Mathematical modeling of fastening, spatial force system

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ANALYTICAL INVESTIGATION OF CARGO DISPLACEMENT DURING THE MOVEMENT OF ROLLING STOCK ON A CURVED SECTION OF A TRACK

Summary. The article gives an account of the results of generalized fastening mathematical modeling of cargo asymmetrically allocated both lengthwise and crosswise the wagon during the rolling stock movement on a curve under the exposure to the spatial force system. On the basis of the particular case of spatial modeling it is possible to obtain mathematical and dynamic fastening models under separate exposures to lengthwise and vertical, crosswise and vertical forces.

АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ СДВИГА ГРУЗА ПРИ ДВИЖЕНИИ ПОДВИЖНОГО СОСТАВА ПО КРИВОМУ УЧАСТКУ ПУТИ

Аннотация. В статье изложены результаты построения обобщённой математической моделей креплений несимметрично размещённого груза как вдоль, так и поперёк вагона при движении подвижного состава по кривому участку пути от воздействия пространственной системы сил. В частном случае из построенной пространственной модели можно получить математические и динамические модели креплений от отдельных воздействии продольных и вертикальных, поперечных и вертикальных сил.

1. FORMULATION OF A PROBLEM

It is common knowledge that [1] flexible fastening elements are mainly intended for pressing the cargo against the wagon floor by this increasing “holding down” forces preventing cargo displacement both lengthwise and crosswise the wagon. These fastening elements are involved in operation only in case of cargo displacement either lengthwise or crosswise the wagon or during the displacement both lengthwise and crosswise the wagon. Only tying devices (rack-mountable shackle) are under the exposure to the fastening elements. If several fastening elements tying different mounting loops are attached to one rack-mountable shackle this might become the reason of its destruction (i.e. its detachment from the wagon frame).

Flexible fastening elements are formed from twisted annealed wires of the given diameters d_i and a number of threads n_i. Fastening elements are made of anisotropic material possessing different rigidities lengthwise and in cross section and also around its own rotation axis. Elasticity modulus E of fastening elements is nearly 20 times less ($E = 1\cdot107$ kN/m^2) than that of an ordinary steel wire ($E = 2.1\cdot108$ kN/m^2). Persistent fastening devices in the form of persistent and spacing wooden bars are intended for keeping the cargo from the displacement lengthwise and crosswise the wagon and also for transferring "shifting" forces from the cargo to the wagon body.

So far there have not been constructed generalized dynamic and mathematical models of cargo fastening asymmetrically allocated both lengthwise and crosswise the wagon as compared to symmetrical allocation during the movement of rolling stock on a curve under the exposure to spatial force system including Carioles inertia forces.

In connection with this fact just as in case of [2] let us assume as the basis of the mathematical model that the exposure to the spatial force system just as in reality is received by the major (wagon) and additional constraints (flexible elastic and persistent wooden fastening means). Let us consider a general case when cargo weighing $\overline{G}$ is asymmetrically (or symmetrically) allocated with respect to lengthwise and crosswise symmetry axes of the wagon the physical model of which is given in Fig. 1, *a*, *b*.

a

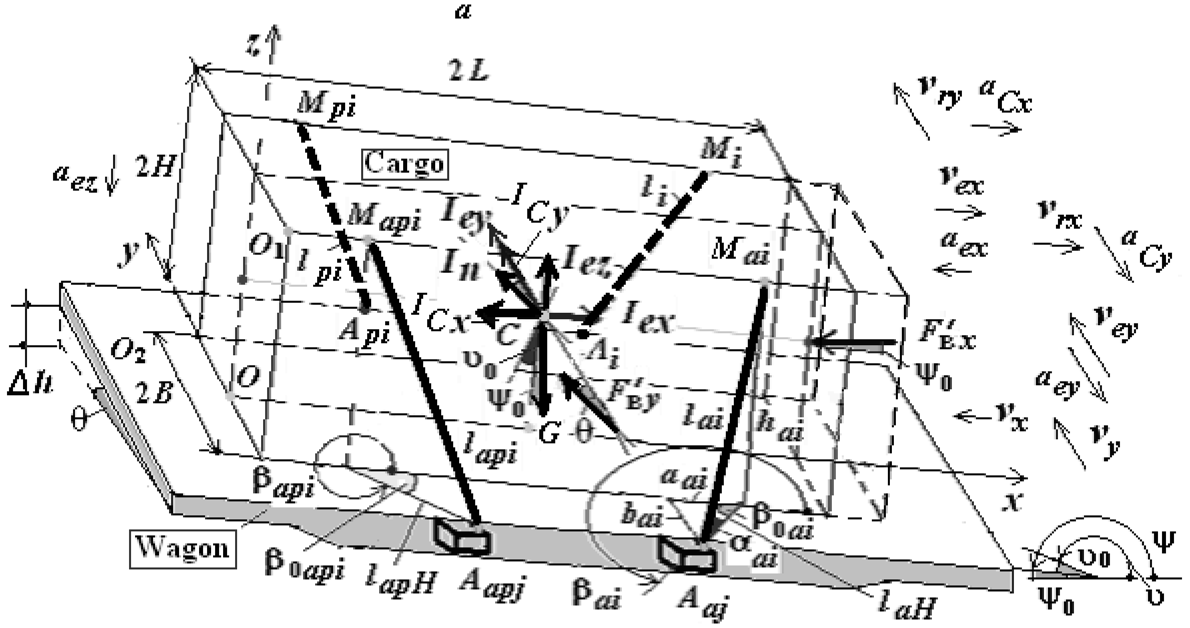

Fig. 1a. Physical modeling allocation cargoes in wagon, running on the curve section of railway descent (side view)

Рис. 1a. Физическая модель размещения груза в вагоне, движущегося по кривому участку пути на спуск (вид сбоку)

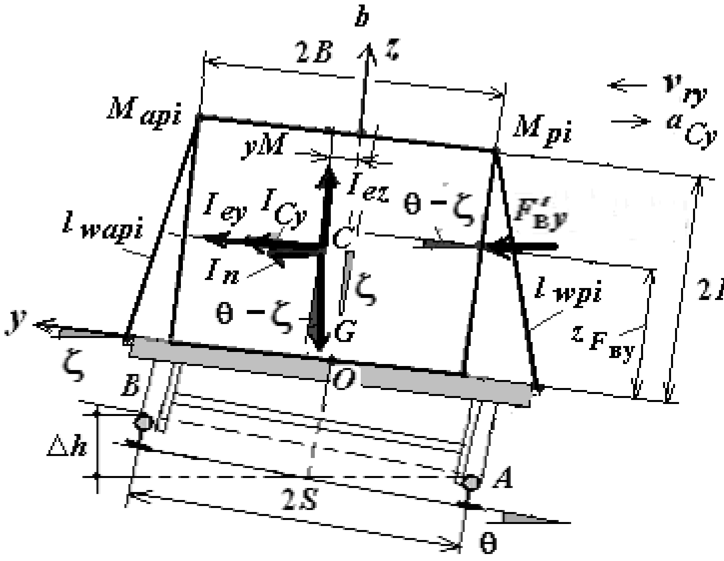

Fig. 1*b*. Physical modeling of cargo allocation in a wagon running on a curved section of railway descent (end view).
Рис. 1б. Физическая модель размещения груза в вагоне, движущегося по кривому участку пути на спуск (вид с торца)

In Fig. 1, as and in [2], the following symbols are accepted j and i are indexes showing the numbers of rack brackets in wagon and elastic fastening elements (i = 1, n_p – a number of flexible elastic fastening elements); $2L$, $2B$ and $2H$ – are cargo length, width and height accordingly; a_i and a_{ai} – are the projections of flexible elastic fastening elements of one direction on the longitudinal wagon axis x (a_{pi} and a_{api} –are also of another direction); b_i and b_{ai} – are projections of flexible elastic fastening elements of one direction on the transversal axis of wagon y (b_{pi} and b_{api} – are also of the other direction); h_i, h_{ai}, h_{pi} and h_{api} – are projections of flexible elastic fastening elements on the vertical axis of wagon z; l_i and l_{ai} – is the length of flexible elastic fastening elements of one direction (l_{pi} and l_{api} – are also of the other direction); l_{wi} and l_{wai} – are projections of the length of flexible eleastic fastening elements of one direction on the transversal axis of wagon y (l_{wpi} and l_{wapi} – are also of the other direction); α_i and α_{ai} –a re the angles which are formed by the fastening elements with the flat surface of the floor of wagon of one direction (α_{pi} and α_{api} – are also of the other direction); β_i and β_{ai} – are the angles that are formed by the projections of fastening elements (l_{iH}, l_{aiH}, l_{piH}, l_{apiH}) on the plane of the floor of the wagon of one direction with axis x (β_{pi} and β_{api} – are also of the other direction (β_{0i}, β_{0ai}, β_{0pi} and β_{0api} – are the same angles only they are acute); Δh – is super elevation; $2S$ –is the distance between wheel rolling circles of wagon wheel set of gauge 1 520 mm ($2S$ = 1 580 mm); θ – is the angle characterizing super elevation; ζ – is the angle taking into account tilting of the frame of the wagon with cargo being displaced on the transversal axis of wagon y by the value$\pm$ yM.

1.1. Man-made assumption

Just because the exposure of the rolling stock will be experienced only by fastening means it was decided to use the fundamental law of dynamics for relative motion of a point which is mathematically presented by the equation

$$M\overline{a}_r = \overline{F} + \overline{R} + \overline{I}_e + \overline{I}_C, \tag{1}$$

where M – is the mass of a material point (cargo); F – active forces; R – constraint reaction; I_e – transferring inertia force; I_C – Carioles inertia force.

It is known that Carioles inertia force if defined as [1]

$$\bar{I}_C = -M\bar{a}_C = -2M\left[\bar{\omega}_e \times \bar{v}_r\right],$$

and its modulus is to be defined as

$$I_C = 2M\omega_e v_r \sin\varphi, \tag{2}$$

where: $\bar{\omega}_e$ – angular velocity vector of transient motion (movable axes $O_1x_1y_1z_1$ along the curvature radius ρ of trajectory of the curve particular point); $\bar{v}_r$ – vector of the point's relative velocity; φ – angle between vectors $\bar{\omega}_e$ and $\bar{v}_r$.

Carioles inertia force is always perpendicular to the relative velocity of movement ($\bar{I}_C \perp \bar{v}_r$), and therefore to the tangent $M\tau$ in respect to the point's relative trajectory. That is why the projection of Carioles inertia force on the tangent $M\tau$ with respect to the point's relative trajectory is always equal to zero ($\bar{I}_{C\tau} = 0$). As applied to the cargo allocated in the wagon, Carioles inertia force appears during the passage of a train from a tangent to a curve and during the movement on a curve (including the passage to the siding track). This makes the cargo shift lengthwise the wagon at velocity $\bar{v}_{rx}$ (while passing the joint) and crosswise the wagon at velocity $\bar{v}_{ry}$.

Now we will show the way of finding the direction of Carioles inertia force. Let us assume that a wagon with cargo with which coordinate system $O_1x_1y_1z_1$ is rigidly linked is running on curve M_1M_2 along trajectory L at transferring velocity $\bar{v}_e$, equal to given train speed $\bar{v}_{\text{w}}$, with respect to fixed coordinate system $Oxyz$. At the moment * let the wagon occupy position M. Through point M we draw tangent τ – τ and normal n – n in the way shown in Fig. 2.

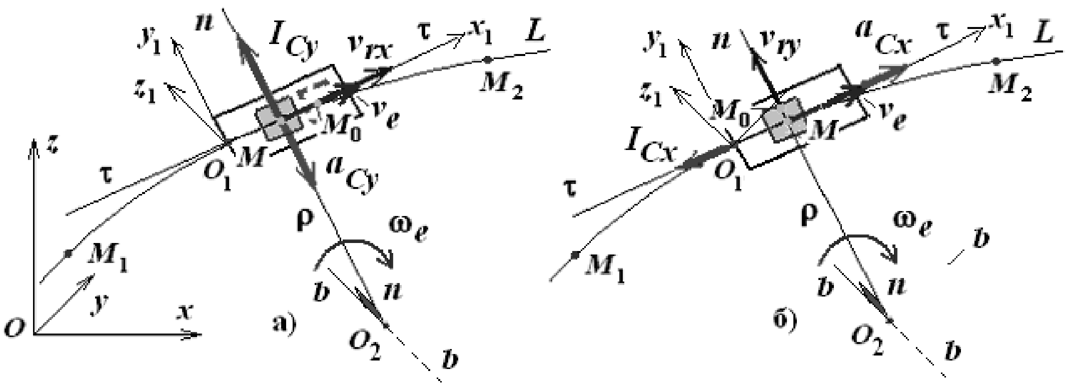

Fig. 2. Finding the direction of the Carioles inertia force
Рис. 2. Нахождение направления силы инерции Кориолиса

Let us consider the case of the cargo displacement that is happening at this moment relative to the wagon at the velocity $\bar{v}_{rx}$ along the tangent line τ – τ at the distance MM_0 (Fig. 2, a). This is accompanied by the appearance Carioles acceleration $\bar{a}_{Cy}$, the modulus of which is defined according to the formula

$$a_{Cy} = 2\omega_e v_{rx}, \tag{3}$$

where: ω_e – angular velocity of the rolling stock running on a curve at the transient velocity $\bar{V}_e$, equal to the train speed $\bar{v}_{\text{w}}$ (a known value), rad/s:

$$\omega_e = \frac{v_e}{\rho}$$

with an allowance for the fact that ρ – curvatures of the considered point *M* of trajectory *L* (for the considered track profile the value is a known one); $\bar{v}_{rx}$ – the velocity of cargo tearing away with respect to the wagon along the lengthwise axis, m/s.

It is necessary to point out that Carioles acceleration $\bar{a}_{Cy}$ is defined according to the value and cargo velocity vector with respect to the wagon along the lengthwise axis $\bar{v}_{rx}$ and is always directed towards the rotation of transient angular velocity ω according to the normal n – n to the curvature centre ρ of the considered point *M* of trajectory *L*. The modulus of Carioles inertia force I_{Cy} is defined according to the formula (2) and the direction is opposite to that of Carioles acceleration $\bar{a}_{Cy}$ (Fig. 2, a).

Let us consider the second case when at a particular moment *t* there happens crosswise displacement of cargo relative to the wagon at velocity $\bar{v}_{ry}$ according to the normal *n* – *n* at the distance MM_0((Fig.2, *b*). This is accompanied by Carioles acceleration $\bar{a}_{Cx}$, *m/s*^{2}, the modulus of which is defined according to the formula

$$a_{Cx} = 2\omega_e v_{ry}, \tag{4}$$

where: $\bar{v}_{ry}$ is the speed cargo tearing away with respect to the wagon along the crosswise axis, m/s.

It should be emphasized that Carioles acceleration is defined according to the value and velocity vector of cargo with respect to the wagon along the crosswise axis $\bar{v}_{ry}$ and is always directed towards the rotation of transient angular velocity ω_e tangentially τ – τ to curvature ρ of the considered point *M* of trajectory *L*. The modulus of Carioles inertia force I_{Cx} is defined according to the formula (2) and the direction is opposite to that of acceleration $\bar{a}_{Cx}$ (Fig 2, *b*).

Let us assume that just as in case of (2) rolling stock is running on the descend at angle ψ_0 both as in the release regime and in the regime of service braking application at velocity *v* on a curve with curvature radius ρ of trajectory at the particular curve point. The cargo is attached to the tying devices of the wagon by the flexible elastic fastening elements in points A_j, A_{aj}, A_{pj} and A_{apj},and to its cargo loops it is attached in points M_j, M_{aj}, M_{pj} and M_{apj} (Fig. 1).

The wagon is moving progressively at velocity $\bar{v}_e$ (i. e. transient progssive motion ($\bar{\omega}_e = 0$)) with the lengthwise $\bar{a}_x = \bar{a}_{ex}$, crosswise $\bar{a}_y = \bar{a}_{ey}$ and vertical $\bar{a}_z = \bar{a}_{ez}$ transient accelerations, caused mainly by track irregularity wave due to aberrations in track maintenance.

Flexible elastic fastening elements and a wagon frame experience as external constrains lengthwise $\bar{I}_{ex} = \bar{I}_x$, crosswise $\bar{I}_{ey} = \bar{I}_y$ and vertical $\bar{I}_{ez} = \bar{I}_z$ transient inertia forces and as well as Carioles inertia force I_{Cy} tending to shift the cargo crosswise the wagon.

Special mention should be made of the fact that Carioles inertia force ICx which is directed along the wagon and which is referred to the class of active forces only because its direction and modulus are known contributes to keeping the cargo from shifting crosswise the wagon without overloading flexible fastening elements .It is due to this reason that we will refer I_{Cx} to the number of reactive i.e. “holding down” forces.

The cargo experiences aerodynamic resistance force F_b. The direction of crosswise air velocity relative $\bar{v}_x = \bar{v}_{ex}$ is opposite to the movement of the rolling stock on a track tangent. Air velocity crosswise component $\bar{v}_y = \bar{v}_{ey}$ effects the lateral surface of the cargo located at the interior side of the rail thread.

In addition we will bear in mind that in crosswise direction there will be nominally applied inertia force normal component $\bar{I}_n$ which will allow for speeding of the rolling stock on a curve [1].

Thus, Carioles inertia force alongside with lengthwise I_{Cy} and crosswise I_{Cx} transient inertia forces as well as aerodynamic resistance force F_b tend to shift the cargo crosswise.

Summing up, it should be emphasized that during the movement of the rolling stock on the curved section of the track there increases the probability of the effect of crosswise forces tending to shift the cargo with respect to the wagon floor crosswise the wagon (Fig. 3).

Fig. 3. Illustration of crosswise forces effect manifested in break-up (displacement) of the cargo crosswise the wagon
Рис. 3. Иллюстрация эффекта эффект действия поперечных сил, выражающегося в развале (сдвиге) грузов поперёк вагона

We believe that spatial force systems (lengthwise, crosswise and vertical) are received by flexible elastic fastening elements which are located opposite the action of these forces, while the fastening elements of the inverse direction sag (i.e. lose their constraint characteristics) (Fig. 4).

2. METHODS OF SOLUTION

Let us apply the principle of clear constraints and the law of relative transient motion [3].

For constructing a dynamic model of cargo (object) in a wagon let us mentally discard external constraints [3] – platform frame as a fundamental constraint and flexible elastic fastening elements as additional constraints. We replace the influence of the discarded constrains by reaction $\overline{R}$ (or its components $\overline{N}$ and $\overline{F}_\tau$ (platform frame) and $\overline{R}_i, \overline{R}_{ai}$ (flexible elastic fastening elements of one direction and $\overline{R}_{pi}, \overline{R}_{api}$ of another direction). We factorize reaction $\overline{R}$ of external constraint (rough surface) into normal $\overline{N}$ and tangent line $\overline{F_\tau}$ components, i.e. $\overline{R} = \overline{N} + \overline{F_\tau}$. We call tangent component $\overline{F_\tau}$, directed along the floor surface of the wagon friction force $\overline{F_\tau}$ defined according to Coulomb law. We want to remind that coordinates x_R, y_R (or x_N, y_N) - application points of external constraint reaction $\overline{R}$ (or $\overline{N}$ and $\overline{F_{\tau y}}$) *are unknown and have to be defined.*

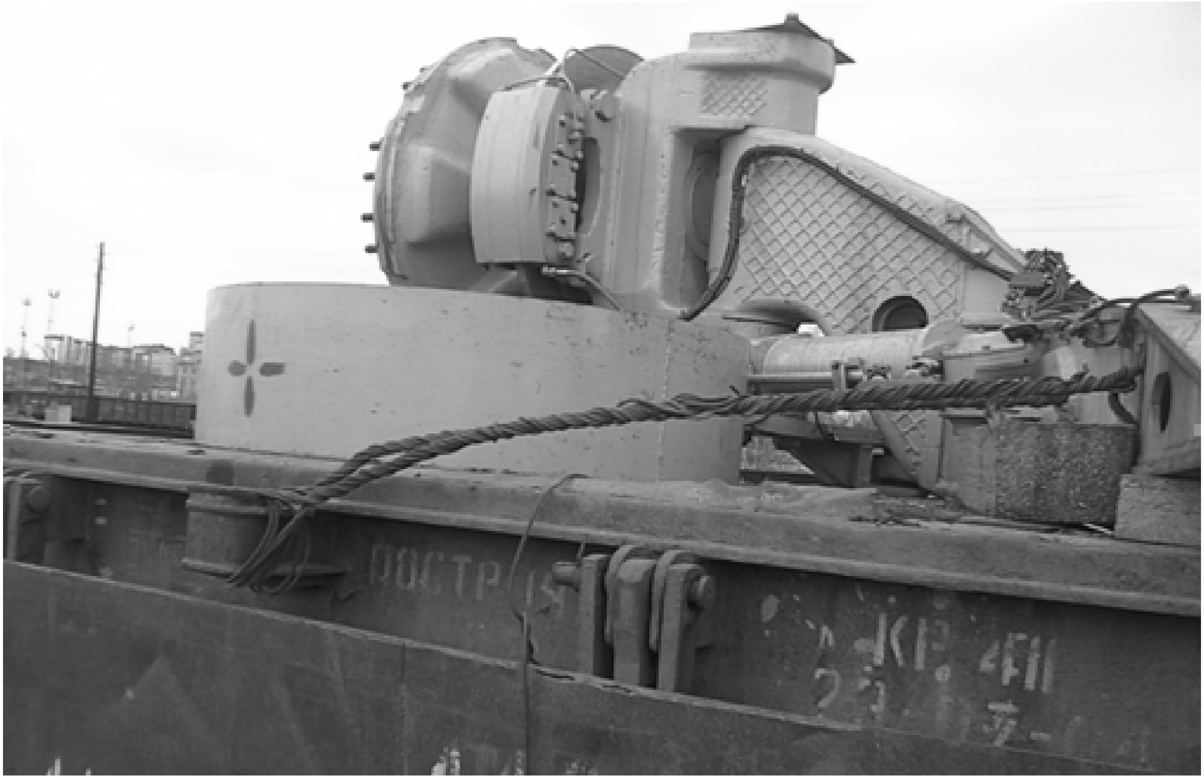

Fig. 4. Loss of constraint of flexible fastening elements of the opposite direction under the exposure to crosswise forces

Рис. 4. Потеря связи гибких элементов креплений противоположного направления от воздействия поперечных сил

Let us apply to the object active ($\overline{G}$, $\overline{I}_{ex}$, $\overline{I}_{ey}$, $\overline{I}_{ez}$, $\overline{I}_{Cy}$ and $\overline{I}_{Cx}$) and reactive ($\overline{F}_w$, $\overline{N}$, $\overline{F}_\tau$ and $\overline{R}_i, \overline{R}_{ai}$ or $\overline{R}_{pi}, \overline{R}_{api}$) forces. As to the active forces $\overline{G}$, $\overline{I}_{ex}$, $\overline{I}_{ey}$, $\overline{I}_{ez}$, $\overline{I}_{Cy}$ and $\overline{I}_{Cx}$ we apply them *nominally* (formally) to the mass centre of the material system (cargo) *C* whereas in reality they are experienced by the external constraints and they are directed from the object while reactive forces $\overline{F}_w$, $\overline{N}$, $\overline{F}_\tau$ are directed to the object and reactive forces of flexible elastic fastening elements of one direction $\overline{R}_i, \overline{R}_{ai}$ ($\overline{R}_{pi}, \overline{R}_{api}$ – of the other direction) are directed from the object. Coordinates $y_{F\text{в}}, z_{F\text{в}}$ – application point of aerodynamic resistance force $\overline{F}_w$ are known: they are located in the geometrical centre of the cargo end surface area. Let us show axis coordinates *x*, *y* and *z*.

It should be noted that the action of lengthwise $\overline{I}_{ex}$, crosswise $\overline{I}_{ey}$ and vertical $\overline{I}_{ez}$ of transient force inertia, $\overline{I}_{Cy}$ – Caroleos inertia forces, and also normal force inertia $\overline{I}_n$ allowing for speeding the movement of the rolling stock on a curve and directed parallel to the horizon are experienced by the external constraints - the wagon (major constraint), flexible elastic fastening elements and persistent wooden bars (additional constraints). That is why it is impossible in reality to point at the application point coordinates of these forces.

3. RESULTS OF SOLUTION

We describe the fundamental law of relative transient car motion in the form of the equation (3.1). Here as applied to the considered dynamic model $\overline{F} \in (\overline{G}, \overline{I}_n)$ – is the active force, $\overline{I}_n$ – normal inertia force allowing only for speeding of the rolling stock on a curve; $\overline{R} = \overline{N} + \overline{F}_\tau$ is the reactive force; $\overline{I}_e \in (\overline{I}_{ex}, \overline{I}_{ey}, \overline{I}_{ez})$ – are lengthwise, crosswise and vertical transient inertia forces the active forces ($\overline{v}_r$) – Carioles inertia force as active force when assumed that cargo velocity ($\overline{v}_{rx}$) is known during the motion with respect to the wagon both lengthwise and crosswise ($\overline{v}_{ry}$) the wagon.

Let us write down the condition of cargo equilibrium in relative equilibrium (at rest):

$$\sum_{k=1}^{n} F_{kx} = 0: \quad I_{ex} + (G_x - F_{\text{в}.x}) - I_{Cx} - F_{ix} - F_{\tau x} = 0 \tag{5}$$

$$\sum_{k=1}^{n} F_{ky} = 0: \quad I_{ey} + I_{Cy} + (I_n + F_{\text{в}.y}) - G_x - F_{iy} - F_{\tau y} = 0 \tag{6}$$

$$\sum_{k=1}^{n} F_{kz} = 0: \quad -(G_z - I_{ex}) + N - (I_{nz} + F_{\text{в}.z}) - F_{iz} = 0 \tag{7}$$

where: $\overline{G}$, $\overline{I}_{ex}$, $\overline{I}_{ey}$, $\overline{I}_{ez}$, $\overline{I}_{Cy}$, $\overline{I}_n$ and $F_{\text{w}} \in (F_{\text{w}x}, F_{\text{w}y}, F_{\text{w}z})$ are active forces; $\overline{I}_{Cx}$, $\overline{N}$, $\overline{F}_\tau$ and $\overline{F}^{(i)}$ are reactive forces, $\overline{N}$ and $\overline{F}_\tau$ being elastic forces(tension) of additional constraint (cargo flexible elastic fastening element). Here and further on designation i raised to the power of elastic force $\overline{F}^{(i)}$ denotes that the force depends on the number of flexible elastic fastening elements but that does not mean that it should be summed according to i. Elastic force $\overline{F}^{(i)}$ has only one meaning.

It should be remembered that Carioles inertia force $\overline{I}_{Cx}$, directed along the wagon is referred to the class of active forces only because its direction and modulus as vector values are known are known. However, it contributes to keeping the cargo from shifting lengthwise the wagon without overloading flexible fastening elements. This is why $\overline{I}_{Cx}$ was previously referred to reactive forces.

Skipping interim mathematical computations just as in [1] we find the dependence of cargo displacement in the wagon floor plane under the exposure to the resultant of the spatial force systems

$$\Delta s = \frac{\Delta F^{(i)}}{c^F_{\text{экв.}}}, \tag{8}$$

where: $c^F_{\text{экв.}}$ – is the equivalent rigidity of flexible fastening elements towards the direction of the action of spatial force *kN/m*.

$$c^F_{\text{экв.}} = 7{,}854 d_i^2 \sum_{i=1}^{n_\text{p}} \frac{n_i}{l_i} B_i \sqrt{\left(C_i \cos\lambda^{(i)}\right)^2 + \left(D_i \sin\lambda^{(i)}\right)^2} \tag{8a}$$

where: d_i, n_i and l_i – is the diameter (mm), the number of threads (pcs.) and the length of fastening wire; B_i, C_i and D_i – are nondimensional variables

$$B_i = \left(\frac{a_i}{l_i}\cos\lambda^{(i)} + \frac{b_i}{l_i}\sin\lambda^{(i)}\right); \; C_i = \left(f\frac{h_i}{l_i} + \frac{a_i}{l_i}\right); \; D_i = \left(f\frac{h_i}{l_i} + \frac{b_i}{l_i}\right).$$

Taking into account that h_i – is the projection of the length of fastening wire l_i on vertical axes; f – is friction coefficient between the surfaces of wagon floor and cargo.

In formula (8) let us present the resultant of spatial forces systems $\Delta\overline{F}^{(i)}$ (Fig 1), received by the flexible elastic fastening elements as

$$\Delta\overline{F}^{(i)} = \Delta F_x^{(i)}\overline{i} + \Delta F_y^{(i)}\overline{j}, \tag{9}$$

where: $\Delta F_x^{(i)}$ и $\Delta F_y^{(i)}$ –the projections of constituent forces on longitudinal and transverse axes [1]:

$$\Delta F_x^{(i)} = \Delta F_x^{0i} - F_\tau^e \cos\lambda^{(i)}; \;\; \Delta F_y^{(i)} = \Delta F_y^{0i} - F_\tau^e \sin\lambda^{(i)}, \tag{10}$$

where $\Delta F_x^{0(i)}$ – is the longitudinal force received by the fastenings of one direction as the difference of „shearing" and „retaining" forces

$$\Delta F_x^{0i} = F_{c\partial.x} - F_{y\partial.ix}; \tag{11}$$

$\Delta F_y^{0(i)}$ – s the transverse force received by the fastenings of one direction as the difference of shearing and retaining forces

$$\Delta F_y^{0i} = F_{c\partial.y} - F_{y\partial.iy}; \tag{12}$$

$\lambda^{(i)}$ –the direction angle where i raised to a power means that the angle depends on the number of flexible elastic fastening elements and it has only one value;

F_τ^e – is friction force

$$F_\tau^e = f\begin{bmatrix} (G\cos(\psi_0+\upsilon_0)\cos\theta - I_{ez}) + \\ + F'_{\text{в}x}\sin(\psi_0+\vartheta_0) + (I_n + F'_{\text{в}y})\sin\theta + \sum_{i=1}^{n_p} R0_{iz} \end{bmatrix}. \tag{13}$$

In expression (11) „shearing" and „retaining" forces are equal

$$F_{c\partial.x} = I_{ex} + G\sin(\psi_0+\upsilon_0); \; F_{y\partial.ix} = I_{Cx} + \sum_{i=1}^{n_p} R0_{ix} + F'_{\text{в}x}\cos(\psi_0+\vartheta_0). \tag{14}$$

In expression (13) „shearing" nd „retainig" forces are equal

$$F_{c\partial.y} = I_{ey} + I_{Cy} + (I_n + F'_{\text{в}y})\cos\theta \;; \quad F_{y\partial.iy} = G\sin\theta + \sum_{i=1}^{n_p} R0_{iy}. \tag{15}$$

On the basis of the value of cargo shear in the plane of the wagon floor (Δs) we find tension R_i in i -x flexible elastic fastening elements according to the formula [1]:

$$R_{\text{упр}.i} = \Delta s 7{,}854 d_i^2 \frac{n_i}{l_i}\left(\frac{a_i}{l_i}\cos\lambda^{(i)} + \frac{b_i}{l_i}\sin\lambda^{(i)}\right) \le [R_i], \tag{16}$$

where: $[R_i]$ – is the admissible value of tension in fastening defined according to specification for cargo placement and fastening in wagons and containers depending on the number of threads n_i and diameter d_i of fastening wire.

4. SUMMARY

Analyzing the received results of mathematical modeling of cargo fastening in a wagon it is necessary to note that for the first time there have been derived generalized formulas for defining cargo displacement and tension in flexible elastic fastening elements during the rolling stock movement on a curved track section under the exposure to the spatial force systems. The deduced formulas take into account physic-geometrical characteristics of elastic elements, the values of external forces, received by the fastenings and cargo and the condition of contacting surfaces of cargo and wagon floor by means of friction coefficient. The derived analytical formulas serve for assessing rolling stock movement safety with asymmetrical and symmetrical allocation of cargo centre- of –mass both lengthwise and crosswise the wagon as a mechanical system „wagon-cargo-fastening".

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