tubular stabilizer bars, body roll

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TUBULAR STABILIZER BARS – CALCULATIONS AND CONSTRUCTION

Summary. The article outlines the calculation methods for tubular stabilizer bars. Modern technological and structural solutions in contemporary cars are reflected also in the construction, selection and manufacturing of tubular stabilizer bars. A proper construction and the selection of parameters influence the strength properties, the weight, durability and reliability as well as the selection of an appropriate production method.

STABILIZATORY RUROWE – OBLICZENIA I KONSTRUKCJA

Streszczenie. W artykule przedstawiono zarys metod obliczeniowych stabilizatorów rurowych. Nowoczesne rozwiązania technologiczno – konstrukcyjne we współczesnych samochodach znajdują również odzwierciedlenie w konstrukcji, doborze i produkcji stabilizatorów. Prawidłowa konstrukcja i dobór parametrów mają wpływ na cechy wytrzymałościowe, ciężar, trwałość oraz niezawodność jak i wybór właściwej metody produkcyjnej.

1. INTRODUCTION

The function of stabilizer bars in motor vehicles is to reduce the body roll during cornering. The body roll is influenced by the occurring wheel load shift and the change of camber angle. Decisive is the steering performance which may be purposefully adjusted towards understeer or oversteer when designing the stabilization. So the stabilizer bars increase the travelling comfort and to a considerable extent the driving safety [10].

Such criteria as spring rate and lifespan in a given installation space are to be met when designing stabilizer bars. The substitution of solid tubular stabilizer bars offers a great weight saving potential. In the simplest approach, the solid stabilizer bar is substituted by a tubular stabilizer bar in which the tubular cross-section is constant over the entire stabilizer bar length. The spring rate of the stabilizer bar results from the total deformation of areas under bending or torsional stress given. In order to obtain the required rate, a specific cross-section (area moment of inertia) is needed. The buckling strength of the tube restricts the reduction of wall thickness. That is why the relationship of outer diameter to wall thickness should not exceed 6,5 to 7,5 mm. Subject to an equal spring rate, outer diameters greater by about 6% and thus accordingly higher strains arise with weight saving of about 40% compared to solid stabilizer bars [10, 11].
2. CALCULATIONS AND CONSTRUCTION

2.1. Design of tubular stabilizer bars

Bent transitions between backs and arms of a stabilizer bar are as a rule the highly stressed areas of this component. Due to sophisticated geometry, in these areas arises increased strains as a result of which the required lifespan may not be achieved [11, 17]. If during analytical calculation of tubular stabilizer bars the strains in the bearing or in elbow areas are too high \( \sigma > \sigma_{\text{zul}} \) (e.g. in case of material 34MnB5, \( R_m \) to 1700 \( \frac{N}{mm^2} \)), the following possibilities are available [10, 17]:

1. Use of higher–strength steel (limited possibilities).
2. Stabilizer bar with variable diameter / wall thickness:
   - If with the use of high–strength steel the maximum permissible strain is also exceeded, the strain energy has to be shifted to less stressed areas. Consequence: stabilizer bar with a non–constant diameter / wall thickness (rotary swaging),
   - Large diameters / wall thicknesses in critical areas (e.g. elbows, bearing surfaces),
   - Thinner diameters / wall thicknesses in the back / arm,
   - Only by diameter / wall thickness reduction in less stressed areas may the required rate be obtained.

When designing tubular stabilizers, attention is to be paid to the fact that the weight must not be reduced at the cost of the component rigidity that may be roughly calculated by the equation (1) [10]:

\[
R = \frac{G_{\text{mod}}}{\eta} I
\]

where:
- \( R \) – rigidity,
- \( G_{\text{mod}} \) – rigidity modulus,
- \( I \) – area moment of inertia,
- \( \eta \) – geometry matrix.

Furthermore, it is important that the target weight reduction is not greater than 40 to 45 % of the weight of a comparable solid stabilizer bar. Weight savings going beyond that lead to no more acceptable wall thicknesses, thus to additional stresses caused by tube warping (Fig. 1). These warping strains have an extremely negative impact on the lifespan of a component. The outer and inner diameters may be calculated using the following equations [10]:

\[
d_{\text{in,tube}} = d_{\text{solid}} \sqrt{\frac{1-k^2}{2k}} \quad (2)
\]

\[
d_{\text{out,tube}} = d_{\text{solid}} \sqrt{\frac{1+k^2}{2k}} \quad (3)
\]

where:
- \( d_{\text{in,tube}} \) – inner or outer tube diameter,
- \( d_{\text{solid}} \) – diameter of the solid stabilizer bar,
- \( k \) – weight portion of the tube e.g. \( k=0,6 \).

Fig. 1. Tubular stabilizer bar – cross–section
Rys. 1. Stabilizator rurowy – przekrój
Moreover, when using tubes, it is to be taken into consideration that the inner surface may be treated and protected against corrosion only to a limited extent. If the calculation results in exceeded permissible stress, then the critical area has to be relieved of strain without changing the rigidity (rate) of the stabilizer bar. Thanks to decreased bending stresses in the back area between bearing surfaces arises the possibility of shifting the rigidity from the back to the area with critical strain. The cross-section of the bearings of the back may be reduced by mechanical processing or by targeted material extrusion, such as swaging process. Here, attention should be paid to the fact that both the diameter and the wall thickness are reduced [10, 11, 17]. A process used to reduce the diameter and the wall thickness was shown in Fig. 2 [17].

Fig. 2. Rotary swaging – a process used to reduce the diameter and the wall thickness
Rys. 2. Kucie – proces mający na celu zmiany średnicy i grubości ścianek

2.2. Calculation und Construction

2.2.1. Stress on tubular stabilizer bars

When twisting the stabilizer bar (Fig. 3), the reaction forces $F_S$ at the ends and $F_L$ at the bearings arise. The arms are stressed by bending. The back is mainly stressed torsionally and in addition by bending.
Shear stress $\tau(M_t)$ resulting from the torsional moment [1–4, 8, 16, 17] is obtained from:

$$\tau(M_t) = \frac{M_t \sqrt{1 - 2\cos \beta + v^2}}{l_p \sqrt{1 - \cos \beta}} \text{[MPa]} \quad (4)$$

$$w = 2 \frac{r}{d} \text{ – elbow ratio} \quad (5)$$

$$v = \frac{1}{2} (w - \sqrt{w^2 - 1}) \text{ – relative eccentricity} \quad (6)$$

Shear stress $\tau(F)$ resulting from the shear force is obtained from:

$$\tau(F) = \frac{F}{A_1 - \left(\frac{d}{d_c}\right)^2} \cos \beta \text{ [MPa]} \quad (7)$$

From these three stresses, the equivalent stress according to Mises, Huber, Hencky [5–8] may be calculated:

$$\sigma_{\text{eq}} = \sqrt{\left(\sigma(M_b)\right)^2 + 3\left[\tau(M_t) + \tau(F)\right]^2} \text{ [MPa]} \quad (9)$$

2.2.2. Stabilizer bar rate

The spring rate of the stabilizer bar results from the sum of deflections at the ends (twisting) and the stabilizer bar force as (Fig. 4):

$$c_{\text{stab}} = \frac{F_{\text{st}} + F_{\text{nr}}}{\Delta s_t + \Delta s_r} \quad (10)$$
2.2.3. Excessive increase of stress on tubular stabilizer bars

When bending tubular stabilizer bars, the following shape deviations (Fig. 5) occur [17, 19, 20]:

- **cross-section ovalization:**
  - diameter $d_1 >$ desired diameter $d_A$,
  - diameter $d_2 <$ desired diameter $d_A$,

- **wall thickness change:**
  - wall thickness $s_1 >$ desired wall thickness (thickening),
  - wall thickness $s_2 <$ desired wall thickness (thinning due to stretching).

Due to real geometry, the highest strains in tubular stabilizer bars occur usually at the outer fiber, and for this reason it is the vulnerable area.

2.2.4. Tube bending

In the process of tube bending [12–15, 18–20], the outer curve of the tube is elongated and the inner curve is upset forged. Furthermore, circumferential stress occurs. The consequences of this complex triaxial stress and deformation condition are displacements of the non–stretched layer towards the bending center, in circumferential direction – elongations of the stretched side and upsetting of the compression side, changes in the cross-sectional shape (ovalization) and an irregular distribution of cold work hardening over the cross–section and unrolling the bend in relation to longitudinal axis. The general definitions relating to bent tube are explained with the aid of Fig. 6.
The elongations during tube bending are determined in the following way \[16, 21\]:

\[
\varepsilon = \frac{y}{R_{th}}
\]

\[
\varepsilon = \frac{dl - dl_0}{dl_0} = \frac{(R_{th} + y)da - R_{th}da}{R_{th}da}
\]  

(12)

According to the equation (12), the elongation in the outer curve rises linearly to distance \(y\) from the center line. The compression in the inner curve increases also linearly: During bending, a linear elongation over the tube cross-section occurs.

This strain distribution results in a characteristic stress distribution. In the elastic area, a linear stress distribution occurs. In the plastic area, a projection of the flow curve occurs. These stress distributions in the elastic and the plastic areas are illustrated in Fig. 7.

2.2.5. The maximum bending moment

For this purpose, the following assumptions are made \[12–15, 18–20\]. There is a uniaxial stress condition (stresses in longitudinal direction of the tubes). The material is characterized by a multi-linear material model. In this case, a linearly elastic and linearly plastic material model is used. The
modulus of elasticity is assumed to be constant. The material is homogenous and has isotropic properties with respect to stresses and strains. The influence of the forming temperature and the rate of forming are disregarded. The Bernoulli’s hypothesis applies. It says that cross-sections perpendicular to the tube axis before deformation will stand vertically on the deformed tube axis also after deformation (assumption of cross-sections remaining plane). Here, a linear elongation process is assumed. This hypothesis is a precondition in case of the beam bending theory. It is equivalent to disregarding shear stresses in the cross-section. Consequently, only the bending moments which cause a curvature in the beam axis and normal stresses in the cross-section have effect. The displacement of the neutral fiber and the inertial forces are disregarded. The bending moment is calculated by:

$$M_B = \delta(y)ydA \text{ with } dA = bdy$$

(13)

Stresses in the cross-section of the tube result from the assumption of a linearly elastic and linearly plastic material, such as in Fig. 8 illustrated. Here, $\delta_f$ is the yield strength ($R_{p0.2}$), $\delta_p$ the stress in the plastic area and $\delta_d$ the edge compression strength.

For the purpose of the bending moment calculation, the stresses in the elastic area and in the plastic area are distinguished. The elastic area is up to the distance $y_e$ from the central axis of the tube.

![Fig. 8. Stresses in the cross-section](image)

Rys. 8. Rozkład naprężeń w przekroju

After it, the plastic area begins:

$$y_e = R_{p0.2} \frac{\delta_f}{E}$$

(14)

To the distribution of stress in the cross-section applies now:

$$\delta(y) = \frac{y}{y_e} \delta_f \text{ for } y \leq y_e$$

(15)

$$\delta(y) = \frac{r_a - y}{r_a - y_e} \delta_f + \frac{y - y_e}{r_a - y_e} \delta_p \text{ for } y \leq y_e$$

(16)

Cross-section surfaces are subdivided into the following two areas (see Fig. 9):

$$d(A) = 2\left(\sqrt{r_a^2 - y^2} - \sqrt{r_i^2 - y^2}\right)dy \text{ for } y \leq y_l$$

(17)

$$d(A) = 2\sqrt{r_a^2 - y^2}dy \text{ for } y \leq r_i$$

(18)
The total moment $M_{B\text{ideal}L}$ consists of an elastic moment $M_{B\text{el}}$ and a plastic moment. Then one has:

$$M_{B\text{ideal}L} = M_{B\text{el}} + M_{B\text{pl}} = \int_{A\text{el}} \delta(y)y\,dA + \int_{A\text{pl}} \delta(y)y\,dA$$  \hspace{1cm} (19)

The ideal bending moment of the tube does not correspond to the bending moment that has to be provided by the tube bending machine. Reasons for this are among others disregarding of the conditions of force transmission and the settings of the bending parameters.

### 2.2.6. Resilience

Contingent on the elastic and plastic properties of metallic materials, the tube springs back by a certain angular amount after each bending procedure [19–21]. Assuming that the strip is curved uniformly, the resilience angle may be calculated from the resilience stress:

$$\frac{1}{r} = \frac{2\sigma_{a\text{el}}}{E}$$  \hspace{1cm} (20)

With $x$ as variable length and $\alpha$ as bending angle, then one has:

$$\alpha_r = \int_{x=0}^{x=r} \frac{2\sigma_{a\text{el}}}{E} \, dx = \frac{2\sigma_{a\text{el}}}{E} r\alpha$$  \hspace{1cm} (21)

If small members are disregarded, then one has:

$$\alpha_r \approx 3 \frac{R_p0.2}{E} \left(\frac{r}{s}\right)$$  \hspace{1cm} (22)

### 3. CONCLUSIONS

In modern vehicles, the tubular stabilizer bars are an increasing „competition“ for solid stabilizer bars. Additional aspects (e.g. deformation and weakening of the cross–section in the bending area) have to be taken into consideration when constructing and designing tubular stabilizer bars. When all construction guidelines are taken into consideration, the tubular stabilizer bars may achieve a comparable (with solid stabilizer bars) strength and durability.

### References


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