flexible fastening elements, thrust bars,
spatial force system, fastening components

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GENERALIZED MATHEMATICAL MODEL OF FASTENING TECHNOLOGY OF CARGO WITH PADS JOINTLY WITH FLEXIBLE AND THRUST ELEMENTS UNDER THE ACTION OF SPATIAL FORCE SYSTEM

Summary. The article gives an account of the results of the constructed generalized mathematical model of fixation of cargo with pads jointly with flexible and thrust fastening elements. Developed for the first time the generalized mathematical model of collateral fixation of cargo with padding by flexible and thrust elements made it possible to identify the conditions providing workability of thrust bars together with pre twisted fastening wires and cargo pads. It should be emphases that it is only when these conditions are not fulfilled that elastic forces of flexible fastening elements are engaged in work on retaining cargo from shifting. The distinguishing characteristics of generalized model under the action of spatial force system as compared to specific models is manifested in its ability to provide workability of flexible fastening elements together with thrust bars and cargo pads according to the criterion of normative value of longitudinal (or lateral) load on a fastening component and allowable pressing tension of the most loaded of the used pads. The investigation results concerning cargo transported by and unprovided for in Specification [3] are now available for researchers and can be looked upon as a new stage in the development of this problem.

ОБОБЩЁННАЯ МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ТЕХНОЛОГИИ КРЕПЛЕНИЙ ГРУЗА С ПОДКЛАДКАМИ СОВМЕСТНО С ГИБКИМИ И УПОРНЫМИ ЭЛЕМЕНТАМИ ПРИ ВОЗДЕЙСТВИИ ПРОСТРАНСТВЕННОЙ СИСТЕМЫ СИЛ

Резюме. В статье изложены результаты построенной обобщённой математической модели креплений груза с подкладками совместно с гибкими и упорными элементами креплений. Разработанная (впервые) обобщённая математическая модели совместного закрепления груза с подкладками гибкими и упорными элементами позволили найти условия, при которых обеспечиваются работоспособность упорных брусков совместно с предварительно скрученными проволоками креплений и подкладками под груз. При этом лишь при несоблюдении таких условии включаются в работу по удержанию груза упругие силы гибких элементов креплений. Отличительная особенность обобщённой математической модели при воздействии пространственной системы сил сравнительно с частными моделями заключается в обеспечении работоспособности гибких элементов креплений совместно с упорными брусками и подкладками под груз по критерию нормативного значения продольной (или поперечной) нагрузки на крепёжную деталь и допустимого напряжения на сжатие из наиболее нагружённой из двух используемых подкладок. Полученные результаты исследований, доступные для разработчиков непредусмотренных ТУ [3] грузов, перевозимых на открытом подвижном составе, является новой ступенью в разработке данной проблемы.

1. FORMULATION OF A PROBLEM

In [1] there has been presented a generalized dynamic model of fixation of cargo with pads together with flexible and thrust fastening means; it has been noted that such model is rather a complicated dynamic system not in the sense of solving differential equations system but in the sense of presenting them as a complex mechanical system "cargo- pads- flexible elements – thrust bars".

In [2] in order to determine the limiting value of tilt angle α of the plane where the cargo is located in accordance to friction coefficient in statics μ under which its relocation will not yet take place, there has been given an equation of force equilibrium (Fig. 3.5.4. "Test of loading methods for transport on road and sea", p. 40-41) in the form:

$$mg(\sin\alpha - \mu\cos\alpha) = mg(a_h - \mu a_v) \qquad (1)$$

where:
m – is cargo mass,
g – gravitational acceleration (9,81 m/s^2),
a_h and a_v – are projections of acceleration onto horizontal and vertical in shares from g.

Having solved the equation (1) we obtain:

$$\alpha = 2\arctan\left[\frac{-1+\sqrt{1+\mu^2-\mu^2 a_v^2+2\mu a_v a_h - a_h^2}}{\mu+\mu a_v - a_h}\right] \text{ at } \mu \neq \frac{a_h}{1+a_v} \qquad (2)$$

$$\alpha = 2\arctan\left[\frac{a_h}{1+a_v}\right] \text{ at } \mu = \frac{a_h}{1+a_v}. \qquad (3)$$

Further, it has been noted that friction coefficient in statics μ 70% larger than in dynamics. Using (2) and /or (3), we built graphical dependence of plane tilt angle from friction coefficient in statics $\alpha = f(\mu)$.

The analysis of the equation (1) shows that in the right part of the equation there was incorrectly introduced g, dimensionality of its right and left parts being not observed. Because of this error formulas (2) and (2, a) prove to be totally inconsistent as dimensionality of values of numerator and denominator are not observed (1 and μ values are dimensionless). Due to this fact graphical dependence $\alpha = f(\mu)$ are of dubious character. When correctly presented the analytical formula in equation (1) expressing $\alpha = f(\mu)$ would be in the form:

$$\alpha = \arctan\left[\mu + \frac{a_h - \mu a_v}{g\cos\alpha}\right] \qquad (4)$$

Besides, in [1, 2] in cargo transportation in the open railway transport (wagon) there have been considered various calculation schemes of application to the object (cargo) of plane force system (longitudinal and vertical, lateral and vertical separately) as well as in [4, 5] by means of fixation either be thrust bars or by flexible elements. For example, there is a calculation scheme presenting cargo fixation only by means of thrust bars. In accordance with D'alamber principle for this scheme there has been set up an equation of force equilibrium in absolute motion in projections onto vertical and longitudinal coordinate axes. The solution of the first equation at the given value of acceleration in the vertical direction a_v there has been defined the reaction of the main constraint (wagon floor) R and as for the second equation the solution has been found according to the allowable value of stress load per one fastening item (nail) F_n (1.25 kN) and the number of nails n there has been determined absolute acceleration in the vertical direction a_h. The number of nails n is to be chosen according to cargo mass m in relation to m/1500. In this case it remains uncertain what the reaction of the main constraint (wagon floor) R was found for. It could only be supposed that acceleration in the vertical direction a_h is further compared with experimentally defined value. Unfortunately, it is necessary to observe that in problems on cargo fastening during transportation in the open rolling stock only equilibrium equations in relative motion (at rest) are used which makes it possible basing on the condition of the absence of thrust bar displacement to find the thrust bar reaction F_h and then according to the given allowable value F_n to determine the number of nails n [3].

In [1, 2] contain calculation scheme presenting cargo fixation only by means of straps. According to D'alamber principle for this scheme there has been set up equation of force equilibrium in absolute motion in projections onto vertical and lateral coordinate axes. By obtaining a direct solution of the first equation at the given value of absolute acceleration in vertical direction a_v there has been found the reaction of the main constraint (wagon floor) R, and by solving the second equation according to the given value of the allowable value of elastic forces of straps F with account for safety factor $\eta = 2$ there has been determined absolute acceleration in the crosswise direction a_t. Value F is accepted according to cargo mass m, thus, for example at impact $F = 32m$, and normally $F = 5m$, dependences $F_t = F\cos(\alpha)\sin(\beta)$ and $F_v = F\sin(\alpha)$ being taken into consideration where α is tilt angle between the direction of the straps and wagon floor plane, β is the angle between projection of the strap onto wagon floor plane and wagon longitudinal axis. Here it is not clear either what the reactions of the main constraint (wagon floor) R have been found for. It can be supposed that further on acceleration in the crosswise direction a_t will be compared with the experimentally found value. Unfortunately, it should be pointed out that normally in problems on cargo fastening during transportation in the open rolling stock only equilibrium equation in relative motion (a rest) are used which makes it possible to find directly elastic force in straps F [3].

Similarly to the previous case, in [1, 2] there has been considered calculation scheme for which there has been set up an equation of force equilibrium in absolute motion in projections onto the vertical and longitudinal coordinate axes. By solving directly the first equation at the given value of absolute acceleration in vertical direction a_v there has been found the reaction of the main constraint (wagon floor) R, and by solving the second equation according to the given value of allowable value of elastic forces of straps F with account for safety factor $\eta = 2$ there has been determined absolute acceleration in the longitudinal direction a_l. Value F is accepted according to cargo mass m, thus, for example during impact $F = 32m$, and normally $F = 5m$. Here, dependences $F_l = F\cos(\alpha)\cos(\beta)$ and $F_v = F\sin(\alpha)$ are taken into account. Here it is not quite clear what the reactions of the main constraint (wagon floor) R have been found for. It can be supposed that further on acceleration in the longitudinal direction a_l will be compared with the experimentally found value.

In [1, 2] also contains a scheme for keeping cargo from shifting in the crosswise direction by means of straps. As a result of solution of the equilibrium force equation in absolute motion in projections onto the vertical and lateral coordinate axes there has been obtained formula for determining crosswise acceleration depending on the value of vertical acceleration, friction coefficient, the number of straps and allowable value of elastic force in straps. It can be supposed that further on acceleration in the crosswise direction a_t will be compared with the experimentally found value.

The analysis of the presented analytical formulas makes it possible to observe that they have been derived in a simplified manner as the researchers failed to take into account the developed technique of calculation of cargo fastening in the open rolling stock under the action of spatial force system.

Thus, there has not been so far constructed a generalized mathematical model of cargo fastening in collateral cargo fixation with cargo pads by flexible and thrust elements during rolling stock movement on a descending grade of a curved track section under the impact of spatial force system (SFS).

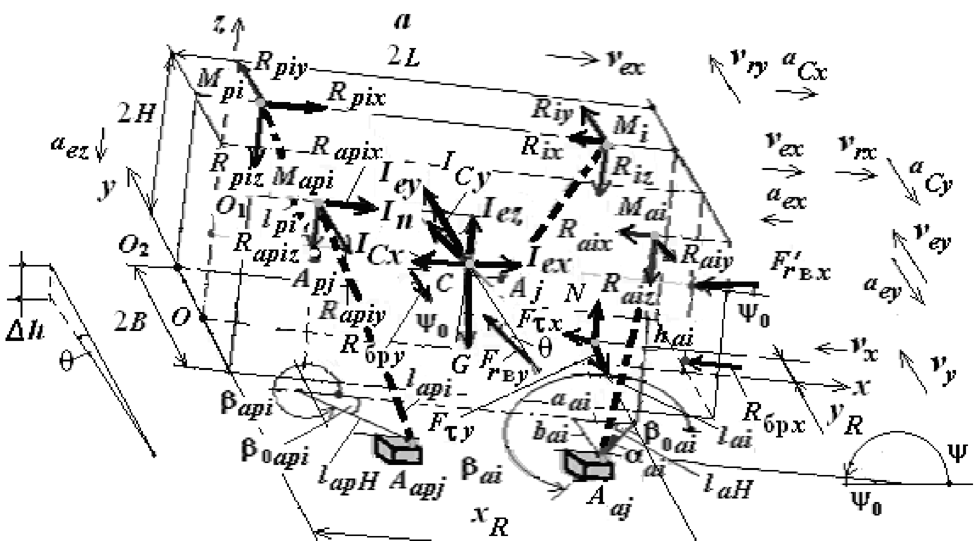
Due to this fact, it is necessary to note that the technical problem of keeping cargo with pads from shifting in relation to the wagon floor in collateral cargo fixation by flexible and thrust fastening elements under the impact of SFS has not been studied yet and it remains unsolved and quite actual for transport science.

1.1. Man-made assumption

As the basis of generalized dynamic model let us take the model (Fig. 1) presented in accordance with the principle of releasing from constraints in [1] where cargo fastening elements together with thrust bars experience the impact of SFS that are perceived by the main constraint (wagon) and additional constraints (flexible, elastic, thrust and supporting wooden fastening means).

In Fig. 1, as and in [3, 6-11], the following table of symbols is accepted: A_j, A_{aj}, A_{pj} и A_{apj} and A_{apj} are points. the cargo is fixed by flexible elastic fastening elements to tying wagon devices; M_j, M_{aj}, M_{pj} and M_{apj} are points corresponding to cargo eyes; I_n is the normal constituent inertia force during absolute motion conditionally applied to the cargo mass center (to be more precise to the wheel pairs together with axle bearings and lateral frames of trucks) that will take into account the accelerated motion of the rolling stock on a curved track section; j and i are indexes showing the number of rack staples and elastic fastening elements ($i = 1, n_\text{p}$ the number of flexible elastic fastening elements); $2L$, $2B$ and $2H$ - are the cargo length, width and height correspondingly; l_{wi} and l_{wai} are projections of the lengths of flexible elastic elements of fastening of one direction onto the lateral axis y (l_{wpi} and l_{wapi} the same of the other direction); Δh is superelevation; $2S$ is the distance between wheel rolling circles of gauge 1 520 mm ($2S$ = 1 520 mm); θ is the angle characterizing superelevation; ζ is the angle allowing for tilting of the wagon frame with cargo during its displacement onto the lateral axis at the value $\pm yM$.

In Fig. 1 the G, I_e, $F_{\text{гв}}$ and R, R_i, R_{thr} of active and reactive forces. Here I_e and $F_{\text{гв}}$ are transferring inertia forces [6-8] and aerodynamic resistance force [9]. Reactive forces $\overline{R} \in (\overline{N}, \overline{F}_\tau)$ consist of components $\overline{R} \in (\overline{R}_1, \overline{R}_2)$ (where $\overline{R}_1 \in (\overline{N}_1, \overline{F}_{\tau 1})$) and $\overline{R}_2 \in (\overline{N}_2, \overline{F}_{\tau 2})$), and reactive forces of flexible elastic fastening elements R_i consist of reaction of one $\overline{R}_i, \overline{R}_{ai}$ as well as the other direction $\overline{R}_{pi}, \overline{R}_{api}$.



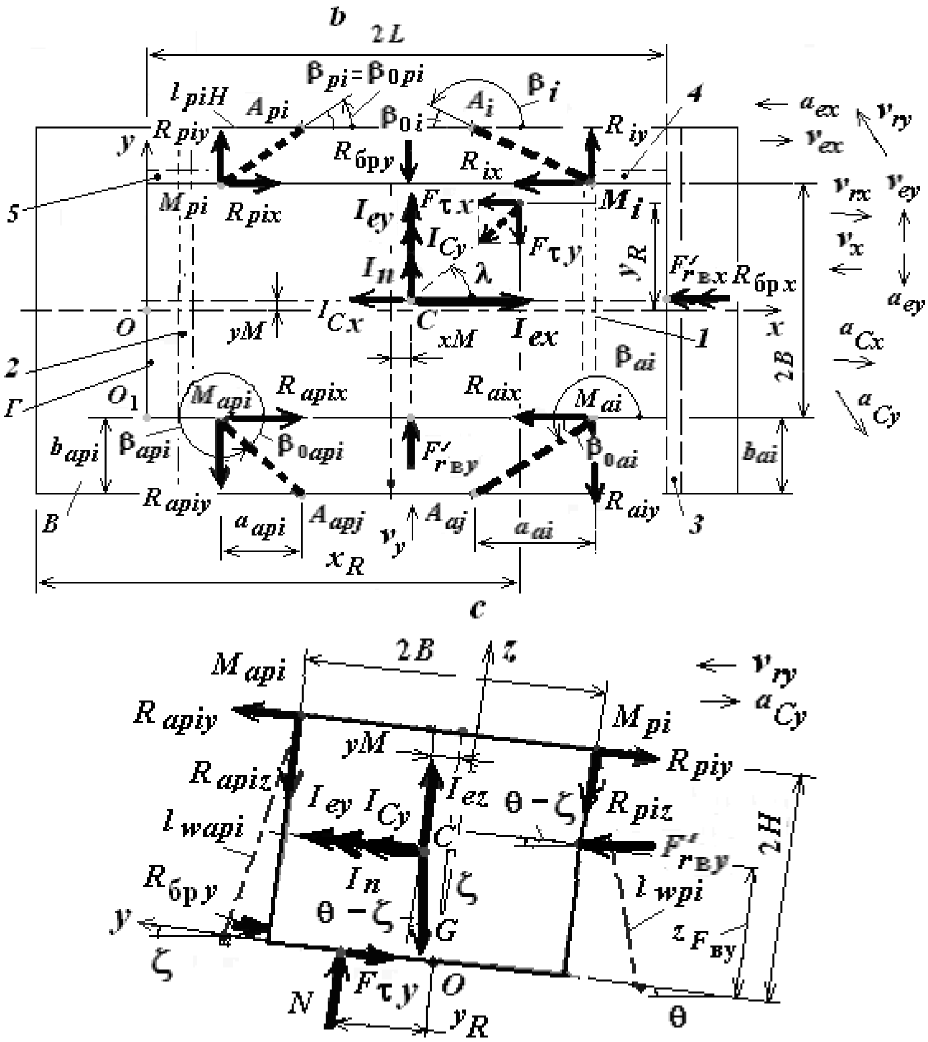


Fig. 1. Dynamic model of cargo with flexible, thrust and supporting fastening means: a) axonometry; b) top view; c) butt view

Рис. 1. Динамическая модель груза с гибкими, упорными и опорными средствами креплений: а) аксонометрия; b) вид сверху; c) вид с торца

2. METHODS OF SOLUTION

It is offered to use an equilibrium equation during relative motion (at rest) in projections onto the coordinate axes under the action of spatial force system (SFS) [12]:

$$\sum_{k=1}^{n} \overline{F}_k + \sum_{k=1}^{n} \overline{R}_k + \bar{I}_e + \bar{I}_C = 0 \qquad (5)$$

where:

M – is the mass of a material point (of cargo),

$\sum_{k=1}^{n} \overline{F}_k$ – active forces (all forces known by value and direction, including the forces of aerodynamic resistance $F_{r\text{в}} \in (F'_{r\text{в}x}, F'_{r\text{в}y}, F'_{r\text{в}z})$)

$\sum_{k=1}^{n} \overline{R}_k$ – reactions of all external constraints,

I_e – transferring inertia force,

I_C – carioles inertia force, which in practical calculations can be neglected due to its insignificant value (it accounts for less than 10 of cargo weight – for example 700 kN – at a train's speed of 100 km/h) [9].

Let us take as the basis of the dynamic model the fact that the cargo fastening elements together with thrust bars experience the impact of SFS which are perceived by the major constraint (wagon) and additional constrains (flexible elastic, thrust and supporting wooden fastening means). In order to construct a cargo dynamic model (object) in presence of thrust and supporting bars during rolling stock movement along a descending grade on a curved track section and according to the principle of releasing from constraints the external constraints - platform frame as a major constraint, and flexible elastic fastening elements and also thrust and supporting bars as additional constraints are mentally discarded from the object.

3. RESULTS OF SOLUTION

Let us write down the condition of cargo equilibrium in relative equilibrium (at rest) in projections onto the coordinate axes [3, 5-11]:

$$I_{ex} + \left(G_x - F_{\text{гв}.x}\right) - I_{Cy} - F_{ix} - F_{\tau x} - R_{\text{бр}.x} = 0 \tag{6}$$

$$I_{ey} + I_{Cx} + (I_{ny} + F_{\text{гв}.y}) - G_x - F_{iy} - F_{\tau y} - R_{\text{бр}.y} = 0 \tag{7}$$

$$-\left(G_z - I_{ez}\right) + N - \left(I_{nz} + F_{\text{гв}.z}\right) - F_{iz} = 0 \tag{8}$$

where:

$F_i \in (F_{ix}, F_{iy}, F_{iz})$ – elastic forces of flexible elastic elements of cargo fastening

$i \in (1, n_{\text{р}})$ – the number of fastening elements normally chosen according cargo weight,

$R_{thr} \in (R_{thr.x}, R_{thr.y})$ – reaction of thrust bars.

It is necessary to remember that F_i includes the elastic forces (efforts) of fastening elements R_i not only due to the impact of external forces but also from the impact of pretensioned wire twists $R0_i$ (normally they are 20 kN [3]), i.e. $F_i \in (R_i, R0_i)$. That is why the unknown values in equation (1) are the efforts (tensions) in flexible fastening elements (R_i) and reactions of thrust bars $R_{thr.x}$ and $R_{thr.y}$.

Let us consider a general case of cargo displacement (Δs) in the direction of the action of resultant SFS $\Delta F^{(i)}$ (Fig. 2) [3, 6-11].

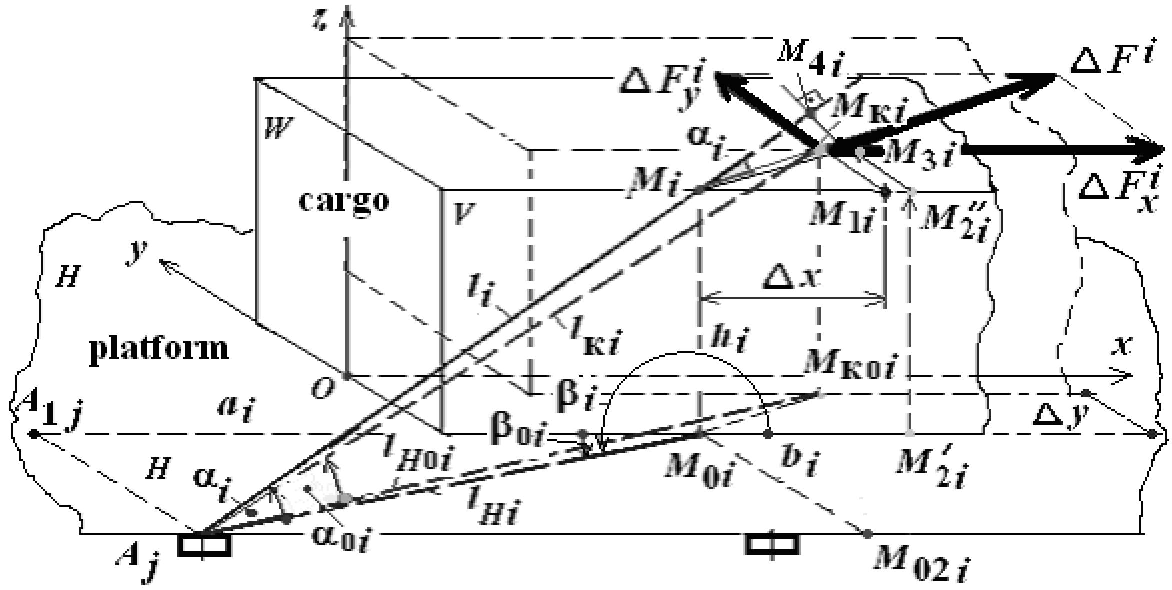


Fig. 2. Generalized model of cargo displacement under the impact of SFS
Рис. 2. Обобщённая модель сдвига груза от воздействия ПСС

In Fig. 2 the following table of symbols is accepted: M_i are points showing lifting lugs and their projections onto horizontal and vertical planes; l_i and a_i, b_i and h_i are lengths of flexible elastic fastening elements and their projections; Δs is cargo displacement in the direction of the action of the resultant of spatial force system $\Delta F^{(i)}$, which is to be found: ξ_i is the angle characterizing cargo displacement in the plane of the wagon floor.

Leaving out intermediate mathematical computations of equation 1) it is possible to determine the dependence of cargo displacement in the wagon floor plane, m, in the direction of the action of the resultant SFS - $\Delta s = f(\Delta F^{(i)}, c^F_{ekv.})$ [3, 6-11], where $c^F_{ekv.}$ is equivalent rigidity of flexible fastening elements as linear rods in the direction of SFS action, kN/m.

In (3) the resultant of spatial force system $\Delta \bar{F}^{(i)}$ (Fig. 2) perceived by cargo flexible elastic fastening elements is presented in the form:

$$\Delta \bar{F}^{(i)} = \Delta F_x^{(i)} \bar{i} + \Delta F_y^{(i)} \bar{j} \tag{9}$$

where:

$\Delta F_x^{(i)}$ and $\Delta F_y^{(i)}$ are projections of force constituents onto the lengthwise lateral axes [3, 6-11]:

$$\Delta F_{ix}^{(i)} = \Delta F_{ix}^{0} - F_\tau^e \cos\lambda^{(i)}, \ \Delta F_y^{(i)} = \Delta F_y^{0i} - F_\tau^e \sin\lambda^{(i)} \tag{10}$$

$\Delta F_x^{0(i)}$ – is the lengthily force perceived by the fastenings of one direction as the difference between “shifting and “retaining” forces:

$$\Delta F_x^{0i} = F_{sh.x} - F_{ret.ix} \tag{11}$$

$\Delta F_y^{0(i)}$ – is the lateral force perceived by the fastenings of one direction as the difference between “shifting and “retaining” forces:

$$\Delta F_y^{0i} = F_{sh.y} - F_{ret.iy} \tag{12}$$

$\lambda^{(i)}$ – is a direction angle where symbol i raised to power means that the angle is dependent on the number of flexible elastic fastening elements and has only one meaning,

F_{τ}^{e} is friction force which is due not only to the impact of external forces but also due to pressing of cargo down to the wagon floor by the fastening previous wire twists:

$$F_{\tau}^{e} = f\left[(G\cos(\psi_0)\cos\theta - I_{ez}) + F'_{\text{гвx}}\sin(\psi_0) + (I_n + F'_{\text{гвy}})\sin\theta + \sum_{i=1}^{n_{\text{p}}} R0_{iz}\right] \quad (13)$$

In (11) “shifting” and “retaining” forces unlike in [3, 6-11] are equal:

$$F_{sh.x} = I_{ex} + G\sin(\psi_0),\ F_{ret.ix} = \sum_{i=1}^{n_{\text{p}}} R0_{ix} + F'_{\text{гвx}}\cos(\psi_0) + F_{thr.x} \quad (14)$$

In (12) “shifting” and “retaining” forces also unlike in [3, 6-11] are equal:

$$F_{sh.y} = I_{ey} + (I_n + F'_{\text{гвy}})\cos\theta,\ F_{ret.iy} = G\sin\theta + \sum_{i=1}^{n_{\text{p}}} R0_{iy} + F_{thr.y} \quad (15)$$

According to (11) and (12) the displacement of cargo will take place under the fulfillments of conditions

$$\Delta F_{ix}^{(i)} > 0,\ \Delta F_{iy}^{(i)} > 0 \quad (16)$$

In this case in the work on retaining cargo from shifting there are being involved flexible elastic fastening elements.

If condition (14) is not fulfilled thrust bars and cargo pads together with pre tensioned fastening wires are able to keep the cargo from shifting in the direction of SFS. Thus, the condition of workability of thrust bars and cargo pads together with pre tensioned fastening wires are:

$$\Delta F_{ix}^{(i)} \le 0,\ \Delta F_{iy}^{(i)} \le 0 \quad (17)$$

In an extreme case, according to the mechanics law of action and reaction taking from (10) – (12) and allowing for (14) and (15) and leaving out interim calculations we will present condition (15) in the form of (16):

$$F_{thr.x} + \sum_{i=1}^{n_{\text{p}}} R0_{ix} \ge I_{ex} + G\sin(\psi_0) - F'_{\text{гвx}}\cos(\psi_0) - F_{\tau}^{e}\cos\lambda^{(i)} \quad (18)$$

$$F_{thr.y} + \sum_{i=1}^{n_{\text{p}}} R0_{iy} \ge I_{ey} + (I_n + F'_{\text{гвy}})\cos\theta - G\sin\theta - F_{\tau}^{e}\sin\lambda^{(i)} \quad (19)$$

Hence, in a particular case, if angle $\lambda^{(i)}$ characterizing the direction of the action of SFS $\Delta F^{(i)}$ (Fig. 2), $\lambda^{(i)} = 0$ it means that fastening means experience the impact of plane force system in the plane Oxz and if $\lambda^{(i)} = \pi/2$ then fastening means experience the impact of these forces in plane Oxy.

Fulfillment of conditions (18) and (19) makes it practicable to use thrust bars and cargo pads together with pre tensioned wires as cargo fastening means being at rest in relation to the platform floor (i. e. $\Delta x = 0$ and $\Delta y = 0$) Thrust elements (bars) and cargo pads together with pre tensioned fastening wires will prevent cargo from shifting in the direction of the action of SFS (i.e. $\Delta s = 0$). In this case it is necessary to take $f = f_{\text{сц}}$, and $R_i = 0$.

We are to take into account the fact that according to Specification the reaction of thrust bars is to be found [3, 4]:

$$R_{thr.x} = n_{fix.x} n_{\text{бр}.x} \lfloor R_{fix.} \rfloor,\ R_{thr.y} = n_{fix.y} n_{\text{бр.y}} \lfloor R_{fix.} \rfloor \tag{20}$$

where:

$n_{\text{fix.}}$ and $n_{\text{thr.}}$ – are the numbers of fastening items (nail) (a parameter to be found) and thrust bars (normally taken according to cargo overall dimension and wagon usable floor area) stacked both lengthwise and crosswise the wagon, item,

$[R_{fix.}]$ – is the normative value of longitudinal (or lateral) load allowable per one fastening part, kN (is to be chosen according to Specification depending on the diameter of the fastening part).

From ratios (17) and (18) and taking into account (20) we find the number of fastening items (nail) [9]:

$$n_{fix.x} \geq \frac{1}{n_{thr.x}\lfloor R_{fix} \rfloor}\left(I_{ex} + G\sin(\psi_0) - \sum_{i=1}^{n_p} R0_{ix} - F'_{\text{гвх}}\cos(\psi_0) - F_\tau^e \cos\lambda^{(i)} \right) \tag{21}$$

$$n_{fix.y} \geq \frac{1}{n_{thr.y}\lfloor R_{fix} \rfloor}\left(I_{ey} + (I_n + F'_{\text{гву}})\cos\theta - G\sin\theta - \sum_{i=1}^{n_p} R0_{iy} - F_\tau^e \sin\lambda^{(i)} \right) \tag{22}$$

In a particular case, when $\lambda^{(i)} = 0$ and $\lambda^{(i)} = \pi/2$, (15) and (16) can be presented in the form:

$$n_{fix.x} \geq \frac{1}{n_{thr.x}\lfloor R_{fix} \rfloor}\left(I_{ex} + G\sin(\psi_0) - \sum_{i=1}^{n_p} R0_{ix} - F'_{\text{гвх}}\cos(\psi_0) - F_\tau^e \right) \tag{23}$$

$$n_{fix.y} \geq \frac{1}{n_{thr.y}\lfloor R_{fix} \rfloor}\left(I_{ey} + (I_n + F'_{\text{гву}})\cos\theta - G\sin\theta - \sum_{i=1}^{n_p} R0_{iy} - F_\tau^e \right) \tag{24}$$

Analyzing (20) and (23) one can observe that tangential component of wagon floor reaction F_τ^e (Fig. 2) called friction force between contacting planes of cargo and wagon floor has the same meaning for the plane force system (PFS).

Then, substituting the accepted values $n_{fix.x}$ and $n_{fix.y}$ according to the calculations results (21) and (22) in accordance with (20) new values $R_{thr.x}$ and $R_{thr.y}$, are found which are to be used in further calculations.

If (17) and (18) – (22) are not fulfilled there will be wrenching away of a thrust element in relation to the platform floor accompanied by pulling of fastening parts (nail) with bending. It should be noted that fastening parts are working only on bending as they will experience the impact of the torque moment $M_{tor} = R_{thr}h_{thr}$, where $h_{thr} = h$ is the half of the thrust bar height [3].

In this case not only will wrenching away with subsequent pulling of fastening parts take place but also there will be drawing away of pre tensioned fastening wires with cargo displacement at same value which is to be defined according to (2). Then tension R_i in i flexible elastic fastening elements should be defined according to the value of cargo displacement in the floor plane of wagon (Δs) only when (10) is not fulfilled according to [3, 6-11]:

$$R_{\text{упр}i} = \frac{EA_i}{l_i}\Delta s\left(\frac{a_i}{l_i}\cos\lambda^{(i)} + \frac{b_i}{l_i}\sin\lambda^{(i)}\right) \le [R_i] \tag{25}$$

where:
$[R_i]$ – is an allowable value of tensions in fastenings defined according to *Specification* depending on diameter d_i and the number of fastening wires lengths n_i,
EA_i – is rigidity of fastening elements as a linear rod working on stretching,
A_i – is cross-sectional area fastening elements $A_i \in (d_i, n_i)$. It is of interest to note that it is sufficient to find R_i only for a short fastening element but not for all of them as is required in *Specification*.

Thus, the criterion of assessment of flexible fastening elements is their allowable value defined according to Table 20 in *Specification* [3].

Besides, analysis (25) suggests that it is possible to solve the reverse problem by defining the allowable value of cargo displacement in the wagon floor plane ($[\Delta s]$) according to the allowable value of elastic forces in each fastening $[R_i]$:

$$[\Delta s] \ge \frac{l_i}{EA_i}\frac{[R_i]}{\left(\frac{a_i}{l_i}\cos\lambda^{(i)} + \frac{b_i}{l_i}\sin\lambda^{(i)}\right)} \tag{26}$$

Then taking into account (26) you can derive a formula of checking calculation of elastic elements of cargo fastening with pads and thrust bars under the action of SFS:

$$\Delta s = \frac{\Delta F^{(i)}}{c^F_{\text{экв.}}} \le [\Delta s] \tag{27}$$

It can easily be seen that that in a particular case when $\lambda^{(i)} = 0$ and $\lambda^{(i)} = \pi/2$, (25) – (27) under the action plane force system (PFS) can be presented:

- lengthwise the wagon

$$R_{\text{упр}i} = \frac{EA_i}{l_i}\Delta x\frac{a_i}{l_i} \le [R_i],\ [\Delta x] \ge \frac{l_i}{EA_i}\frac{l_i}{a_i}[R_i],\ \Delta x = \frac{\Delta F_x^{(i)}}{c^F_{\text{экв.}x}} \le [\Delta x] \tag{28}$$

- crosswise the wagon

$$R_{\text{упр}i} = \frac{EA_i}{l_i}\Delta y\frac{b_i}{l_i} \le [R_i],\ [\Delta y] \ge \frac{l_i}{EA_i}\frac{l_i}{b_i}[R_i],\ \Delta y = \frac{\Delta F_y^{(i)}}{c^F_{\text{экв.}y}} \le [\Delta y] \tag{29}$$

Thus, the problem of cargo fixation on the open rolling stock can also be solved by comparing calculation value of cargo displacement in the wagon floor plane (Δs) with allowable value ($[\Delta s]$) not resorting to defining elastic force R_i in each i-fastening element as is required in Specification.

After $R_i = R_{el.i}$, have been defined it is possible to normal constituent N of external constraint (pads) reaction from the third equation (3) in the form [11]:

$$N = (G_z - I_{ez}) + (I_{nz} + F_{\text{гв.}z}) + F_{iz} \tag{30}$$

where:
G_z – is the projection of gravity force onto axis z:

$$G_z = G\cos\psi_0\cos\theta; \tag{31}$$

$F_{гв.z}$ – projections of aerodynamic forces $F'_{гв.x}$ and $F'_{гв.y}$ onto axis z:

$$F_{гв.z} = F'_{гв.x}\sin\psi_0 + F'_{гв.y}\sin\theta \tag{32}$$

F_{iz} – is the projection of elastic forces $F_{el.iz}$ and previous twists of fastening elements $F0_{iz}$ onto axis z:

$$F_{iz} = F_{упр.iz} + F0_{iz} \tag{33}$$

with allowing for the fact that:

$$F_{elas.iz} = \sum_{i=1}^{n_р} R_{elas.iz},\ F0_{iz} = \sum_{i=1}^{n_р} R0_{iz}, \tag{34}$$

or in projections onto axis z.

Working out a force moment balance equilibrium around axis Oy (Fig. 1) we find application coordinates x_R of normal constituent N of reaction of external constraints in the form:

$$x_R = \frac{1}{N}\left(\begin{array}{l}(G_z - I_{ez} + I_{nz})x_C + (I_{ex} - I_{Cx} - F'_{гв.x}\cos\psi_0)H + \\ + F'_{гв.x}\sin\psi_0 2L + F'_{гв.y}\sin\theta L - \left(\sum_{i=1}^{n_р} R_{упр\,i}\frac{a_i}{l_i} + \sum_{i=1}^{n_р} R0_i\frac{a_i}{l_i}\right)2H + \\ + \left(\sum_{i=1}^{n_р} R_{упр\,i}\frac{h_i}{l_i} + \sum_{i=1}^{n_р} R0_i\frac{h_i}{l_i}\right)x_{Mi}\end{array}\right) \tag{35}$$

Taking into account equation $N = N_1 + N_2$ and $N_1 / N_2 = (x_R - x_{R2})/(x_{R1} - x_R)$ [1], and having made elementary computations we find [11]:

$$N_1 = \frac{N(x_R - x_{R2})}{(x_R - x_{R2}) + (x_{R1} - x_R)},\ N_2 = N - N_1 \tag{36}$$

Similarly to x_R it is possible to find coordinate y_R of application N crosswise the wagon (Fig. 1).

Taking into consideration the fact that reaction N_1 is applied to the side of the action of SFS resultant, $N_1 > N_2$, checking calculation should be made only for cargo pad *1* (Fig. 1) which, for instance, makes it possible to find the height according to its width.

Thus, the assessment of cargo pad loading is to be done according to the criterion of allowable pressing tension (*Specification* – Table 23 in [3]) of the most loaded of the two cargo pads used.

The reliability of the obtained results under the impact of SFS is confirmed by the exact fit of solution results of definite tasks of cargo fastening with cargo being located symmetrically in relation to the wagon symmetry axis as in a particular case, concerning the results of previous research – for example, formulas

(1)-(4) *Appendix 8* in [3, 4] and digital modeling in MathCAD [3] under the impact of plane force system as a test task.

Summing up the results of the analytical research conducted it is necessary to point out that developed for the first time ever generalized dynamic and mathematical models of collateral cargo fixation with padding by flexible and thrust elements made it possible to identify conditions providing workability of thrust bars together with pre twisted fastening wires and cargo pads. It should be noted that it is only when these conditions are not fulfilled that elastic forces of flexible fastening elements are engaged in work on retaining cargo from shifting. From a generalized dynamic and mathematical model there can be obtained specific models for all known techniques of cargo fastening under the action of both plane and spatial force systems [3, 6–11].

4. SUMMARY

Summing up the results of carried out research it should be observed that the technical problem of collateral fixation of cargo with padding by flexible and thrust fastening elements has been properly solved in a general way on the basis of equilibrium equation at relative motion (at rest) (similarly to the method of kinetostatics [12].

Developed for the first time the generalized mathematical model of collateral fixation of cargo with padding by flexible and thrust elements made it possible to indentify the conditions providing workability of thrust bars together with pre twisted fastening wires and cargo pads. It should be emphased that it is only when these conditions are not fulfilled that elastic forces of flexible fastening elements are engaged in work on retaining cargo from shifting.

In a particular case, on the basis of the constructed generalized mathematical model there can be obtained analytical formulas for calculating collateral cargo fixation by flexible and thrust [10], flexible and supporting (pads) [11] fastening means separately and also by flexible fastening elements without thrust bars and pads in accordance of worked out technique of allocation and fixation of cargo in the wagon.

Obtained analytical formulas for calculating stress loading of cargo fastening means are to be used for safety assessment of moving rolling stock under all possible cargo transportation conditions (for example, during rolling stock movement both on a tangent and curved track section as well as in case of collisions and abrupt braking.

The distinguishing characteristics (a feature of novelty) of generalized model under the action of SFS as compared to specific models [3, 6-11] is manifested in its ability to provide workability of flexible fastening elements together with thrust bars and cargo pads according to the criterion of normative value of longitudinal (or lateral) load on a fastening component and allowable pressing tension of the most loaded of the used pads. The investigation results concerning cargo transported by and unprovided for in Specification are now available for researchers and can be looked upon as a new stage in the development of this problem.

In perspective the results obtained can be used for working out normative documents on fixation of cargo under the action of PFS unprovided for by technical conditions.

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