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WAITING TIME FOR ENTRY INTO THE INTERSECTION AS A DISTURBING FACTOR OF TRAFFIC STREAM OF SUBORDINATED VESSELS

Summary. The presented paper concerns the disturbed traffic of vessels, the passage through the waterway intersection. To determine a time between subordinated vessels leaving the intersection a convolution operation has been used. The main factor disturbing the traffic stream of subordinated units is speed reduction on the intersection area. To model times needed to speed changes a normal distribution has been used.

CZAS OCZEKIWANIA NA WEJŚCIE NA SKRZYŻOWANIE, JAKO CZYNNIK ZABURZAJĄCY STRUMIEŃ RUCHU STATKÓW PODPORZĄDKOWANYCH

Streszczenie. Niniejszy artykuł dotyczy zaburzonego ruchu statków, przechodzących przez skrzyżowanie dróg wodnych. Do wyznaczenia czasu pomiędzy jednostkami podporządkowanymi, opuszczającymi skrzyżowanie wykorzystano operację splotu. Głównym czynnikiem zaburzającym strumień ruchu jednostek podporządkowanych jest ograniczenie prędkości na obszarze skrzyżowania. Do modelowania niezbędnych czasów, w których następują zmiany prędkości wykorzystano rozkład normalny.

1. INTRODUCTION

Out of two vessels approaching each other on crossing routes, always one of them is privileged in relation to the other. A traffic stream of subordinated vessels crossing an intersection is disturbed by traffic of vessels with priority. When vessel with priority is on the intersection the subordinated unit can't enter into the intersection, she must proceed in such a way as to let the privileged one pass. When the units are close to the intersection, the subordinated vessel has to perform a manoeuvre, in order to avoid a collision situation with the unit proceeding crosswise. She must reduce her speed in such a way which would enable the ship to safely pass the unit sailing crosswise [6]. This speed reduction drives to a delay, which can be called a waiting time for entry into the intersection.

2. SPEED CHANGES OF SUBORDINATED UNIT

The subordinated unit's movement in intersection region can be divided into 5 phases. The first one is the subordinated vessel's movement at full speed. The second phase is speed reduction. The third phase is the subordinated vessel's movement at safe speed. The fourth is gathering speed to reaching full crossing speed. And last phase is the subordinated vessel's movement at full speed. Figure 1 shows speed changes of the subordinated unit.

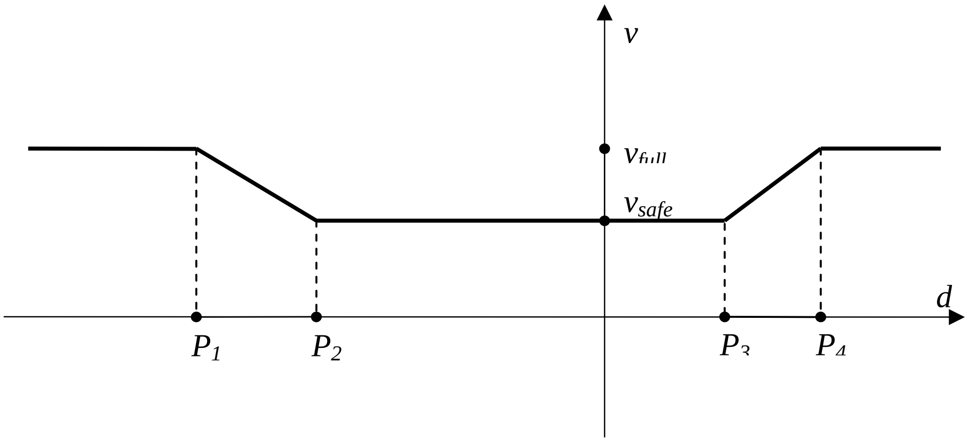

Fig. 1. Graph of velocity of subordinated vessel in intersection region
Rys. 1. Wykres prędkości jednostki podporządkowanej w rejonie skrzyżowania

Let vessel A will be a subordinated unit. Four points were signed in this graph. P_1 is a place before intersection area, where unit A starts braking. P_2 is a place before intersection area, where unit A reaches the safe speed [6]. P_3 is a limit of intersection area, where unit's A speed starts to gather. P_4 is a place out of the intersection area, where unit A reaches full speed. The times of covering particular stages of the crossing are random variables.

3. PROCESS OF REPORTING WHILE ENTERING AND LEAVING THE INTERSECTION

When the subordinated vessel arrives to the intersection, vessel with priority can be at any point of the intersection area (with probability p) or can be out of the intersection area (with probability q). Similarly, when the next successive subordinated vessel arrives to the intersection, vessel with priority can be at any point of the intersection area (with probability p) or can be out of the intersection area (with probability q). Hence we have four different cases crossing the intersection by two successive subordinated units (fig.2).

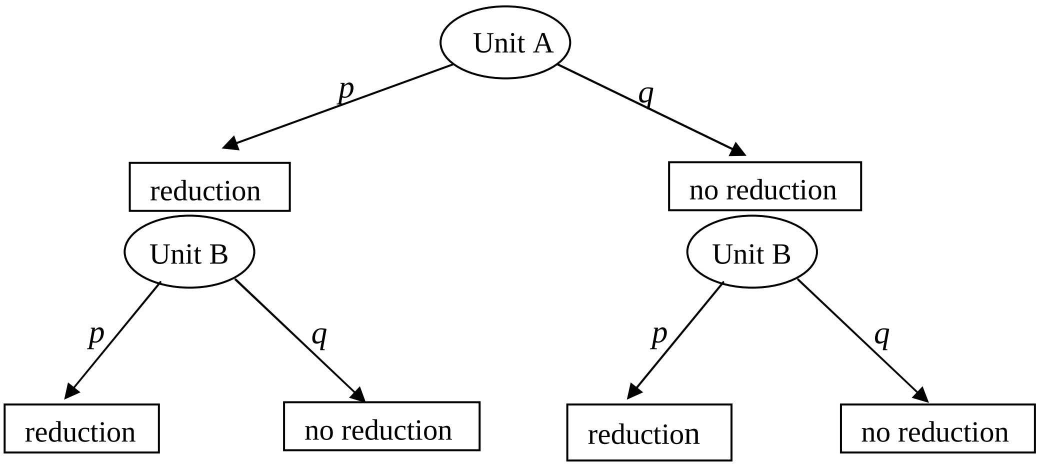

Fig. 2. An event tree of speed changes during crossing the intersection
Rys. 2. Drzewo zdarzeń zmian prędkości podczas przechodzenia przez skrzyżowanie

3.1. Case 1

Let X denotes the waiting time for the reporting of the successive subordinated vessel before the intersection, Y denotes the time needed to reduce speed, W denotes the time needed to cover the distance P_2P_3 with safe speed, T denotes the time needed to cover the distance P_2P_3 with full speed, Z denotes the time needed to cover the distance P_1P_3 with full speed. If a fairway unit A arrives at the point P_1 at the time t_0, reduces of the speed during the time Y_A and covers the intersection in the time W_A than she leaves the intersection at the time $t_0 + Y_A + W_A$. A next fairway unit (B) leaves the intersection at the time $t_0 + X + Y_B + W_B$. Therefore the time between leavings the intersection by ships is equal to $R = X + (Y_B - Y_A) + (W_B - W_A)$ where X is an exponential random variable, Y_B and Y_A are normal random variables $N(m_Y; \sigma_Y)$, W_B and W_A are normal random variables $N(m_W; \sigma_W)$ [8]. Using the convolution operation [8, 9, 12], the formula of density function $t_1(u)$ of random variable R was obtained:

$$t_1(u) = \frac{\lambda}{2}\exp\left(\lambda^2\sigma_Y^2 + \lambda^2\sigma_W^2 - \lambda u\right)\cdot\left(1 + \operatorname{erf}\left(\frac{u - 2\lambda^2\sigma_Y^2 - 2\lambda^2\sigma_W^2}{\sqrt{2\sigma_Y^2 + 2\sigma_W^2}}\right)\right) \tag{1}$$

where: erf(x) – the error function used in *Mathematica* program in probabilistic problems [2], λ – parameter of exponential distribution, σ_Y – standard deviation of the time necessary to reduce the speed, σ_W – standard deviation of the time necessary to cover the distance P_2P_3 with safe speed.

3.2. Case 2

When first unit brakes and the second doesn't brake we have the following situation: unit A arrives at the point P_1 at the time t_0, and leaves the intersection at the time $t_0 + Y_A + W_A$; next fairway unit arrives at the point P_1 at the time $t_0 + X$ and leaves the intersection at the time $t_0 + X + Z_B$. Z_B is time at which unit B covers the distance P_1P_3. Therefore the time between leavings the intersection by ships is equal to $R = (X - Y_A) + (Z_B - W_A)$. The formula of density function $t_2(u)$ of random variable R is the following:

$$\begin{aligned} t_2(u) = \frac{\lambda}{2}\exp\left(\frac{\lambda^2\sigma_Y^2 + \lambda^2\sigma_W^2 + \lambda^2\sigma_Z^2}{2} + \lambda m_Z - \lambda m_Y - \lambda m_W - \lambda u\right)\cdot \\ \cdot\left(1 - \operatorname{erf}\left(\frac{\lambda^2\sigma_Y^2 + \lambda^2\sigma_W^2 + \lambda^2\sigma_Z^2 + m_Z - m_Y - m_W - u}{\sqrt{2\sigma_Y^2 + 2\sigma_W^2 + 2\lambda^2\sigma_Z^2}}\right)\right) \end{aligned} \tag{2}$$

where: σ_Z – standard deviation of the time necessary to cover the distance P_1P_3 with full speed, λ – parameter of exponential distribution, m_Y – mean time necessary to reduce the speed, m_W – mean time necessary to cover the distance P_2P_3 with the safe speed, m_Z – mean time to cover the distance P_1P_3 with full speed.

3.3. Case 3

Let a fairway unit A doesn't brake and leaves the intersection at the time t_0. A next fairway unit (B) leaves the intersection at the time $t_0 + X + Y_B + T_B - W_B$. Hence the time between leavings of ships is a sum of three random variables $R = X + Y_B + (T_B - W_B)$, where X is an exponential random variable and all others are normal random variables with different parameters. Using the convolution operation, the formula of density function $t_3(u)$ of random variable R was obtained:

$$t_3(u)=\frac{\lambda}{2}\exp\left(\frac{\lambda^2\sigma_Y^2+\lambda^2\sigma_W^2+\lambda^2\sigma_T^2}{2}+\lambda m_T+\lambda m_Y-\lambda m_W-\lambda u\right)\cdot$$
$$\cdot\left(1-\mathrm{erf}\left(\frac{\lambda^2\sigma_Y^2+\lambda^2\sigma_W^2+\lambda^2\sigma_T^2+m_T+m_Y-m_W-u}{\sqrt{2\sigma_Y^2+2\sigma_W^2+2\lambda^2\sigma_Z^2}}\right)\right) \qquad (3)$$

where: σ_T – standard deviation of the time necessary to cover the distance P_2P_3 with full speed, m_T – mean time necessary to cover the distance P_2P_3 with full speed.

3.4. Case 4

When both units don't brake we deal with Poisson process [1, 3, 7, 10]. Therefore the time between leavings the intersection by ships is an exponential random variable X [4, 11].

4. RESUME

Using convolutions different random variables, allows taking into consideration different factors which disturb the traffic stream of subordinated units. It allows connecting the concept of control area of entry into the intersection [6] with the traffic intensity of subordinated vessels leaving the area of the intersection. Very important problem is a determination of probability of an event when the intersection is occupied by privilege units (*p* in fig.2). But this is a separate problem, which will be considered in my next work.

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