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TWO–LANE TRAFFIC ANALYSIS BY MEANS OF CELLULAR AUTOMATA SOLUTIONS WITHIN A HIGHWAY MODEL

Summary. A discrete model to simulate two–way traffic flow is introduced. The well known cellular automata Nagel-Schreckenberg model is extended by adding another road lane. New sets of state rules is developed to provide lane change maneuver for vehicle overtaking and returning to lane designated for slower traffic. Results of numeric simulations are consistent with the so-called fundamental diagram (flow vs. density), as is observed in the real free-way traffic.

MODELOWANIE AUTOMATEM KOMÓRKOWYM RUCHU NA DWUPASMOWEJ AUTOSTRADZIE

Streszczenie. W Artykule przedstawiono dyskretny model ruchu drogowego. Znamy model Nagela-Schreckenberga oparty na automatach komórkowych został rozszerzony o dodatkowe pasmo ruchu. Opracowano nowy zestaw reguł zmiany stanów umożliwiający manewr zmiany pasa ruchu-wyprzedzania oraz powrót na pas przeznaczony do jazdy z mniejszą prędkością. Wyniki numerycznych symulacji są zgodne z podstawowym diagramem fundamentalnym (przepływ versus gęstość), zależnością obserwowaną w ruchu rzeczywistym.

1. MOTIVATION – TRAFFIC FLOW QUALITY

Modeling traffic transport problem is very interesting and important for its dynamics and serious dramatic consequences in real life. The main goal of traffic flow control and road network design is to provide a qualitative description of traffic flow, especially to answer the question whether the traffic flow is equal to demand flow level over network in time and space [3]. The models, we examined, can be useful to provide proper tools to perform simulations for various scenario i.e. closed lane segment, lane speed limit, accidents, start-stop condition.

2. TRAFFIC FLOW PARAMETERS

Before we introduce basic traffic flow models, it is essential to point out a group of traffic stream related parameters: speed u, flow q and density k. Let $N = \sum n_i$ denotes the total number of vehicles traversing dx in time T then:

$$k = \frac{Ndt}{Xdt}, \quad u = \frac{\sum n_i x_i}{Ndt}, \quad q = \frac{\sum n_i x_i}{Xdt} \tag{1}$$

The mean flow q of vehicles traveling over a section of a roadway of length X during a time interval T is the total distance traveled on the roadway by all vehicles which were on this section during any part of time T divided by XT, the area of the space-time domain observed.

The mean concentration k of vehicles traveling over a roadway section of length X during a time interval T is the total time spent by all the vehicles on X during a time T divided by XT.

The mean speed u of vehicles traveling over a roadway section of length X during time T is the total distance traveled on the section by all vehicles that were on it for any part of time T divided by the total time spent by all vehicles on the section during time T [1]. Many researchers use other formula Eq. (2) to compute flow. The time-averaged flow q between i and $i+1$ is defined by:

$$q = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{i,i+1}(t) \tag{2}$$

where: $n_{i,i+1}(t) = 1$ if the car motion is detected between sites i and $i+1$ [7]. One can find some analysis and discussion upon various methods of flow computations in [13].

In our research we focus on the main flow-density relationship which is the most important to reflect the traffic dynamic. This dependency is known as a Fundamental Diagram and is postulated as a certain function used for approximation of observational data (see Fig. 1).

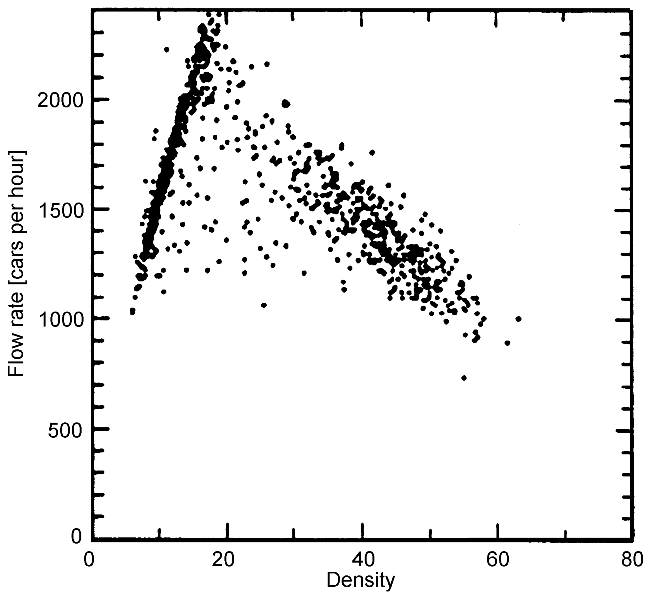

Fig. 1. Real-life observation
Rys. 1. Obserwacje rzeczywiste

With the basic relationships among traffic flow, speed and density, a special attention can now be directed toward the scale of view of traffic flow: macroscopic or microscopic.

3. CLASSICAL APPROACH

In the classical approach traffic is mainly modeled as aggregated vehicle counts or traffic streams. The macroscopic treatment views traffic as fluid moving along a duct which is road lane. The microscopic treatment considers the movement of individual vehicles as they interact with each other. In both approaches partial differential equations or delay differential equations are used.

3.1. Macroscopic approach

The macroscopic treatment views the traffic as a continuum akin to a fluid along a duct which is a highway. The discussed traffic along a reasonably crowded road has no appreciable gaps between individual vehicles. in such cases traffic may be viewed as continuum, and its characteristic correspond to the physical characteristic of the imaging fluid. Macroscopic traffic flow models do not distinct vehicle-driver individual behaviour. This is the main issue which makes a lot of phenomena cannot be observed during simulations [1].

First the classical macroscopic Lighthill-Whitham traffic flow model Eq. (3) is introduced, where k is the cars concentration on the road and q is he traffic flow [1].

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{3}$$

The traffic flow q is regarded as a function of concentration $k : q = q(k)$, thus after differentiating Eq. (3) becomes :

$$\frac{\partial k}{\partial t} + V \frac{\partial k}{\partial x} = 0 \tag{4}$$

where: $V = \partial q / \partial k$ is an average discontinuity speed at point k. Dependence $q = q(k)$ is defined as an arbitrary function and relates to used model. Presented equation (4) as the first order, scalar continuity equation of hyperbolic type, can be solved using Godunov approximation scheme, also known as up wind scheme [5].

3.2. Microscopic approach

Microscopic traffic flow models aim to describe the behaviour of individual vehicle-driver unit with respect to other vehicles in the traffic stream. Microscopic models are very suitable for the description of multiple user-class flow. However, the more realistic microscopic flow models are very complex. What is more, it has been argued that the assumptions underlying the equations describing the motion of each individual car are difficult to validate, since human behaviour in real-life traffic is difficult to observe and measure. This is unfortunate, since for reliable simulation, the microscoping parameters have to be just right. Consequently, many researchers and traffic flow management software use macroscopic traffic flow models instead [3].

Car following model

In car-following model (CFM) we postulate that an individual car's motion only depends on the car ahead [4]. Analysing driver behaviour, one can discover that human being has a time lag in reacting to any input stimulus. Decision of using brake pedal has some delay [1]. Observations being a basis for theories of this sort help us to define the simplest linear CFM equation:

$$\frac{d^2 x_{n+1}(t+T)}{dt^2} = \lambda\left[\frac{dx_n(t)}{dt} - \frac{dx_{n+1}(t)}{dt}\right] \tag{5}$$

Equation (5) is a class of second order of neutral difference-differential equation, namely NDDEs. It is important to notice that there is no universal numerical method to solve NDDEs.

After linearization, we assume that acceleration and deceleration occurs instantaneously. Now, the equation (6) forms a simple system of ODEs which can be numerically solved using 4th order Runge-Kutta method.

$$\frac{d^2 x_{n+1}(t)}{dt^2} = \lambda\left[\frac{dx_n(t)}{dt} - \frac{dx_{n+1}(t)}{dt}\right] \tag{6}$$

Making another assumption: in steady state all vehicles are equidistant apart and move with the same velocity, one can develop from Eq. (6) analytical solution, and consequently, velocity-density relationship:

$$u = \lambda\left(\frac{1}{k} - \frac{1}{k_{max}}\right) \tag{7}$$

where: u is mean speed, and k is density. All simplifications which stand behind Eq. (5) have a dramatic consequence: we lost possibilities of modelling individual driver behavior.

4. DISCRETE MODELS

In the discrete models, the continuous quantities such as positions and velocities of a car are approximated by (discontinuous) integers numbers. In our opinion, however, considered models have the ability to show phenomena observed both at macroscopic and microscopic level.

4.1. Cellular Automata–traffic flow models

We focus on the cellular automata approach (CA) instead of on the classical ones (fluid–dynamics approach [1]) because of one important property of cellular automata, namely the lack of stability, i.e. very small changes in transition rules or states can have very dramatic consequences [6]. The biggest advantage of CA is that each cell of the automaton can reflect individual object characteristics. Recently the cellular automaton approach is chosen more often in different area, e.g. for biological models, modeling spread and movement of an oil slick, simulation of behavior of vivid entities population and modeling predator-prey systems or to model physical problems like heat conductivity [6, 8]. Because cellular automata are used widely in various disciplines, many definitions exist. We quote one of them [8].

Def. 1 *Cellular automata are dynamical systems in which space and time are discrete. A cellular automaton consists of a regular grid of cells, each of which can be in a finite number of k possible states, updated synchronously in discrete time steps according to local, identical interaction rules. The state of a cell is determined by the previous states of surrounding neighborhood of the cell.*

We summarize the physical and evolutionary properties of cellular automata:

CA develops in space and time.

CA is a discrete simulation method, hence Space and Time are defined in discrete steps.

CA is built up from cells, that are lined up in a string for one-dimensional automata arranged in a two or higher dimensional lattice for two- or higher dimensional automata.

The number of states of each cell is finite.

The states of each cell are discrete and all cells are identical.

The future state of each cell depends only of the current state of the cell and the states of the cells in the neighborhood.

The development of each cell is defined by the same set of deterministic or probabilistic rules.

4.2. Basic Nagel-Schreckenberg model

Known as NaSch cellular automata model was originally defined by Nagel and Schreckenberg [11] in 1992. The model concerns only one lane with periodic boundary condition. This means the total number of vehicles is constant. The cell is empty or occupied by vehicle. All the cells are updated simultaneously. We use notation as follow: x_i denotes position of the vehicle, v_i is speed of the vehicle and g_i is a gap between leader and follower, $g_i = x_{i+1} - x_i - 1$.
Then the set of rules is defined:

Acceleration of free vehicles: $v_i < v_{max} \wedge g_i > v_i + 1 \rightarrow v_{i+1} = v_i + 1$

Slowing down due to other vehicles: $v_i > g_i - 1 \rightarrow v_i = g_i$

Random braking (noise): $v_i > 0 \rightarrow v_{i+1} = v_i + 1$ with probability p

Vehicle Motion: $x_{i+1} = x_i + v_i$

Basic NaSch model assumes constant p for the third rule. It is insufficient for modeling some traffic flow phenomena i.e. start-stop state. Other researchers extended ”random braking” rules and proposed velocity-depended randomization (VDR) approach. It is a simple idea, the probability p is a function of the vehicle speed $p = p(v(t))$. In the simplest case the probability function is defined as follows:

$$p(v) = \begin{cases} p_0 & \text{for } v = 0 \\ p & \text{for } v > 0 \end{cases} \tag{8}$$

This makes the flow characteristics closer to realistic values. The phenomenon of uncertainty of vehicle behavior (which depends on p) in the one- and two-lane models is discussed in [9].

The basic limitation of NaSch models is that all drivers behave in the same manner. One should take this into account to develop model where different vehicles are assigned different maximum speeds. The tolerated gap between vehicles is driver dependent or is a function of speed.

5. TWO-LANE CELLULAR AUTOMATA TRAFFIC MODEL

Presented approach is based on CA model which is described as follows: on a ring with L sites every site can either be empty or occupied by one vehicle with velocity $V \in (0,1,...,V_{max})$. At each discrete time step the arrangement of N cars is updated in parallel, according to the set of rules. In the multi-lane case, we simply take single lane and place each one alongside the other. We consider a two-lane model with periodic boundary conditions, where additional rules defining the exchange of vehicles between the lanes are introduced. It is clarified that this extension can be made without changing the basic properties of the single-lane model.

Basic set of rules

For our model, we adapt NaSch set of rules to provide vehicles movement. We intentionally, have not included "random braking" rule. Such model is known as deterministic NaSch traffic flow model [7]. Because we are using now a deterministic, reversible and finite CA model with periodic boundaries, the corresponding traffic system is periodic in its system states.

Addition set of rules

After many numerical experiments, we simplified our model and introduced no-accident condition. It means the set of rules have to preserve against situations, where more than one car could occupy the same cell. We realize this approach help us to define some generic rules–further research is under development.

Overtaking maneuver uses an extended neighborhood and covers sites behind and ahead of the vehicle, on both lanes. We assume the driver only detects the space occupancy of his neighborhood. The speed of the other vehicle on the highway remains unknown for him. In consequence, some other strong assumptions have to be made to assert "no accident condition". We require empty neighborhood behind the car to ensure that only one car overtakes the considered cell. The need of the empty neighborhood behind the car on the left lane protects against collisions with the vehicles that drive along the adjoining lane. All required conditions are below:

$$V_{0,j} < V_{max} \wedge D^{-}_{0,j} \geq D_{max} \wedge D^{-}_{1,j} \geq D_{max} \wedge D^{+}_{1,j} \geq D \rightarrow L_{1,j+(D-1)} = V_{0,j} + 1 \tag{9}$$

where: $V_{0,j}$ – vehicle speed on the right lane, at relative position j, V_{max} – vehicle preferred maximum speed, $D^{-}_{0,j}$ – distance to the nearest follower at the right lane, at relative position j, $D^{-}_{1,j}$ – distance to the nearest follower at the left lane, at relative position j, D_{max} – distance cover by a vehicle at maximum speed (per one iteration), $D^{+}_{1,j}$ – distance to the nearest leader at the left lane, at relative position j, D – distance cover by a vehicle at spot speed, $L_{1,j+(D-1)}$ – value of cell at relative position $j + D - 1$, at the next time step

Returning maneuver satisfies requirement that left lane should be mainly used for overtaking purposes. The rule is similar to that one used in overtaking maneuver.

$$D^{-}_{0,j} \geq D_{max} \wedge D^{-}_{1,j} \geq D_{max} \wedge D^{+}_{0,j} \geq D \rightarrow L_{1,j+(D-1)} = V_{0,j} \tag{10}$$

The rules applying schema

The exchange rules are defined by the following two criteria: first, a vehicle needs an incentive to change a lane; second, a lane change is only possible if some safety constraints are fulfilled. Rules are processed in following priority order:
returning maneuver $\rightarrow$ overtaking maneuver $\rightarrow$ moving (NaSch rules' set)
but only one single rule is applied per iteration In most situations, vehicles just keep moving on the same lane.

The above set of rules is minimal in the sense that leads to a realistic behavior and the so-called fundamental diagram, i.e. the relation between flow and density, is reproduced correctly. Unfortunately some phenomena like spontaneous jamming will not occur in such system. One of the solutions to perform more realistic simulation using deterministic cellular automata model is to introduce stochastic boundary conditions [14] or open boundary conditions as well.

6. THE RESULT OF THE SIMULATIONS

The results [15] are obtained from simulations on a lattice of 2*500 sites with random initial configurations of vehicles. The population of N cars were randomly distributed in on both lanes around of complete loop with initial speed sampled from $(0, V_{\max})$. The sensitivity analysis was done. The size of automata and number of iterations equal 1000 is sufficient for the system to reach a stationary states.

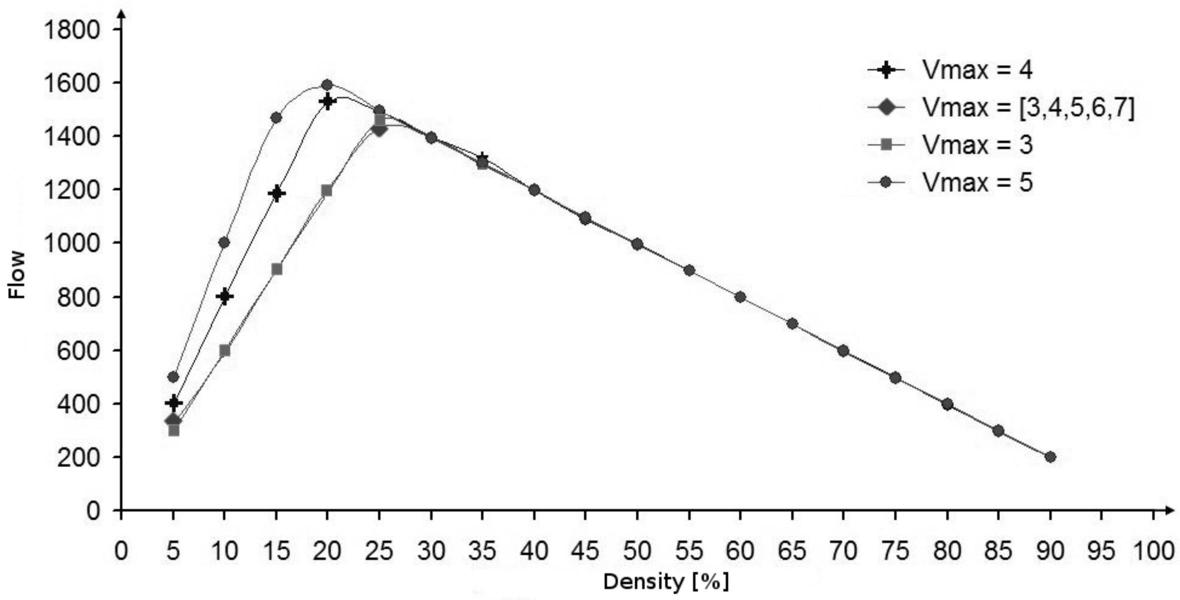

Fig. 2. Numeric simulations for various preferred speed distribution
Rys. 2.Wyniki symulacji numerycznych dla różnych rozkładów prędkości preferowanej

We studied a homogeneous state, where preferred speed distribution is uniform or constant. At this point of our research, we investigate some relations between preferred speed distribution and extrema point on the fundamental diagram (see fig. 2). At higher density, flow is stabilized and does not depend on driver comfortable speed preferences.

7. CONCLUSION

The proposed model is based on the Nagel-Schreckenberg cellular automata model without VDR. The solution of highway traffic dynamics is partially agreed with real-life traffic. The lack of some stochastic noise influences the model behaviour. Some class of phenomena – spontaneous, unstable state, i.e. jam creation, kinematic waves, will not be reproduced in strictly deterministic CA model. On the other hand, in further studies we are planning to verify how the lane changing maneuver changes the fundamental diagram. There is a need to introduce some kind of algorithm (in substeps or at a random choice) to fulfill no-accidents requirement and make driver behavior less preservative. In most times, there are no cars exchanging between lanes in discussed model. Cars are moving along the same lane regardless of density. The two-lane traffic model behaves rather like a two-independent lane traffic model. In the near future, we are going to pay more attention to the set of rules that governs lane changing maneuvers.

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