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## **VEHICLE WHEELS VIBRATION SUPPRESSION BY DYNAMIC VIBRATION ABSORBER**

**Summary.** The article deals with the methods of calculation and optimization of dynamic processes in vibroexcited constructions with dynamic absorbers. The improved constructions of such absorbers for vehicle wheels vibration suppression are discussed.

## **ZMNIEJSZENIE WIBRACJI KÓŁ POJAZDU POPRZEZ DYNAMICZNY ABSORBER WIBRACJI**

**Streszczenie.** Artykuł zajmuje się metodami obliczeń i optymalizacji procesów dynamicznych w konstrukcjach wibrujących z dynamicznymi absorberami. Omówiona została poprawiona konstrukcja takiego DVA dla zmniejszenia wibracji kół pojazdu.

### **1. INTRODUCTION**

To be able to use a moderate damping coefficient for the suspension dashpot and yet avoid its undesirable consequence of having an underdamped wheel mode which results in wheel hopping, the use of a dynamic vibration absorbers (DVA) has been suggested. The most effective way to solve the problem of vibration decreasing is to apply optimally designed dynamic absorber or a set of such absorbers. It is desirable to eliminate unwanted vibration in many applications. One of the most visible applications is transportation. In automobiles, aircraft, and watercraft vibration can cause irritation and even motion sickness to the occupants, and can also cause accelerated wear and mechanical fatigue to the vehicle hardware. Other applications include manufacturing equipment, where imbalances of rotating machinery can cause eccentric rotation, which can degrade surface finishing and machining tolerances. As the speeds of electronic information storage devices, such as compact discs, DVDs, and hard drives increase, the effect of imbalance on the rotating discs becomes increasingly important. Still it is possible to give another dozen of examples of machines and constructions where DVA application is expedient.

The main aim of the paper is the improved DVA design in taking into account complex rotating machines dynamic. It is often impossible to balance the rotating elements to reduce the vibration to an acceptable level. The paper contemplates the provision of DVA or number of such DVA. Such originally designed DVA reduces vibration selectively in maximum mode of vibration without introducing vibration in other modes. By installing DVA, one can minimize excitation virtually at the source. In order to be more effective, a vibration absorption system should react in all frequency domains. The present absorber also has as an advantage that it can be constructed such that it has a

wide-range vibration absorption property. This construction allows for the easy connection above the car wheel equipment.

## 2. METHODOLOGY OF MACHINE AND ABSORBER MODELING

In order for optimal parameters of DVA to be determined the complete modeling of dynamics of rotating machine is obvious. The two degrees of freedom model is totally inadequate to calculate the vibration frequencies of the construction with accuracy and therefore, for a sufficiently accurate determination of its dimensional characteristics so as to determine such frequencies. It is therefore necessary in practice to construct the dimension through more complex modeling. In particular, concentrated mass and rigidity calculation methods may be adopted based on an even more accurate theoretical determination.

### 2.1. MACHINE MODELING

The numerical schemes (NS) row is considered for the complex vibro-exitated constructions. Methods of decomposition and the NS synthesis are considered on the basis of new methods of modal synthesis. Complex NS are led of discretely-continuum type that enables in the adaptive mode to calculate tension not only in the continue elements, but in places of most tension concentration in joints. The absorbers in accordance with this project may be applied not only to electric machines or aeronautic structures, but also to any other type of vibro-exitated structure, such as cars, chisel installation, optical, magneto-optical disks, washing machines, refrigerators, vacuum cleaners, etc. Rotating machinery will typically introduce both acoustic and vibration energy into any fluids or structures surrounding the machinery. Both random and deterministic processes related to the operation of the machinery can cause acoustic and vibration energy. Random processes result in noise or vibration that is spread over a wide band of frequencies. Deterministic processes, on the other hand, often generate energy that is confined to a family of distinct frequencies radiated as "pure" tones.

Large rotating elements, particularly such elements as exhaust fan rotors used in electric power generating plants or in gas compression, are unbalanced in operation due to their exposure to varying factors. It is often impossible to balance the rotating elements to reduce the vibration to an acceptable level. The two degrees of freedom model described in [1-3] is totally inadequate to calculate the natural frequencies of vibration of the rotor with accuracy and therefore, for a sufficiently accurate determination of its dimensional characteristics so as to determine such frequencies. It is therefore necessary in practice to dimension the rotor through more complex modeling [4-6]. In particular, concentrated mass and rigidity calculation methods may be adopted based on an even more accurate theoretical determination. The numerical schemes (NS) row is considered for such complex vibro-exitated constructions. Methods of decomposition and the NS synthesis are considered on the basis of new methods of modal synthesis [7-10]. Complex NS are provided of discretely-continual type that enables in the adaptive mode to calculate tension not only in the stratified elements, but in places where their concentration is the most – in joints. It is considered also numerical schemes for research of halving elements that also are on the basis of kinematics hypotheses. On the basis of simple and more complex NS research of local tensions on the verge of stratified structure at the different kinds of its fixing is conducted. Traditional design methodology, based on discontinuous models of structures and machines is not effective for high frequency vibration. Present research develops a modern prediction and control methodology, based on complex continuum theory and application of special frequency characteristics of structures. Complex continuum theory allows to take into consideration system anisotropy, supporting structure strain effect on equipment motions and to determine some new effects that are not described by ordinary mechanics of the continuum theory.

In Fig.1 a discretely typical scheme of DVA is presented [1]. In Fig.2 the improved scheme of DVA is presented.

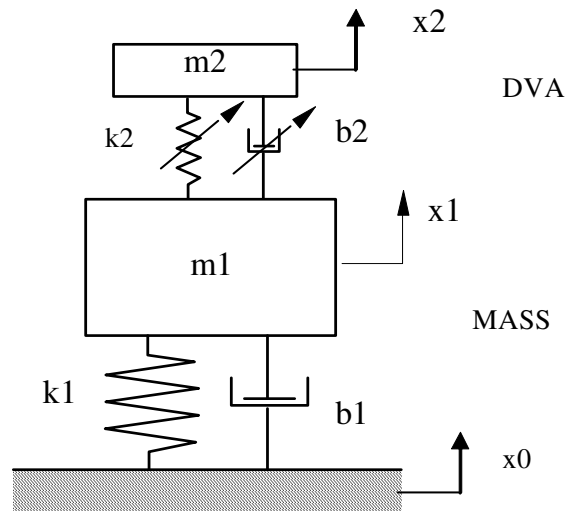


Fig. 1. Classical DVA scheme  
Rys. 1. Klasyczny schemat DVA

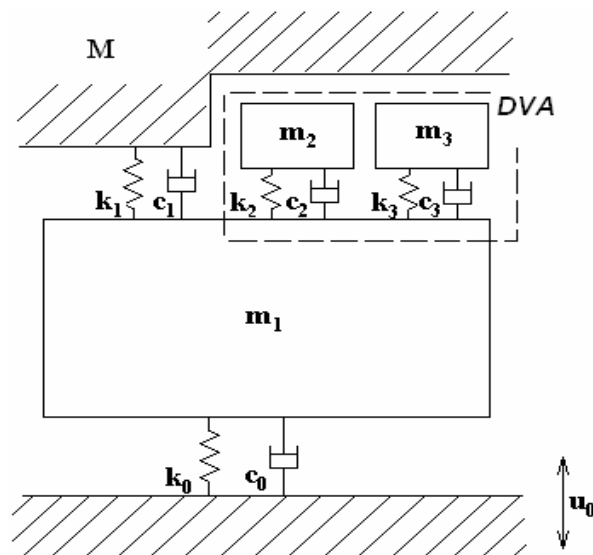


Fig. 2. Improved DVA scheme  
Rys. 2. Ulepszony schemat DVA

Governing equations for these both cases are:

a)

$$m_1 \ddot{x}_1 + (\dot{x}_1 - \dot{x}_2)b_2 + (\dot{x}_1 - \dot{x}_0)b_1 + (x_1 - x_0)k_1 + (x_1 - x_2)k_2 + B1 \cdot \dot{x}_1 = 0 \quad (1)$$

$$m_2 \ddot{x}_2 + (\dot{x}_2 - \dot{x}_1)b_2 + (x_2 - x_1)k_2 - B1 \cdot \dot{x}_1 = 0$$

b)

$$m_1 \ddot{x}_1 + (\dot{x}_1 - \dot{x}_0)c_0 + (x_1 - x_0)k_0 + (x_1 - x_0)k_0 + (\dot{x}_1)c_1 + (\dot{x}_1 - \dot{x}_2)c_2 + (x_1 - x_2)k_2 + (\dot{x}_1 - \dot{x}_3)c_3 + (x_1 - x_3)k_3 = 0$$

$$m_2 \ddot{x}_2 - (\dot{x}_1 - \dot{x}_2)c_2 - (x_1 - x_2)k_2 = 0 \quad (2)$$

$$m_2 \ddot{x}_3 - (\dot{x}_1 - \dot{x}_3)c_3 - (x_1 - x_3)k_3 = 0$$

## 2.2. WHEEL SHAFT MODELING

Let's consider dynamics of the rotating wheel shaft taking into account flexibility of the shaft both on curving and on torsion. We shall proceed from a variation principle of Hamilton. We shall accept kinematics hypotheses for deviations of an axis from a certain statically counterbalanced position of the shaft on bearings. We suppose that the center of masses of the given cross-section of the shaft coincides with its center of weight in the immovable frame rigidly connected to statically equilibrium position of the shaft on elastic support. Then the kinetic energy will consist of two items.

$$T = T_w + T_{rot} \quad (3)$$

$T_w$  – bending component,  $T_{rot}$  – rotation one. The first item is equal to

$$2T_w = \int_0^L \rho V_C^2 dz = \int_0^L \rho (V_r^2 + V_\varphi^2) dz \quad (4)$$

( $\rho(z)$  - a linear mass of a shaft,  $V_C^2$  – quadrate of tangential velocity of shaft equal the sum of quadrates of radial  $V_r$  and tangential  $V_\varphi$  velocities of the center of the given cross-section of the shaft, multiplied on its mass of relative movement will be equal to:

$$2T_{rot} = \int_0^L \rho R^2 \gamma_\varphi^2 dz \quad (5)$$

Here  $R(x)$  - radius of the given part of the shaft,  $\gamma_\varphi$  - axial angular velocity. Here gyroscopic composites [1-3] are neglected.

Let's accept kinematics hypotheses for deformation of the shaft. We assume, that in an immovable frame  $(z, r, \varphi)$ , the shaft realizes elastic rotating oscillations with a changeable angle of twisting:

$$\varphi = q_\varphi(t) \lambda_\varphi(z) \quad (6)$$

and radial oscillations in a mobile frame  $(z, r, \varphi)$ , rigidly connected to shaft in orthogonal planes [1]:

$$W_1 = \sum_i q_i^1(t) \lambda_i(z), \quad W_2 = \sum_i q_i^2(t) \lambda_i(z) \quad (7)$$

Here  $q_i^j(t)$  – are time dependent unknown functions and multipliers by its  $\lambda_i(z)$  – known coordinate functions.

Let's note kinetic and potential energy. For the last we assume resistance of the shaft to be linearly elastic. According to (3) - (7) we obtain [7-10]:

$$(M^c \ddot{q}^c + \bar{K}^c \cdot q^c) \cdot \delta q^c + \sum (M_i q_i^n) \delta q_i^n = 0 \quad (8)$$

Here  $M^c$  – the mass matrix of continua elements,  $M_i$  – the discrete elements masses,  $\delta q_i^n$  – independent time functions variations. By equating the terms by these variations we obtain the system of ordinary differential equations [7-10].

### 3. DYNAMIC ABSORBER MODELING

Some example of beam-like DVA with prescribed damping properties are presented in Fig. 3.

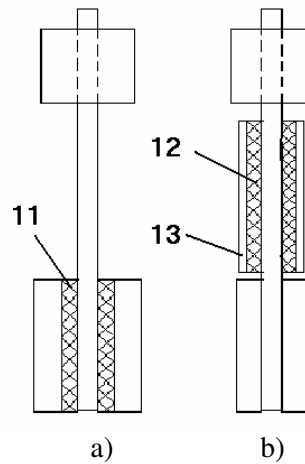


Fig. 3. DVA with: a) damping layer in the joint (11); b) attached constrained (13) damping layer (12)  
 Rys. 3. DVA z a) warstwą tłumiącą w złączu (11); b) zamocowana przez utwierdzenie (13) warstwa tłumiąca (12)

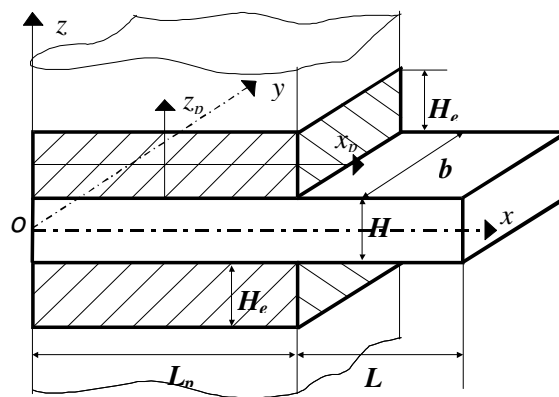


Fig. 4. Elastic joint of beam to base  
 Rys. 4. Elastyczne połączenie belki do złącza

Consider plain bending of plate in elastic joint (Fig. 4). The detail of elastic and damping property evaluation may be found in [11-13]. Material of plate is assumed more rigid than that of interlayer (Fig. 4). Material of plate is anisotropic, with elasticity modules  $C_{xx}$ ,  $C_{xz}$ ,  $C_{zz}$ ,  $G_{xz}$ . Material of interlayer is isotropic and incompressible (rubber-like material). Such joint is widely used in industry. Equilibrium equations are derived on the basis of such kinematical hypothesis [13]

$$u = u_{ij} \cdot x^i \cdot z^{j-1}, \quad w = w_{ij} \cdot x^i \cdot z^{j-1} \quad (9)$$

or

$$u = u \cdot x^{i-1} \cdot z^{j-1}, \quad w = w \cdot x^{i-1} \cdot z^{j-1} \quad (10)$$

for unclamped left corner of plate. Put (9) or (10) to the (11)

$$\int_{V_p} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{zz} \delta \varepsilon_{zz} + \tau_{xz} \delta \varepsilon_{xz}) dV + \int_0^L (K^+(x) w^+ + K^-(x) w^- \delta w^-) dx +$$

$$+ \int_0^L (K_u^+(x) u^+ \delta u^+ + K_u^-(x) u^- \delta u^-) dx - \int_{-H_p}^{H_p} (N(z) \delta u + T(z) \delta w) \cdot dz = 0. \quad (11)$$

Here  $K^+$ ,  $K^+$  are respectively normal and tangential elastic coefficient of elastic interlayer us Winkler foundation one. There may be found by the same method [13].

#### 4. DESCRIPTION OF ACHIEVED RESULTS

On the Fig.4 some numerical results are presented of rigidity of plate elastically clamped.

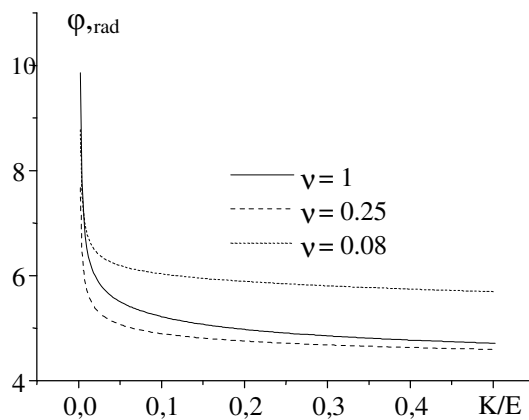


Fig. 5. Value of the angle of plate bending for various coefficient of anisotropy  $\nu = C_{zz}/C_{xx}$   
Rys. 5. Wartość kąta ugięcia płyty wyginanej przy różnym współczynniku anizotropii  $\nu = C_{zz}/C_{xx}$

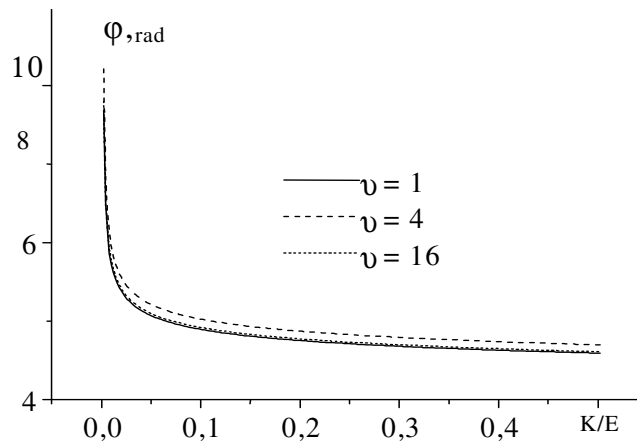


Fig. 6. Angle of plate bending for various values of  $C_{xx2}/C_{xx1}$  for outer and inner plate layer ( $H1/H = 0.2$ )  
Rys. 6. Kąt płyty giętej przy różnej wartości  $C_{xx2}/C_{xx1}$  dla zewnętrznej i wewnętrznej warstwy płyty ( $H1/H = 0.2$ )

In Fig. 7 is presented the result of rotating machine optimization in frequency domain with DVA. The evolutionary minimization process was used [14]. For these purposes the linearized system of ordinary differential equations (8) was provided. The modified one mass model for rotor and case was considered [10]. The DVA's were attached to the wheel case.

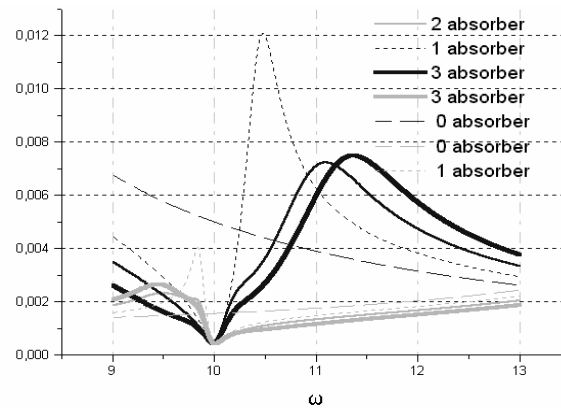


Fig. 7. Dynamic response with and without absorbers  
Rys. 7. Odpowiedz dynamiczna z i bez tłumików drgań

Here  $M_a$  is the mass of absorber and  $M_o$  – mass of the case. The frequency response lines  $M_a=0$  and  $M_a=0.115M_o$  presents the case of vibration: levels A without and with DVA with optimal mass. The result may be much improved by changing the special form of the DVA [10]. Forces obtained on the basis of the given calculation schemes may be used further to determine deformed conditions of cases, bases and to define vibrations levels for elements of the rotating machine. Algorithms adduced in [7-10], and known programs – such as ANSYS, NASTRAN, COSMOS may be used by calculation of bearings and shaft stresses at the given above loading.

## 5. CONCLUSION

In order that optimal parameters of DVA are determined the complete modeling of dynamics of rotating machine should be made. Traditional design methodology, based on discontinuous models of structures and machines are not effective for high frequency vibration. They do not give the possibility to determine strains and to predict durability. Present research develops a modern prediction methodology, based on complex discrete-continuum theory. This allows to take into consideration system anisotropy, supporting structure strain effect on equipment motions and to determine some new effects that are not described by ordinary mechanics, namely concentration of strain in junctions and chaotic oscillations.

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