operations. The purpose of this article is to create models for particular stages of take-off operation and to identify elementary models of these stages based on parameters recorded by the on board flight recorder. The created mathematical and computer models (for simulation research), reproducing the aircraft's real operations in the area of the airport, shall be used for automation of the operations conducted in the airport's area. For the purpose of the analysis comparison of two models has been done to show the effects of conducting mathematical normalization of parameters prior to the analysis.

## **METODY MODELOWANIA MATEMATYCZNEGO OPERACJI STARTU SAMOŁOTU KOMUNIKACYJNEGO**

**Streszczenie.** Obserwując dynamiczny wzrost ruchu lotniczego istotnym staje się zagadnienie ciągłej kontroli i monitorowania poszczególnych operacji wykonywanych podczas procesu przelotu. Jedną z ważniejszych operacji w tym procesie jest operacja startu samolotu. Na lotniskach komunikacyjnych operacje te odbywają się z częstotliwością od kilkudziesięciu sekund do kilku minut. Poprawność wykonywania tych założonych procedur w czasie tej operacji ma istotny wpływ na przepustowość drogi startowej, liczbę operacji wykonywanych w rejonie lotniska a przede wszystkim na bezpieczeństwo pasażerów. Badanie i analiza tych procesów z przyczyn oczywistych nie mogą być wykonywane na obiektach w rzeczywistych warunkach. W związku z tym dąży się do wykorzystania w analizie operacji zachodzących w rejonie lotniska narzędzi i metod komputerowych. Aby wykorzystać możliwości symulacji komputerowej koniecznym jest posiadanie modeli matematycznych tych operacji. Celem artykułu jest zamodelowanie poszczególnych etapów operacji startu a następnie identyfikacja elementarnych modeli tych etapów w oparciu o parametry zarejestrowane przez

pokładowy rejestrator parametrów lotu samolotu. Zbudowane modele odwzorowujące rzeczywiste operacje samolotu w rejonie lotniska, będą mogły być wykorzystane w praktyce lotniczej.

## 1. INTRODUCTION

A computer parameters' identification depends on the mathematical model based on the results of measurements carried out on a real object, [4]. The identification is based upon the assignment of the dependence describing the given variable in the form of a mathematical model. The next step is the coefficients' identification for function's form determined in the range of segments within the field of interest. In case of a take-off operation, segmentation of the phase has been made but the separate segments are not an exact reflection of the following phases: take-off run, lift-off, take-off climb. Their length is determined by the operations carried out by the crew. Segments were chosen to minimize the problem of function continuity on the intervals limits; which is the most significant defect of the computer identification method. There are six segments. After determining the function form within all the segments for all the described variables, and after evaluation of this analysis through calculation of quality and identification coefficient, the model of the take-off phase is ready to be used.

This particular analysis allows to analyze various parameters, with respect of demand and specific use of the model. Using it one can make complex identification of aircraft's movement dynamics at the take-off phase through identification of forces and moments of force, which affect the aircraft at take-off. This kind of projection can be used to build for example flight simulators, used for training of air staff, [4]. In the example shown below, the model consists of indicated air speed (IAS) and vertical speed ( $w$ ) of an aircraft at the take-off phase, which gives a basic representation of an aircraft's movement, and its altitude. This analysis is sufficient to investigate the runway occupancy or to make other type of investigations aiming at the increase in the capacity of airports and safety of motion within the airport area.

## 2. MODEL IDENTIFICATION

In the example shown, parameters recorded by an on-board flight recorders of Embraer 170 [6] aircraft are being analyzed. Within the parameters, there are: indicated air speed (IAS) and ground speed, altitude, geographical coordinates, course, pitch and roll angle, longitudinal and normal accelerations, Mach number, thrust, the position of flaps and undercarriage and total weight of the aircraft. The flight parameters, used to create the model, were recorded during flights from the Warsaw Okęcie Airport, runway 29 for the same route and with the use of the same procedure of standard instrument departure (SID) TITAC 1G. The data were recorded until the moment, when the aircraft reached IAS of 250 knots. This way the end of the take-off phase has been determined for the analysis' purposes. Fulfilling of the conditions mentioned above means that all the assumptions made for the analysis have been met.

After IAS flights analysis, it has been accepted that function representing IAS and vertical speed in the take-off phase consists of algebraic polynomials, which can be described as:

$$\begin{aligned} V_{mj}(t) &= a_{2j} \cdot t + b_{2j} \cdot t^2 + c_{2j} \cdot t^3 + d_{2j} \cdot t^4 + e_{2j} \cdot t^5 + f_{2j} \cdot t^6 \\ w_{mj}(t) &= a_{3j} \cdot t + b_{3j} \cdot t^2 + c_{3j} \cdot t^3 + d_{3j} \cdot t^4 + e_{3j} \cdot t^5 + f_{3j} \cdot t^6 \end{aligned} \quad (1)$$

where:

$V_{mj}(t)$  – the IAS of the aircraft at  $t$  moment for its  $j$ -segment,

$w_{mj}(t)$  – the vertical speed of the aircraft at  $t$  moment for its  $j$ -segment,

$a_{2j}, b_{2j}, c_{2j}, d_{2j}, e_{2j}, f_{2j}$  – polynomial coefficients for adequate segments of the take-off phase for the indicated air speed of the aircraft,

$a_{3j}, b_{3j}, c_{3j}, d_{3j}, e_{3j}, f_{3j}$  – polynomial coefficients for adequate segments of the take-off phase for the vertical speed of the aircraft,

$j$  – the take-off phase segment's number,

$t$  – time measured in seconds.

On fig. 1 and tab. 1, the division of the take-off phase into segments has been shown as well as the description of the area covered by particular segments. The division is also visible on fig. 2 and 3, which demonstrate the time course of the IAS and vertical speed of the aircraft at take-off.

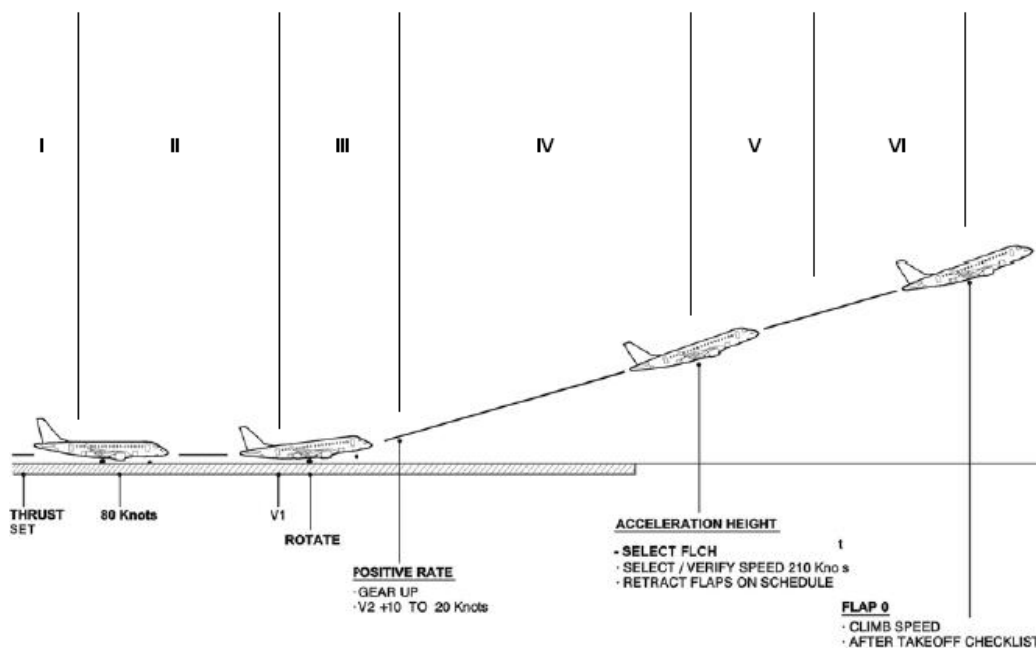


Fig. 1. The aircraft's take-off phase diagram and the division of segments

Rys. 1. Faza startu samolotu z podziałem na segmenty

Table 1

Description of individual take-off phase segments of an aircraft

Segment	Segment's description
I	Beginning of take-off run
II	Take-off run until the moment of reaching the decision speed ( $V_1$ ), then the rotation speed ( $V_R$ ) until the moment of lift-off and take-off climb at safe speed
III	Further take-off climb and undercarriage retracting
IV	Change in the position of flaps up to 0
V	Flight from the moment of accomplishing flaps 0 until the moment of reaching the IAS of 210 knots
VI	Flight until the moment of reaching the IAS of 250 knots

The identification has been made for three selected flights made on 14<sup>th</sup>, 29<sup>th</sup> and 31<sup>st</sup> of December 2010. For each segment there were values of polynomial coefficients chosen, identified for one of the three flights mentioned above, with respect to the value of quality identification coefficients.

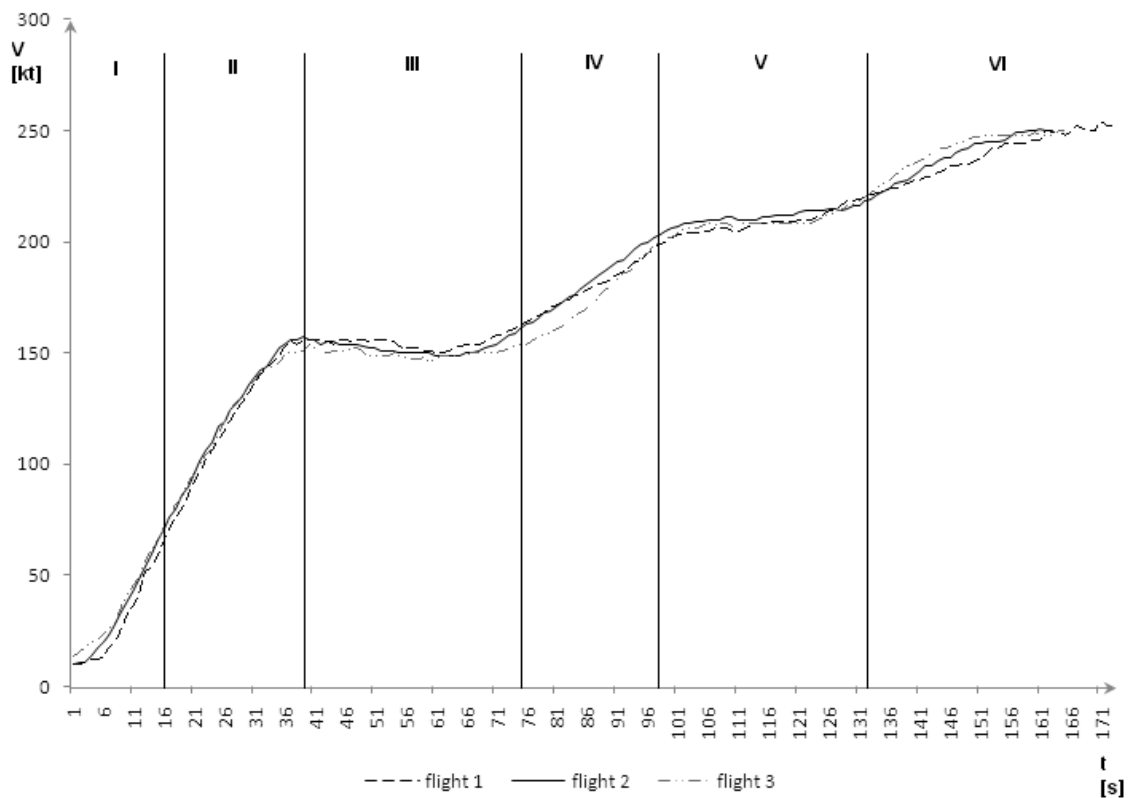


Fig. 2. The indicated air speed of the aircraft for three flights  
 Rys. 2. Prędkość przyrządowa samolotu dla trzech lotów

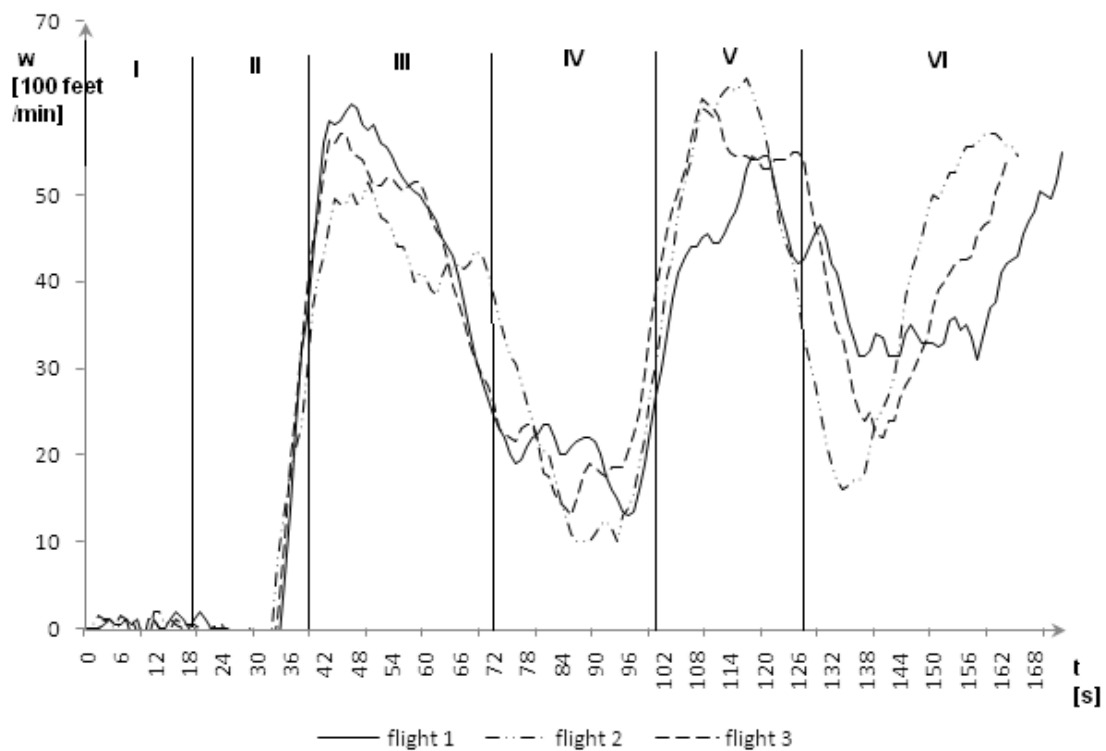


Fig. 3. The vertical speed of the aircraft for three flights  
 Rys. 3. Prędkość pionowa samolotu dla trzech lotów

## 2.1. Segment I

As the result of the identification of the polynomial coefficients (tab. 2 and 3), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the segment I of the take-off phase, based on the model flights of the EMB 170 aircraft, for flight1 from December 14, 2010 (fig. 4).

Table 2

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment I of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m1}$	$w_{m1}$	$V_{m1}$	$w_{m1}$	$V_{m1}$	$w_{m1}$
$a_{21}/a_{31}$	2,26E+00	5,46E-02	3,29E+00	-4,53E-02	8,69E+00	-2,13E+00
$b_{21}/b_{31}$	2,37E+00	-2,37E-01	3,40E+00	1,51E-01	3,08E+00	4,09E+00
$c_{21}/c_{31}$	-	1,40E-01	-	-5,63E-02	-	-1,95E+00
$d_{21}/d_{31}$	-	-1,11E-00	-	3,83E-02	-	3,91E-01
$e_{21}/e_{31}$	-	-2,73E-04	-	-6,65E-03	-	-3,51E-02
$f_{21}/f_{31}$	-	2,52E-05	-	3,19E-04	-	1,16E-03

Table 3

Coefficients for the quality identification of the segment I in the EMB 170 aircraft's take-off phase

	14.12.2010		29.12.2010		31.12.2010		
	$V_{m1}$	$w_{m1}$	$V_{m1}$	$w_{m1}$	$V_{m1}$	$w_{m1}$	
Generalized coefficient of correlation $\rho$	0,9043	0,9050	0,9693	0,6152	0,9688	0,8704	
Remnant variance $\sigma^2$	4,46E+00	0,37E+00	3,68E+00	7,51E-01	3,06E+00	5,65E-01	
Confidence intervals	$a_{21}/a_{31}$	5,49E+00	3,33E+00	3,86E+00	1,47E+00	3,21E+00	1,10E+00
	$b_{21}/b_{31}$	7,46E-01	4,42E+00	5,46E-01	2,86E+00	4,54E-01	2,15E+00
	$c_{21}/c_{31}$	-	1,91E+00	-	1,64E+00	-	1,23E+00
	$d_{21}/d_{31}$	-	3,56E+00	-	3,62E-01	-	2,72E-01
	$e_{21}/e_{31}$	-	2,97E-02	-	3,37E-02	-	2,54E-02
	$f_{21}/f_{31}$	-	9,08E-04	-	1,12E-03	-	8,42E-04

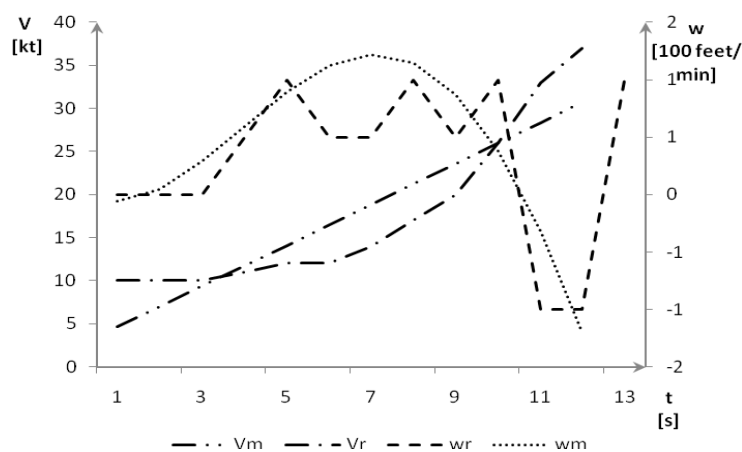


Fig. 4. IAS (V) and vertical (w) speed recorded in an actual flight ( $V_r$  and  $w_r$ ) and from aircraft model ( $V_m$  and  $w_m$ ) in segment I take-off phase for flight 1

Rys. 4. Prędkość przyrządowa (V) i pionowa (w) zarejestrowane w rzeczywistym locie ( $V_r$  i  $w_r$ ) i wynikające z modelu ( $V_m$  i  $w_m$ ) w segmencie I fazy startu dla lotu 1

## 2.2. Segment II

As the result of the identification of the polynomial coefficients (tab. 4 and 5), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the segment II of the take-off phase, based on the model flights of the EMB 170, for flight 2 from December 29, 2010 (fig. 5).

Table 4

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment II of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m2}$	$w_{m2}$	$V_{m2}$	$w_{m2}$	$V_{m2}$	$w_{m2}$
$a_{22}/a_{32}$	-1,18E+01	-1,18E+02	2,11E+00	1,94E+02	6,79E+00	2,41E+02
$b_{22}/b_{32}$	4,60E+00	2,81E+01	4,38E+00	-3,49E+01	4,16E+00	-4,53E+01
$c_{22}/c_{32}$	-	-2,70E+00	-	2,11E+00	-	3,04E+00
$d_{22}/d_{32}$	-	1,31E-01	-	-4,13E-02	-	-8,30E-02
$e_{22}/e_{32}$	-	-3,21E-03	-	-3,23E-04	-	6,08E-04
$f_{22}/f_{32}$	-	3,10E-05	-	1,35E-05	-	5,39E-06

Table 5

Coefficients for the quality identification of the segment II in the EMB 170 aircraft's take-off phase

	14.12.2010		29.12.2010		31.12.2010		
	$V_{m2}$	$w_{m2}$	$V_{m2}$	$w_{m2}$	$V_{m2}$	$w_{m2}$	
Generalized coefficient of correlation $\rho$	0,9965	0,9058	0,9955	0,9531	0,9930	0,9599	
Remnant variance $\sigma^2$	0,30E+01	1,85E+00	3,08E+00	1,51E+00	3,50E+00	1,62E+00	
Confidence intervals	$a_{22}/a_{32}$	4,27E+00	-	4,63E+00	-	5,26E+00	-
	$b_{22}/b_{32}$	1,64E-01	-	1,82E-01	-	2,7E-01	-
	$c_{22}/c_{32}$	-	-	-	-	-	-
	$d_{22}/d_{32}$	-	-	-	-	-	-
	$e_{22}/e_{32}$	-	-	-	-	-	-
	$f_{22}/f_{32}$	-	-	-	-	-	-

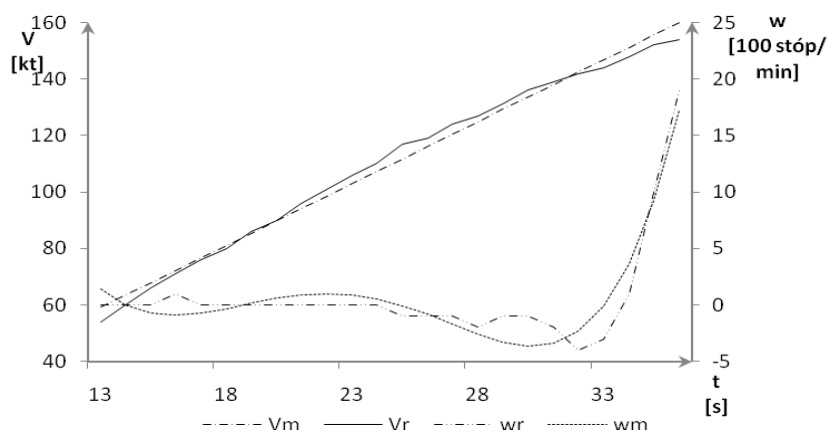


Fig. 5. IAS ( $V$ ) and vertical ( $w$ ) speed recorded in actual flight ( $V_r$  and  $w_r$ ) and from aircraft model ( $V_m$  and  $w_m$ ) in segment II take-off phase for flight 2

Rys. 5. Prędkość przyrządowa ( $V$ ) i pionowa ( $w$ ) zarejestrowane w rzeczywistym locie ( $V_r$  i  $w_r$ ) i wynikające z modelu ( $V_m$  i  $w_m$ ) w segmencie II fazy startu dla lotu 2

### 2.3. Segment III

As the result of the identification of the polynomial coefficients (tab. 6 and 7), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the III segment of the take-off phase, based on the model flights of the EMB 170 aircraft, for flights 2 and 3 from December 29 and 30, 2010 (fig. 6 and fig. 7).

Table 6

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment III of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m3}$	$w_{m3}$	$V_{m3}$	$w_{m3}$	$V_{m3}$	$w_{m3}$
$a_{23}/a_{33}$	1,64E+02	-	1,67E+02	-1,16E+00	1,57E+02	-9,87E+02
$b_{23}/b_{33}$	-1,90E-01	-1,75E+01	-2,89E-01	1,03E+02	-1,41E-01	5,49E+01
$c_{23}/c_{33}$	-	8,34E-01	-	-2,11E+00	-	-9,56E-01
$d_{23}/d_{33}$	-	-7,08E-03	-	1,08E-02	-	7,00E-03
$e_{23}/e_{33}$	-	-1,08E-03	-	1,28E-04	-	-5,37E-05
$f_{23}/f_{33}$	-	1,32E-06	-	-1,22E-06	-	4,64E-07

Table 7

Coefficients for the quality identification of the segment III in the EMB 170 aircraft's take-off phase

	14.12.2010		29.12.2010		31.12.2010		
	$V_{m3}$	$w_{m3}$	$V_{m3}$	$w_{m3}$	$V_m$	$w_{m3}$	
Generalized coefficient of correlation $\rho$	0,7613	0,9460	0,9628	0,9213	0,7102	0,9867	
Remnant variance $\sigma^2$	1,43E+00	2,92E+00	7,41E-01	2,31E+00	1,28E+00	1,21E+00	
Confidence intervals	$a_{23}/a_{33}$	3,34E+00	-	1,63E+00	-	12,82E+00	-
	$b_{23}/b_{33}$	6,33E-02	-	3,12E-02	-	5,40E-02	-
	$c_{23}/c_{33}$	-	-	-	-	-	-
	$d_{23}/d_{33}$	-	-	-	-	-	-
	$e_{23}/e_{33}$	-	-	-	-	-	-
	$f_{23}/f_{33}$	-	-	-	-	-	-

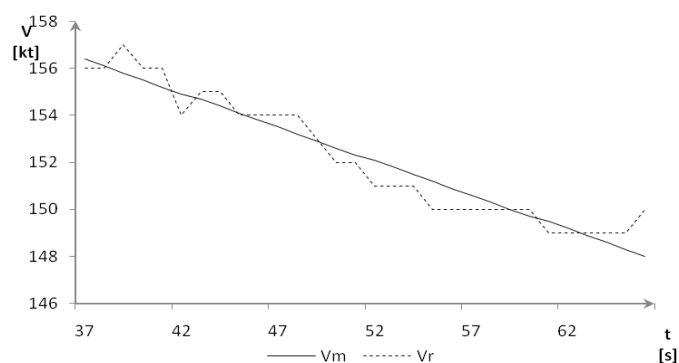


Fig. 6. Indicated air speed ( $V$ ) recorded in actual flight ( $V_r$ ) and from aircraft model ( $V_m$ ) in segment III take-off phase for flight 2

Rys. 6. Prędkość przyrządowa ( $V$ ) zarejestrowana w rzeczywistym locie ( $V_r$ ) i wynikająca z modelu ( $V_m$ ) w segmencie III fazy startu dla lotu 2

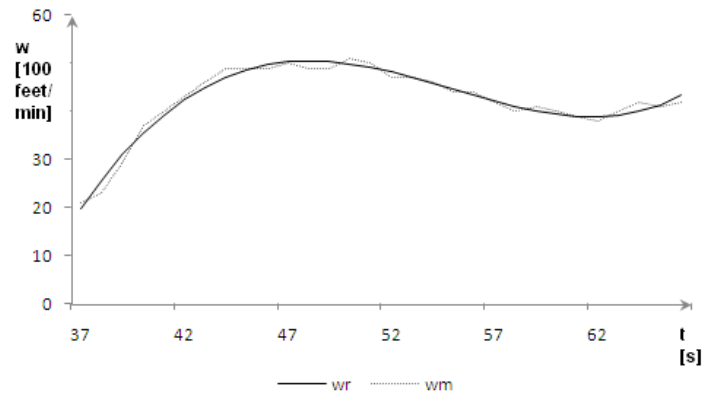


Fig. 7. Vertical speed ( $w$ ) recorded in IAS flight ( $w_r$ ) and from aircraft model ( $w_m$ ) in segment III take-off phase for flight 3

Rys. 7. Prędkość pionowa ( $w$ ) zarejestrowana w rzeczywistym locie ( $w_r$ ) i wynikająca z modelu ( $w_m$ ) w segmencie III fazy startu dla lotu 3

#### 2.4. Segment IV

As a result of the identification of the polynomial coefficients (tab. 8 and 9), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the IV segment of the take-off phase, based on the model flights of the EMB 170 aircraft, for flights 1 and 3 from December 14 and 31, 2010 (fig. 8 and fig. 9).

Table 8

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment IV of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m4}$	$w_{m4}$	$V_{m4}$	$w_{m4}$	$V_{m4}$	$w_{m4}$
$a_{24}/a_{34}$	5,93E+01	9,05E+02	3,19E+01	-1,47E+01	5,66E+01	-6,44E+01
$b_{24}/b_{34}$	1,37E+00	-8,80E+00	1,73E+00	1,78E+00	1,32E+00	-
$c_{24}/c_{34}$	-	-2,20E-01	-	9,53E-03	-	-2,09E+02
$d_{24}/d_{34}$	-	-2,11E-05	-	-2,10E-04	-	4,01E-03
$e_{24}/e_{34}$	-	6,39E-05	-	-7,59E-06	-	-7,60E-05
$f_{24}/f_{34}$	-	-4,22E-05	-	7,49E-08	-	3,90E-07

Table 9

Coefficients of the quality identification in segment IV of the EMB 170 aircraft's take-off phase

		14.12.2010		29.12.2010		31.12.2010	
		$V_{m4}$	$w_{m4}$	$V_{m4}$	$w_{m4}$	$V_{m4}$	$w_{m4}$
Generalized coefficient of correlation $\rho$		0,9977	0,9647	0,9969	0,9519	0,9633	0,9933
Remnant variance $\sigma^2$		6,57E-01	1,47E+00	1,00E+00	1,98E+00	2,72E+00	1,23E+00
Confidence intervals	$a_{24}/a_{34}$	3,28E+00	-	4,68E+00	-	1,26E+01	-
	$b_{24}/b_{34}$	4,13E-02	-	5,93E-02	-	1,60E-01	-
	$c_{24}/c_{34}$	-	-	-	-	-	-
	$d_{24}/d_{34}$	-	-	-	-	-	-
	$e_{24}/e_{34}$	-	-	-	-	-	-
	$f_{24}/f_{34}$	-	-	-	-	-	-



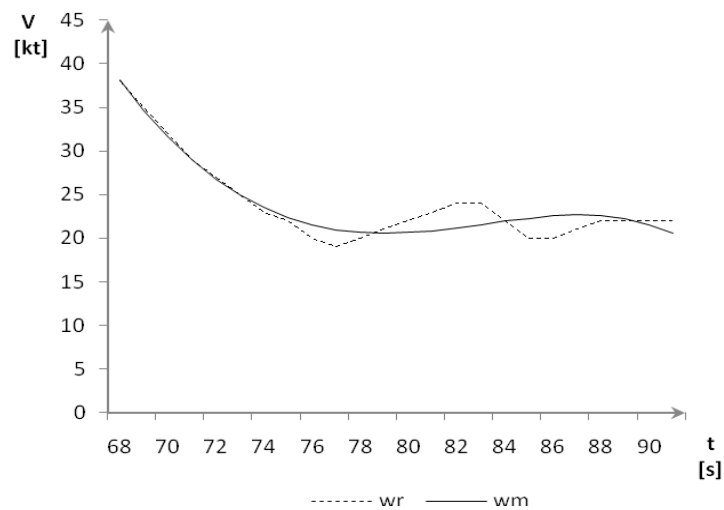


Fig. 8. Indicated air speed ( $V$ ) recorded in IAS flight ( $V_r$ ) and from aircraft model ( $V_m$ ) in segment IV take-off phase for flight 1

Rys. 8. Prędkość przyrządowa ( $V$ ) zarejestrowana w rzeczywistym locie ( $V_r$ ) i wynikająca z modelu ( $V_m$ ) w segmencie IV fazy startu dla lotu 1

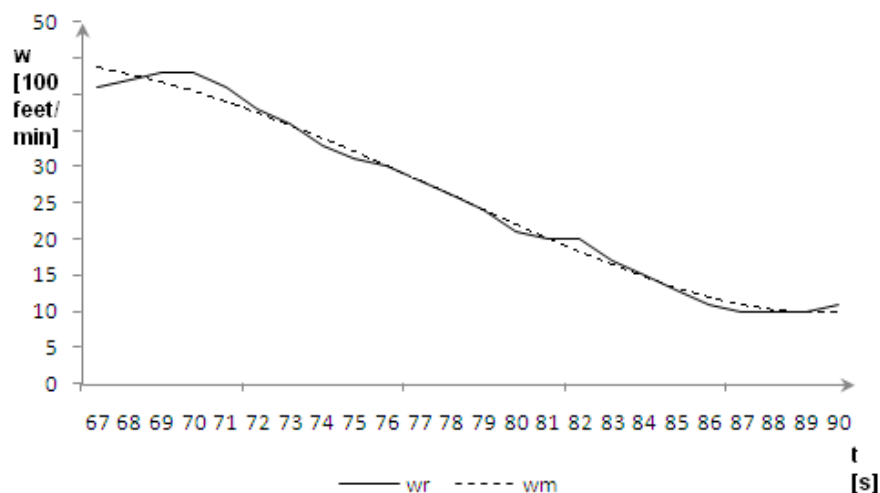


Fig. 9. Vertical speed ( $w$ ) recorded in IAS flight ( $w_r$ ) and from aircraft model ( $w_m$ ) in segment IV take-off phase for flight 3

Rys. 9. Prędkość pionowa ( $w$ ) zarejestrowana w rzeczywistym locie ( $w_r$ ) i wynikająca z modelu ( $w_m$ ) w segmencie IV fazy startu dla lotu 3

## 2.5. Segment V

As the result of the identification of the polynomial coefficients (tab. 10 and 11), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the V segment of the take-off phase, based on the model flights of the EMB 170 aircraft, for flights 1 and 3 from December 14 and 31, 2010 (fig. 10 and fig. 11).

Table 10

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment V of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m5}$	$W_{m5}$	$V_{m5}$	$W_{m5}$	$V_{m5}$	$W_{m5}$
$a_{25}/a_{35}$	1,29E+02	8,76E+02	1,44E+02	2,32E+02	1,32E+02	1,51E+03
$b_{25}/b_{35}$	6,89E-01	-1,42E+01	5,91E-01	-5,52E+00	6,70E-01	-2,39E+01
$c_{25}/c_{35}$	-	-4,20E-02	-	-1,28E-02	-	-8,61E-02
$d_{25}/d_{35}$	-	9,65E-04	-	3,22E-04	-	1,30E-03
$e_{25}/e_{35}$	-	5,78E-06	-	7,01E-06	-	1,86E-05
$f_{25}/f_{35}$	-	-5,53E-08	-	-5,40E-08	-	-1,38E-07

Table 11

Coefficients of the quality identification in segment V of the EMB 170 aircraft's take-off phase

	14.12.2010		29.12.2010		31.12.2010		
	$V_{m5}$	$W_{m5}$	$V_{m5}$	$W_{m5}$	$V_{m5}$	$W_{m5}$	
Generalized coefficient of correlation	0,9075	0,9666	0,8895	0,9699	0,8277	0,9878	
Remnant variance $\sigma^2$	3,0824E+00	3,77E+00	2,95E+00	3,95E+00	4,40E+00	2,71E+00	
Confidence intervals	$a_{25}/a_{35}$	1,27E+01	-	1,21E+01	-	1,80E+01	-
	$b_{25}/b_{35}$	1,18E-01	-	1,13E-01	-	1,68E-01	-
	$c_{25}/c_{35}$	-	-	-	-	-	-
	$d_{25}/d_{35}$	-	-	-	-	-	-
	$e_{25}/e_{35}$	-	-	-	-	-	-
	$f_{25}/f_{35}$	-	-	-	-	-	-

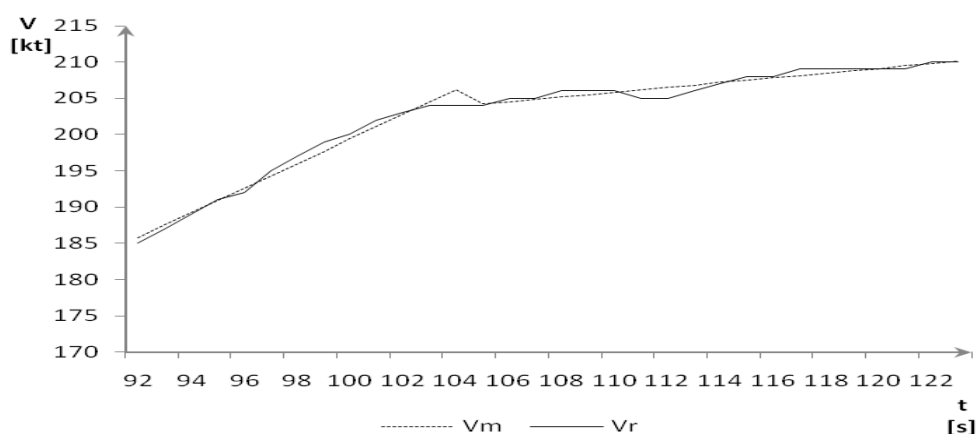


Fig. 10. Indicated air speed ( $V$ ) recorded in IAS flight ( $V_r$ ) and from aircraft model ( $V_m$ ) in segment V take-off phase for flight 1

Rys. 10. Prędkość przyrządowa ( $V$ ) zarejestrowana w rzeczywistym locie ( $V_r$ ) i wynikająca z modelu ( $V_m$ ) w segmencie V fazy startu dla lotu 1

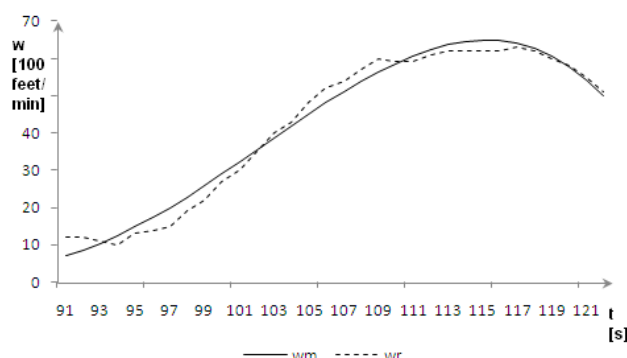


Fig. 11. Vertical speed ( $w$ ) recorded in IAS flight ( $w_r$ ) and from aircraft model ( $w_m$ ) in segment V take-off phase for flight 3

Rys. 11. Prędkość pionowa ( $w$ ) zarejestrowana w rzeczywistym locie ( $w_r$ ) i wynikająca z modelu ( $w_m$ ) w segmencie V fazy startu dla lotu 3

### 2.6. Segment VI

As a result of the identification of the polynomial coefficients (tab. 12 and 13), as well as the verification of the value of the quality identification's coefficients, we obtained the functions describing the change in the aircraft's IAS and vertical speed in the segment VI of the take-off phase, based on the model flights of the EMB 170 aircraft, for flight 1 from December 14, 2010 (fig.12).

Table 12

Coefficients of the IAS and vertical speed function of the EMB 170 aircraft in segment VI of the take-off phase

coeff.	14.12.2010		29.12.2010		31.12.2010	
	$V_{m6}$	$w_{m6}$	$V_{m6}$	$w_{m6}$	$V_{m6}$	$w_{m6}$
$a_{26}/a_{36}$	1,02E+02	3,75E+02	7,54E+01	1,94E+03	8,18E+01	3,80E+03
$b_{26}/b_{36}$	8,85E-01	-3,76E+00	1,10E+00	-1,78E+01	1,07E+00	-4,10E+01
$c_{26}/c_{36}$	-	1,06E-02	-	-5,51E-02	-	-7,31E-02
$d_{26}/d_{36}$	-	-3,22E-05	-	3,56E-04	-	7,64E-04
$e_{26}/e_{36}$	-	5,70E-08	-	0,45E-05	-	1,04E-05
$f_{26}/f_{36}$	-	8,15E-10	-	-1,94E-08	-	-5,04E-08

Table 13

Coefficients of the quality identification in segment VI of the EMB 170 aircraft's take-off phase

	Flight from 14.12.2010		Flight from 29.12.2010		Flight from 31.12.2010		
	$V_{m6}$	$w_{m6}$	$V_{m6}$	$w_{m6}$	$V_{m6}$	$w_{m6}$	
Generalized coefficient of correlation	0,9922	0,9455	0,9871	0,9360	0,9517	0,9089	
Remnant variance $\sigma^2$	1,69E+00	2,21E+00	2,19E+00	3,32E+00	4,42E+00	3,83E+00	
Confidence intervals	$a_{26}/a_{36}$	4,82E+00	-	8,30E+00	-	1,57E+01	-
	$b_{26}/b_{36}$	3,22E-02	-	5,79E-02	-	1,09E-01	-
	$c_{26}/c_{36}$	-	-	-	-	-	-
	$d_{26}/d_{36}$	-	-	-	-	-	-
	$e_{26}/e_{36}$	-	-	-	-	-	-
	$f_{26}/f_{36}$	-	-	-	-	-	-

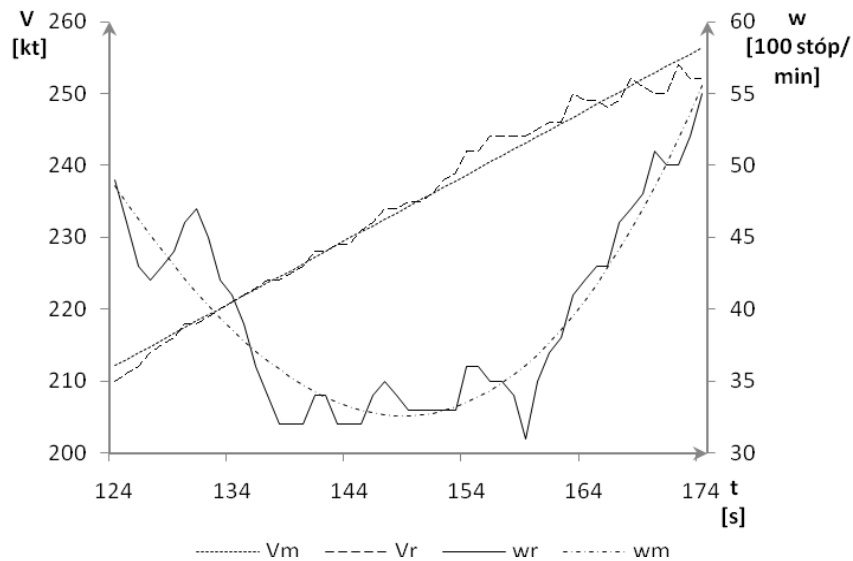


Fig. 12. The IAS ( $V$ ) and vertical ( $w$ ) speed of the aircraft recorded during a IAS flight ( $V_r$  and  $w_r$ ) and from model ( $V_m$  and  $w_m$ ) in segment VI of the take-off phase for the flight number 1

Rys. 12. Prędkość przyrządowa ( $V$ ) i pionowa ( $w$ ) zarejestrowane w rzeczywistym locie ( $V_r$  i  $w_r$ ) i wynikające z modelu ( $V_m$  i  $w_m$ ) w segmencie VI fazy startu dla lotu 1

Taking into account identification results of equations discussed, fig. 13 presents time courses of IAS ( $V$ ) and vertical ( $w$ ) speed of aircraft in take-off phase.

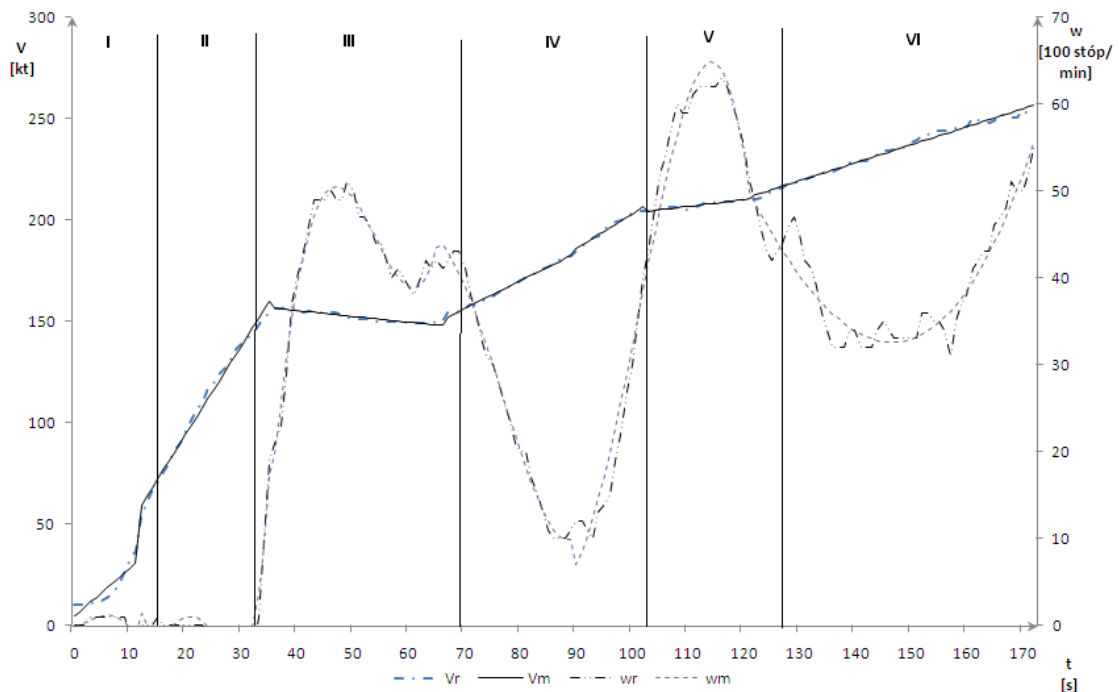


Fig. 13. The result of the identification of the model time course of the IAS and vertical speed in the take-off phase for the Embraer 170 aircraft

Rys. 13. Rezultaty identyfikacji modelu prędkości przyrządowej i pionowej samolotu względem czasu dla fazy startu samolotu Ebraer 170

### 3. CONCLUSIONS

The presented example proves that the methods of computer identification may very well be used in order to reproduce the aircraft's flight. While analyzing a sufficient number of parameters, it is possible to reproduce exactly the dynamics of the aircraft's flight in a given phase of the flight, or in its whole course. Such an aircraft's flight, described by a mathematical model, may serve for many purposes, for example - creation of flight simulators. However, the computer identification method's main flaw is the necessity to divide the area of analysis into stages of parameters' variability, which may generate problems in continuity of the examined parameters on the limits of these intervals.

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