Rotational mass in car, moment of inertia

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PROBLEMS OF ROTATIONAL MASS IN PASSENGER VEHICLES

Summary. This study presents an overview of methods for calculation of inertia mass in vehicles which impact on inertial resistance. Different opinions in references on rotational mass (engine with clutch mechanism and the road wheels) provided the author an impulse to verify them with respect to currently manufactured vehicles.

PROBLEM MAS WIRUJĄCYCH W SAMOCHODACH OSOBOWYCH

Streszczenie. W pracy dokonano przeglądu metod obliczeń mas wirujących w pojeździe, wpływających na opory bezwładności ruchu. Zróżnicowane dane literaturowe na temat mas wirujących (silnik z mechanizmem sprzęgłowym i koła jezdne) skłoniły autora do ich weryfikacji we współcześnie produkowanych samochodach osobowych.

1. INTRODUCTION

The highest motion resistance results from considerable rise in inertial resistance during vehicle's acceleration [1, 2]. Inertial resistance component is reversible within a short range of acceleration values (dv/dt) [3]. According to the author, its boundary value (i.e. reversibility), in terms of scalar values, equals to acceleration in neutral. Hence general tendency towards limitation of maximal value of acceleration within the range of higher values, whose excess is irreversibly lost in the form of rise in fuel consumption Q [4].

Vehicle acceleration is strongly correlated with unit power of additional motion resistance (a*v) with also reversible climbing resistance force. Economical driving should cause reduction in unit power (a*v) to 6 W/kg, which was discussed in detail in references [3, 4, 5].

Inertial resistance during accelerating of a vehicle using engine does not only depend on vehicle weight but also on rotational masses present in the car. They include all the accelerated rotational masses, starting form crank system parts to vehicle wheels. Their inertial resistance depends not only on the value of rotational mass but also on the radius of gyration. There have been a variety of approaches to rotational mass inertial resistance in vehicles and the related discrepancies in calculation results [6, 7, 8, 9]. Therefore, in the present study, the author made attempts to present the details of this problem.

2. LITERATURE OVERVIEW AND THEORETICAL ANALYSIS

In classical mechanics, mass moment of inertia in physical body in relation to the axis of rotation is typically given by the equation [10]:

$$I_z = \int r^2 \cdot dm = \int \rho \cdot r^2 \cdot dV, \tag{1}$$

where: r – radius of rotation of quantum of mass *dm,* m; ρ – material density, 7.8 g/cm^3 for steel, 1.2 g/cm^3 for rubber, 4.0 g/cm^3 for cord, representative part of tire R14 ρ_{op} = 1.27 g/cm^3 ; dV – volume of quantum of mass *dm* rotating with radius r, m^3.

Hence, the value of radius of gyration for rotating body can be calculated:

$$r = \sqrt{\frac{I_z}{m}}, \tag{2}$$

or, reduced mass:

$$m_{red} = \frac{I_z}{r^2}. \tag{3}$$

During calculations of mass moment of inertia I_z in rotational bodies in relation to the axis of rotation z there are no problems with cutting this body into imaginary sleeves with thickness of Δr and the radius of gyration (r+Δr/2). The total of products of squared radii of gyration and its mass gives approximated value of I_z for a part of rotational element.

Calculation of mass moment of inertia for the rotational crank system in relation to crankshaft rotation is much more complex. Firstly, the mass of a part of connecting rods which perform rotational motion connected with the axis of the crankpin must be taken into consideration. Its measurement can be made by means of one or two scales measuring connecting rod mass, performing complex rotational motion with crankshaft journal [11].

Next, using Steiner's theorem, mass moment of inertia should be calculated for the masses concentrated around the axis of crankpin (not connected with the axis of crankshaft journal) in relation to crankshaft centerline. The calculations proved that mass moment of inertia for the engine results in 80-90% from distribution of the mass of flywheel assembly (the flywheel combined with clutch mechanism).

Even better results can be obtained through calculations of mass moment of inertia by means of *Catia V5* software [12]. In both methods, the control values included densities and the measured masses of crankshafts, flywheels with clutch mechanisms and the wheels.

It is necessary for calculations of mass inertia of a vehicle with mass m to find rotational mass coefficient δ:

$$F_b = m \cdot \delta \cdot a \tag{4}$$

where: m – gross vehicle weight, kg; a = (dv/dt) – vehicle acceleration, m/s^2.

There are a few reasons for discrepancies in the results of measurements as compared to literature data of [13, 14, 15]. Main reason for this is evolution of vehicle structure towards minimization of its weight, including rotational masses. Throughout the past 25 years, the weight of crank systems in passenger vehicles has been dramatically reduced [16]. However, due to more popular dual-mass structure of flywheels, its mass has increased in the past 10 years, which is illustrated by Isuzu 1.7 TDI engine with single- or dual-mass flywheel (Tab. 1).

Among the rotating parts in driving system of a vehicle, the most significant inertia can be observed for the engine with clutch mechanism and the wheels. Wheels have also been modified in the past 25 years: on the one hand, their mass has been considerably reduced (tubeless tires, disc brakes); on the other hand, there is a tendency towards increase in wheel size (width and diameter). It is generally accepted that δ rotational mass coefficient is given by the equation [14, 15]:

$$\delta = 1 + \delta_s + \delta_k \tag{5}$$

$$\delta = 1 + \delta_s' \cdot i_b^2 + \delta_k \tag{5a}$$

Through exclusion of reduction ratio in gearbox in equation (5a), δ_s is constant for a given vehicle (for transmission $i_b = 1.0$ – col. 10 Tab. 2) where:

$$\delta_s = \frac{I_s \cdot i_g^2 \cdot \eta_m}{m \cdot r_d^2} \cdot i_b^2, \tag{6}$$

and for the wheels:

$$\delta_k = \frac{4 \cdot I_k}{m \cdot r_d^2}, \tag{7}$$

m – gross vehicle weight, kg; i_b – reduction ratio for a given gear, i_g – final drive ratio, η_m – efficiency of power transmission system, I_s – mass moment of inertia for moving engine parts reduced to crankshaft centerline, kg·m^2, I_k – mass moment of inertia for wheel, kg·m^2, r_d – dynamic wheel radius, m.

Table 1

Comparison of measurements and calculations by means of Catia V5 software for mass moments of inertia in selected car engines

No	Type of engine	Measured weight [kg]				Mass moment of inertia I_z, kg ·m^2		
		Crank -shaft	flywheel assembly	connecting rod		crankshaft with connecting rod	flywheel assembly	Engines ΣI_z
				Assembly	rotational part[1)]			
0	1	2	3	4	5	6	7	8
1.	ZI 1.0XE	9.67	14.3	0.375	0.266	0.014	0.135	0.149
2.	ZI 1.2XE	10.05	11.3	0.375	0.266	0.015	0.110	0.125
3.	ZS 1.3TDI*	-	15.0	-	-	-	0.145	-
4.	ZS 1.7TDi	16.72	12.94	0.707	0.505	0.027	0.152	0.179
5.	ZS 1.7TDi*	16.72	17.50	0.707	0.505	0.027	0.184	0.211
6.	ZS 2.0 TDI*	15.43	18.13	0.701	0.500	0.027	0.219	0.246

*- dual-mass flywheel

In English-language references, rotational mass coefficient is sometimes defined as [9]:

$$\delta = \frac{m + m_e}{m}, \tag{8}$$

m_e – rotational masses in vehicle reduced to wheel dynamic radius r_d, kg.

As a result of acceleration of rotating masses in vehicle, a part (often considerable) of energy generated by the engine is absorbed by inertia forces, which, in consequence, causes reduction in engine torque under non-stationary conditions [9]. This is given by the equation:

$$I_m \frac{dn}{dt} = T_e - T_l \tag{9}$$

where: I_m – mass moment of inertia of power unit, kg·m^2, dn/dt – increase in engine speed per time unit, s^{-2}, T_e – effective engine torque, Nm, T_l – engine torque under load, Nm

Value of mass moment of inertia of power unit I_m strongly depends on crank system with clutch mechanism and the wheels. Tab. 1 and 2 present detailed values.
In the developed form, with coefficient of reduction in engine power under conditions of unsteady operation υ, this equation is given [15]:

$$\frac{\vartheta \cdot T_s \cdot i_c \cdot \eta_m}{r_d} - \frac{I_s \cdot \ddot{\varphi}_s \cdot i_c \cdot \eta_m}{r_d} = F_\psi + F_p + F_b + \frac{\sum I_k \cdot \ddot{\varphi}_k}{r_d} \tag{10}$$

where: ϑ - coefficient reduction in engine power under non-stationary conditions, T_s – engine torque in unsteady conditions, i_c – final drive ratio in power unit, η_m – mechanical efficiency of power transmission, r_d – wheel dynamic radius, I_s – moment of inertia of rotating parts of engine reduced to crankshaft centerline, φ_s – crank angle, F_ψ – road resistance for moving on flat road, equal rolling resistance force, F_p – drag resistance force, F_b – inertia force, m – vehicle weight, I_k – mass moment of inertia of wheels, φ_k – wheel angle, x – vertical coordinate of center of mass in the vehicle.

Inertia force is given by the equation:

$$F_b = m \cdot \left(1 + \delta_s^{'} \cdot i_b^2 + \delta_k\right) \cdot \ddot{x} \tag{11}$$

where: $\ddot{x}$ - vehicle acceleration dv/dt, m/s^2.

As mentioned before, rotational masses in vehicle are reduced to masses of two systems: engine/clutch and road wheels, i.e. $m_e = m_s + m_k$. Taking equation (8) into consideration, the expression in brackets from equation (11) can be given by:

$$\delta = \frac{m + m_s + m_k}{m}. \tag{12}$$

After replacing the expression in brackets with equation (12) and after transformations the inertia force is given by:

$$F_b = \left(m + m_s + m_k\right) \cdot \ddot{x} \tag{13}$$

Hence, instead of coefficient of rotational mass [2] and rotating masses of inertia [9] which is difficult to be evaluated, masses reduced to the radius of drive wheels of two assemblies of rotating masses, connected with crankshaft centerline and the wheels, can be used. First of them, depending on the transmission ratio used, is subject to considerable fluctuations, which is presented in Table 2. It would be beneficial to determine rough dependence between reduced engine masses for one of the transmission ratios and the engine's rotating mass (masses). This would prevent calculation errors which exceed the value ranged from a few tens to a few hundred percent.
Equation (12) can be transformed to:

$$\delta = 1 + \frac{m_s^{'}}{m} \cdot i_b^2 + \frac{m_k}{m}, \tag{14}$$

(where: $m_s^{'}$ - engine mass reduced per wheel axis for i_b = 1,0)

in which, through exclusion of transmission ratio, the value of (m_s/m) is constant (col. 10 in Tab. 2).
As results from equations (5) and (10):

$$\delta_s = \frac{m_s}{m}, \tag{15}$$

$$\delta_k = \frac{m_k}{m}. \tag{16}$$

Since the significant rotating masses in the vehicle are formed by the engine and the wheels ($m_e = m_s + m_k$), the following dependence is obtained from equations (5), (8), (10) and (12):

$$\frac{m_s}{m} = \frac{I_s \cdot i_g^2 \cdot \eta_m}{m \cdot r_d^2} \cdot i_b^2.$$

After transformations:

$$m_s = \left(\frac{I_s \cdot i_g^2 \cdot \eta_m}{r_d^2} \right) \cdot i_b^2.$$

From equations (7) and (12):

$$\frac{m_k}{m} = \frac{4 \cdot I_k}{m \cdot r_d^2}.$$

After transformations:

$$m_k = \frac{4 \cdot I_k}{r_d^2}.$$

The results of calculations for mass moments of inertia I for the crank system are presented below for several SI and CI engines with single and dual mass flywheels (with division into crankshaft $I_{s,w}$ and flywheel set $I_{s,kz}$) and four with typical dimensions of the wheels with division into the tire and the rim. The calculations were performed by means of linearlization and using Catia V5 software [12].

3. RESULTS OF CALCULATIONS FOR REDUCED ROTATING MASSES IN VEHICLES

3.1. Engine / Clutch Mechanism Assembly

In terms of rotating masses, engine has two comparable masses: crank system and flywheel combined with clutch mechanism. Due to small radius of rotating masses in crank system, its mass moment of inertia $I_{s,wk}$ amounts to barely ca. 15-20% of the total mass moment of engine inertia I_s. Remaining 80-85 % are taken by the mass moment of inertia of freewheel with clutch mechanism.

Coefficient of engine rotating masses δ_s rises in proportion to the square of rise in engine speed in relation to the wheels. It results from total power transmission system reduction ratio $i_c = i_g \cdot i_b$, which is reflected by the equation (6). The values of rotating masses in the engine, reduced to dynamic radius index for the selected vehicles are compared in Tab. 2. Reduced masses were calculated for five transmission ratios in gearbox and the ratio of i_b = 1,0 (col. 10).

Comparison of rotating masses in engine with the calculated reduced masses does not allow for conclusions concerning their interdependences to be used for checking the assumed or calculated I_s [15]. Impact of the square of total ratio and the dynamic radius index calls for a higher number of samples for investigations of SI and CI engines with single and dual mass flywheels. Therefore, more probable solution is to find correlation between the masses connected with crank system col. (11) and one of the reduced masses from col. (5) to (10).

3.2. Road Wheels

The approximated mass moment of inertia for road wheels can be easily calculated from the chart presented in Fig. 1b [1]. It mainly depends on the wheel size (dynamic radius index). The relevance of these data can be checked with the example of four randomly chosen wheels of different sizes and manufacturers. By means of Catia V5 software or linearization of cross sections of the wheel, mass moment of inertia for the rim and the tubeless tire can be calculated. Brake disc and the wheel mounting flange can be neglected as insignificant for calculation results (small diameter), which is presented in Tab. 3.

Table 2

Reduced masses in engine / clutch mechanism assembly (assumed η_m= 0,95)

nr	vehicle/ class	engine/ r_d	I_s kg·m^2	$i_{c,1\ldots5}$[2)]/i_g -	m_s kg i=1	2	3	4	5	$\frac{I_s i_g \cdot \eta_m}{r_d^2}$	$m_{s,wk}+m_{s,kz}$ kg
1	2	3		4	5	6	7	8	9	10	11
1	Fiat Doblo	1.6 ZI* 0.293	0.145	15.1; 8.34; 5.72; 4.33; 3.56/3.865	366	112	53	30	***20***	24/ 24.1	No data
2	Astra II B/K	1.6 ZI* 0.298	0.152	13.95; 8.00; 5.27 4.53; 3.33/3.74	316	104	45	33	***18***	22.8/ 23.7	***n.d.***
3	Astra II B/K	1.7 TDI 0.292	0.142	13.94; 7.32; 4.87 3.54; 2.846	307	85	38	20	***13***	22.1/ 21.5	***12.94+18.7 31.6***
4	Astra III B/K	1.7TDI* 0.298	0.181	13.94; 7.32; 4.87 3.54; 2.846/3.74	378	104	46	24	***16***	24/ 23.8	***17.5+18.7 36.2***
5	Fiat Marea/C	1.6 0.288	0.143	14.9;8.56;5.5; 3.93; 3.334/3.82	366	120	50	25	***18***	24/ 24.1	***n.d.***
6	Corsa[1)] 1.2 ZI	1.2XE 0.284	0.134	14.68; 8.42;5.57; 4.42; 3.51/3.94	340	112	49	31	***19***	24.5/ 23.5	***11.3+11.4 22.7***
7.	Corsa 1.2 ZI	1.2XE 0.284	0.134	15.58; 8.18;5.45; 3.95; 3.18/4.18	383	106	47	25	***16***	27.6/ 24.0	***11.3+11.4 22.7***

[1)] – Opel Corsa with 1.2 SI engine was made in two versions of gearboxes and final drives: CR5 - i_g=3.94 i WR5 - i_g = 4.18, [2)]- $i_c = i_g \cdot i_b$

The used methods allow for calculation of mass moments of inertia for each part (rim, tire) with comparison for bigger group of masses located at the biggest distance from the axis of wheel rotation (rim, tire end face). Additionally, total moment of inertia $I_{k,c}$ was compared with the selected one on the basis of the wheel dynamic radius index r_d calculated from the chart in Fig. 1 [15].

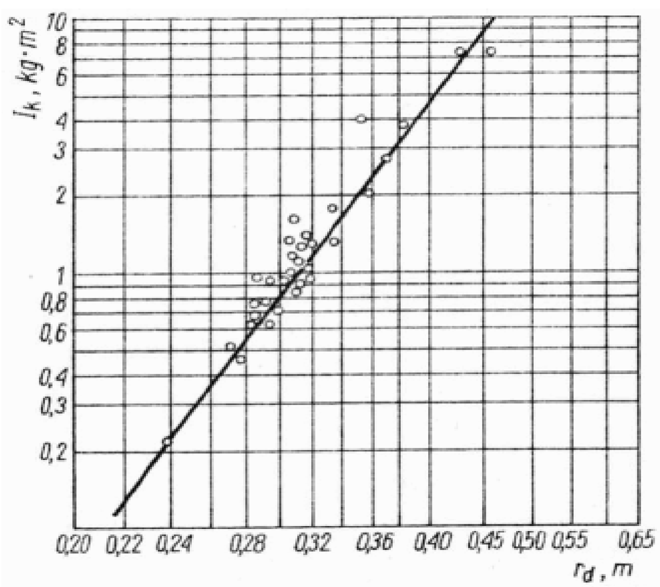

Fig. 1. Characteristics $I_{k,c}$ of vehicle wheels

Rys. 1. Charakterystyka I_k kół jezdnych samochodu

From the standpoint of mass moment of inertia (mass distribution throughout the radius), the wheel is composed of the rim with mounting flange and the end face with sides of the tire.

As results from the calculations, 75-80% of mass moment of inertia in steel rim is taken by the outside flange. However, tire end face (cylindrical part with the tread) takes over 70-75% of share in mass moment of tire inertia. This results from concentration of a considerable mass on the radius of the road wheel. Hence share of the tire in the I_k for the wheel with steel rim amounts to 72-77%. Therefore the conclusion that mass moment of wheel inertia will decrease with tire tread wear is justified. The convergence of the calculated and the measured wheel dynamic radius index, despite considerable mileage, should be pointed out.

Table 3

The results of calculation of mass moments of inertia I_k typical of passenger car wheels by means of superposition and Catia V5 software (line 3')

No.	Size/ manufacturer	r_d/i [1)] cm	rim + flange/rim			tire/ tire end face			$I_k = (I_o + I_{op})$ kg·m²	I_k [2)] kg·m²
			m_{ot} kg	I_o kg·m²	I_{o-t} kg·m²	m_{op} kg	I_{op} kg·m²	I_{cz} kg·m²		
1	2	3	4	5	6	7	8	9	10	11
1.	165/70R13 T[3)]	27.3/13	5.2	0.091	0.073	5.5	0.300	0.210	0.391	0.435
2.	175/65R14 T[4)]	28.2/13	7.7	0.180	0.135	6.0	0.424	0.317	0.604	0.56
3.	175/70R14[5)]	29.1	7.0	0.191	0.149	7.2	0.518	0.380	0.71	0.69
3'[7)]	***175/70R14[5)]***	***29.1***	***7.0***	***0.169***	**-**	***7.2***	***0.492***	**-**	***0.661***	***0.690***
4.	195/60R15 H[6)]	29.8/14	7.1	0.191	0.152	7.8	0.583	0. 42	0.774	0.77

[1)]- number of calculation ranges for the method of linearization for mass moments of inertia (Δr = 2 cm), [2)]- the values from the chart in Fig. 1 [15], [3)]- Mabor, mileage of ca. 20,000 km, [4)]- Pirelli, mileage of ca. 30,000 km, [5)] – all-season Michelin tire, mileage of ca. 30,000 km, [6)] - Michelin, mileage of ca. 30,000 km, [7)]- Catia V5 method

(Continuation/c.d.)

No.	$\lvert\delta\rvert = \Delta I_{k,w}/ I_k$	I_{o-t}/I_o	I_{cz}/I_{op}	I_{op}/I_k	$r_{d,ob.}/r_{d,z}$, mm
1	12	13	14	15	16
1.	0.10	0.80	0.700	0.767	272/272
2.	0.07	0.75	0.750	0.702	280/282.3
3.	0.03	0.78	0.734	0.744	291.3/288.5
3'	***0.04***	**-**	**-**	***0.744***	**-**
4.	0.009	0.76	0.720	0.753	298/298

Subscripts explanation: o- rim, o-t – rim without flange, op – tire, cz – tire end face, k – wheel, w - I_k from the chart [15], ob- calculated, z- measured

4. CONCLUSIONS

On the basis of the presented calculation results, one can conclude that:

1. The biggest contribution (80-90%) to the mass moment of inertia in SI and CI engines with engine displacement of 1.0-2.0 dm³ in crank system is taken by the flywheel with clutch mechanism.
2. In the four cases presented in Tab. 2, engine masses of inertia reduced to the road wheel dynamic radius index for each power transmission system reduction ratio are barely related to actual masses of the crankshaft with connecting rods and the flywheel assembly.

3. In order to evaluate mass moment of inertia for road wheels with steel rim, the $I_k = f(r_d)$ characteristics, presented in Fig. 1, can be employed. Rise in wheel size impacts on accuracy of determination of mass moment of inertia I_k from 10 (R13 wheel) to 1 % (R15 wheel).

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