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MATHEMATICAL MODELLING OF FASTENING WITH CARGOES
DISPLACEMENT TRANSVERSE TO WAGON

Summary. The article has considered the situation when the mechanical system “wagon – fastening – cargo” is influenced by transverse and vertical forces. It has given the results of mathematical modeling construction of cargo in wagon running on the curve section of track. It has stated the conclusion of analytical formula of cargo displacement transverse to wagon and tension in fastening from exposure to transverse and vertical forces.

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ КРЕПЛЕНИЙ ПРИ СДВИГЕ ГРУЗА ПОПЕРЁК ВАГОНА

Аннотация. В статье рассмотрено ситуация, когда механическая система “вагон - закрепленный груз” находится под действием поперечных и вертикальных сил. Это удалось в результате математического моделирования груза в вагоне, движущемся на кривом участке пути. Этому способствовал вывод аналитической формулы сдвига груза поперёк вагона и натяжений в креплениях.

1. FORMULATION OF A PROBLEM

We take a particular situation [1], when cargo with weight $G$, which has asymmetrical (or symmetrical) allocation relatively longitudinal axis of the truck symmetry being kept from displacement transverse to wagon by flexible elastic fastening elements (binding) $A_{ai}M_{ai}$ and $A_{aj}M_{aj}$, and bar persistent allocated at some distance $\delta_y$ from the lateral surface of cargo (fig. 1). Preliminary stretching of flexible elastic fastening elements equals to $R_0 = 20$ kN.

The truck runs with transverse $\ddot{a}_y = \ddot{a}_{ey}$ and vertical $\ddot{a}_z = \ddot{a}_{ez}$ acceleration. The fastenings experience transverse $\ddot{T}_e = \ddot{T}_y$ and vertical $\ddot{T}_c = \ddot{T}_z$ transient inertia forces. Acceleration of the wagon motion with cargo on the curve section of the track is taken into account by the normal inertia force $\ddot{T}_n$.

The cargo is effected by the aerodynamically resistance force $\overline{F}_a$. In solving some practical problems it is assumed that maximum normative meanings of transverse transient acceleration $\ddot{a}_{ey}^{\text{max}} = \ddot{a}_{ey}$ equal to $a_{ey} = (0.4 - 0.46)g$ and those of vertical transient acceleration $\ddot{a}_{ez}^{\text{max}} = \ddot{a}_{ez}$, appearing due
to track maintenance standards deflection \( a_{e_2} = (0.46 - 0.66)g \). Accordingly the meanings of transverse and vertical transient inertia forces are taken as \( l_{ey} = (0.4 - 0.46)G \) and \( l_{ez} = (0.4 - 0.66)G \).

It is considered that active transverse and vertical forces are perceived by pair flexible elastic fastening elements \( (\alpha \text{ and } \beta) \), placed opposite to transverse forces action and fastening elements of the opposite direction \( (\alpha \text{ and } \beta) \) sag.

2. METHODS OF SOLUTION

One may use the principle clear constrains and the law of relative transient motion \([2, 3]\).

One must take into account that lengthening of flexible elastic fastening element is directly proportional to cargo displacement transverse to wagon \([2, 3]\):

\[
\Delta l_i = \Delta y \cdot \frac{b_i}{l_i}.
\]  

3. RESULTS OF SOLUTION

Leaving out some intermediate mathematical calculations \([1 – 3]\), one can get a formula to determine cargo displacement transverse to wagon.
\[ \Delta y = \frac{\Delta F_y^{(i)}}{7.854 d_i^2 \sum_{i=1}^{n_e} \frac{n_i}{l_i} \left( \frac{f}{l_i} + \frac{b_i}{l_i} \right) b_i}. \] (2)

In the formula (2) \( \Delta F_y^{(i)} = \Delta F_n \) is a transverse force perceived by flexible elastic fastening elements and determined as the difference between displacement and keeping forces
\[ \Delta F_y^{(i)} = F_{ei} - F_{yo,i}, \] (3)
where:
\[ F_{ei} = I_{ey} + (I_n + F_e); F_{yo,i} = f(G - I_{ey}) + \sum_{i=1}^{n_e} R_{0,y}. \] (3, a)

**Statement 1.** Cargo displacement transverse to wagon will take place (that is \( \Delta y > 0 \)) provided \( \Delta F_y^{(i)} = \Delta F_n > 0 \).

**Definition 1.** Cargo displacement transverse to wagon \( \Delta y \) is that distance from lateral cargo surface which can provide joint work of flexible and persistent fastenings if a bar persistent is fixed to wagon’s bottom off the lateral cargo side at the distance less than \( \Delta y \).

Thus one should note that on the basic of the results mathematical fastening modeling with asymmetrical (or symmetrical) allocation in wagon the analytical formula has been derived to determine cargo displacement transverse to wagon (\( \Delta y \)) taking into account physics and geometrical characteristics of elastic elements (that is \( E, n, d, l \)), meanings of external forces (\( \bar{G}, \bar{T}_{ey}, \bar{T}_n, \bar{T}_{ce}, \bar{F}_e \)), perceived by fastenings and cargo and contacted cargo and bottom surfaces state taking into consideration coefficient friction (\( f \)).

**Statement 2.** The rupture of flexible elastic elements won’t take place provided the condition is kept \( \delta_y = \Delta y \leq \Delta y \), where \( \Delta y \) is admissible meaning of cargo displacement transverse to wagon (mm) determined by the meaning \( [R_i] \) depending on quantity of threads \( n_i \) and diameter \( d_i \) of wire fastenings.

With the meaning of cargo displacement transverse to wagon (\( \Delta y \)) and the formula (2) one can determine strain (effort) in \( i^{th} \) flexible elastic fastening element
\[ R_{wp,i} = 7.854 d_i^2 \Delta y \sum_{i=1}^{n_e} \frac{n_i}{l_i} \frac{b_i}{l_i} \leq [R_i]. \] (4)

The strain (effort) in flexible elastic fastening element according to the axiom of process equality to opposition equals the reaction in this element.

**Statement 3.** In the situation when cargo is kept from displacement transverse to wagon by bar persistent nailed to wagon’s bottom closely to the cargo lateral surface (that is when \( \Delta y = 0 \)) strain (effort) \( R_i \) in \( i^{th} \) flexible elastic fastening elements equals to zero.

**4. CONCLUSION**

On the basis of mathematical fastening modeling results of asymmetrical (or symmetrical) cargo allocation in wagon one can note that the analytical formula has been derived to determine cargo displacement transverse to wagon taking into account physics and geometrical characteristics of elastic elements, meanings of external forces perceived by fastenings and cargo and contacted cargo and bottom surfaces state which are taken into account by the friction coefficient. The obtained analytical
formula serve to estimate safety of rolling stock traffic with asymmetrical or symmetrical allocation of cargo mass centre transverse to wagon as mechanical system “wagon – fastening – cargo”.

Bibliography

2. Туранов Х.Т., Бондаренко А.Н.: Теоретическая механика в задачах погрузки-выгрузки и перевозки грузов в вагонах. – Екатеринбург: УрГУПС, 2006.

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